Numerical Investigation of thin liquid films over a granular bed: Kapitza waves, ripples and dunes formation

João L. S. Almeida

Abstract
A mathematical model of a gravitational flow of a liquid film above a granular bed based on the Shallow Water Equations together with Exner [2] and Meyer-Peter and Muller [4] equation was studied and computationally implemented in a FORTRAN code. The PDE system was solved with the MacCormack finite difference method and the results were compared with the field observations.

Key words: liquid film, roll-waves, numerical investigation

Introduction
When the bed on which a perturbed liquid flows is composed by granular elements, concurrently with the waves propagation phenomenon (roll waves) occurs the erosion and deposition of sediments that can be modelled by means a coupled partial differential equations system.

Results and Discussion
The shallow waters equations in its conservative form are as follows [3]:

\[
\frac{\partial h\overrightarrow{u}}{\partial t} + \frac{\partial (h\overrightarrow{u}^2)}{\partial x} + gh \cos \theta \frac{1}{2} \frac{\partial h^2}{\partial x} = -C_f \frac{(\overrightarrow{u})^2}{2} + gh \sin \theta \tag{1}
\]

\[
\frac{\partial h}{\partial t} + \frac{\partial h\overrightarrow{u}}{\partial x} = 0 \tag{2}
\]

Image 1. Mass flow \((h\overrightarrow{u})\) profile, with \(t = 15\) s.

To complete the system equations of the problem is necessary to add the Exner equation:

\[
\frac{\partial \delta}{\partial t} + \frac{1}{p - 1} \frac{\partial q_x}{\partial x} = 0 \tag{3}
\]

Where \(\delta\) represents the bed curvature. The grain discharge along the horizontal axis \(x\) is obtained by correlation 4:

\[
q_x = A\overrightarrow{u}^2 \tag{4}
\]

\[
\frac{\partial q_x}{\partial x} = \frac{q_{sat} - q_x}{L_{sat}} \tag{5}
\]

It is a semi-empirical relation, where \(q_{sat}\) is the discharge concerning the stationary erosion/deposition state, determined with the Meyer-Peter and Muller correlation

Conclusions
The waves profile as seen in the image 1 is similar to the results shown in [1] and [3], what indicates a reasonable consistency between this result and the literature. The erosion and deposition, at first sight, seem to respect the physics in a point; the sediments are accumulating in the regions with low velocities, and occurs an eroding process in the higher flow intensity areas.

Acknowledgements
I am deeply grateful to the professors Ph.D Erick de Mores Franklin and Ph.D William Roberto Wolf by attention and support.


DOI: 10.19146/pibic-2016-51707