Effective Field Theory for Dissipative Hydrodynamics

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Abstract
In our work, we present a Lagrangian Field Theory compatible with hydrodynamics. We start with a coarse-grained system specifying the fundamental symmetries and establishing the “dictionary” with its thermodynamics in the language of the effective field theory. As the lagrangian formalism restricts the work to non-dissipative systems, the contact to realistic fluids is severely limited. As an alternative, it is shown that doubling the sets of coordinates it is possible to develop a self-consistent lagrangian description for non-conservative systems. In this new context, we rewrite the equations for the conserved quantities using the Closed-Time-Path (CTP) scheme, in particular the energy-momentum tensor, which is approximately conserved near equilibrium. In this way we obtain a consistent description of dissipative fluids via effective field theory.

Key words:
Field Theory, Dissipative Hydrodynamics, Fluid Mechanics.

Introduction
Our main goal in this work is to establish a consistent description of dissipative fluids using the effective field theory language. Initially, we focused on describing ideal hydrodynamics using a lagrangian field theory1, which is compatible with the system’s symmetries and its thermodynamics. Using the technique developed by Galley2, which describes the mechanics of non-conservative systems via lagrangian formalism, we extend the conservative effective field theory description to dissipative fluids3,4.

Results and Discussion
We begin by extending the main aspects of classical fluid mechanics to a relativistic field theory background, also rewriting the equations that dictate the dynamics of classical systems, e.g., Euler’s equation. The establishment of a general lagrangian and a “dictionary” between field theory and thermodynamics allowed us to study the fluid near equilibrium. The effects of including higher order derivatives around equilibrium are neglected by appropriate field redefinitions. This new lagrangian $F$ for a perfect fluid carrying a conserved charge can be seen as a non-usual thermodynamic potential depending on the entropy density $s$ and the chemical potential $\mu$, as follows:

$$ F(s, \mu) = n(s, \mu) \mu - \varepsilon(s, \mu) $$

where $n(s,\mu)$ is the particle number density and $\varepsilon(s,\mu)$ is the energy density. Furthermore, the energy-momentum tensor $T_{\mu\nu}$ for a perfect charged fluid with velocity field $u_{\mu}$ is found to be:

$$ T_{\mu\nu} = (F_{\mu} - F_s s) u_{\mu} u_{\nu} + (F - F_s s) \eta_{\mu\nu} $$

Despite the fact that the results are consistent with the description of perfect hydrodynamics, they are not suitable to model dissipative effects. In order to introduce the latter in our theory, we explored the classical mechanisms of non-conservative systems2. It is possible to rewrite the action functional of a physical system doubling the set of coordinates in a way that one of them travels forward in time and the other backward in time. The action, then, has independent contributions coming from each new set and an interaction term, which is responsible for the coupling of the variables.

Furthermore, we rewrite the Euler-Lagrange equations in terms of the coupling part of the action, which could be regarded as a “non-conservative potential”. We then revisit the Noether theorem, leading to new corrections to the energy-momentum tensor, and the Noether currents representing the interaction of open systems with eliminated or inaccessible degrees of freedom.

With this formalism, known as Closed-Time-Path (CTP) scheme, new lagrangians can be written in the first two orders in the gradient expansion of the fluid’s conserved current5,6:

$$ L_{CTP}^{(0)} = F(K^i K^{3i}) - F(K_0 K^{0i}) + G(K^i K^{ij}) $$

$$ L_{CTP}^{(1)} = \sum_{l,j,k} f_{ijl} (K^i K^{lj}) K^{mj} \partial_\rho K^{lj} $$

where the superscript indicates the order in the gradient expansion, $K^{ij}$ is a conserved current commoving to the velocity field of the fluid. $G$ is the dissipative lagrangian.

Their variation with respect to the doubled variables returns the energy-momentum balance equation, which contains a 2-tensor that is not conserved due to the interaction between the fluid and the environment. It is shown, however, that this 2-tensor is conserved near equilibrium and is identified as the energy-momentum tensor of the fluid. Going back to the energy-momentum balance equation, we recall the continuity equation and the Navier-Stokes equation.

Conclusions
In this work, we rely on previous works1,2,3,4 to develop a self-consistent description to dissipative hydrodynamics using the already known equations of motion (continuity, Euler and Navier-Stokes) and introducing the CTP field theory formalism. This treatment to dissipation can be extended to quantum field backgrounds and is consistent with phenomenological relativistic hydrodynamics.

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1Dubovsky, Sergei; Hui, Lam; Nicolis, Alberto; Son, Dam Thanh; PhysRevD.85.085029 2012.
3Grozdanov, Saio; Polonyi, Janos; PhysRevD.91.105031 2015.
4Torrieri, Giorgio; Montenegro, David; arXiv:1604.05291[hep-th]

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