Numerical Investigation of thin liquid films over a granular bed: Kapitza waves, ripples and dunes formation

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Abstract

A mathematical model of a gravitational flow of a liquid film above a granular bed based on the Shallow Water Equations together with Exner [2] and Meyer-Peter and Muller [4] equation was studied and computationally implemented in a FORTRAN code. The PDE system was solved with the MacCormack finite difference method and the results were compared with the field observations.

Key words: liquid film, roll-waves, numerical investigation

Introduction

When the bed on which a perturbed liquid flows is composed by granular elements, concurrently with the waves propagation phenomenon (roll waves) occurs the erosion and deposition of sediments that can be modelled by means a coupled partial differential equations system.

Results and Discussion

The shallow waters equations in its conservative form are as follows [3]:

$$\frac{\partial h\bar{u}}{\partial t} + \frac{\partial (h\bar{u}^2)}{\partial x} + g \cos \theta \frac{1}{2} \frac{\partial h^2}{\partial x} = -C_f \left( \frac{\bar{u}^2}{2} \right) + g h \sin \theta$$  \hspace{1cm} (1)

$$\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} = 0$$  \hspace{1cm} (2)

Image 1. Mass flow ($h\bar{u}$) profile, with $t = 15$ s.

To complete the system equations of the problem is necessary to add the Exner equation:

$$\frac{\partial \delta}{\partial t} + \frac{1}{p - 1} \frac{\partial q_x}{\partial x} = 0$$  \hspace{1cm} (3)

Where $\delta$ represents the bed curvature.

The grain discharge along the horizontal axis $x$ is obtained by correlation 4:

$$q_x = A\bar{u}^2$$  \hspace{1cm} (4)

$$\frac{\partial q_x}{\partial x} = \frac{q_{sat} - q_x}{L_{sat}}$$  \hspace{1cm} (5)

It is a semi-empirical relation, where $q_{sat}$ is the discharge concerning the stationary erosion/deposition state, determined with the Meyer-Peter and Muller correlation

$$q_{sat} = k \left( \frac{\bar{u}}{L} \right)^n$$

where $k$, $n$, and $L$ are characteristic parameters of the phenomenon [4], and $L_{sat}$ is a characteristic parameter of the phenomenon [4].

Image 2. Bed curvature ($\delta$) profile, with $t = 15$ s.

A and $c$, are appropriated physical parameters choose according to the problem characteristics, and $p$ are the porosity of the granular bed [2]. Performing the simulation of the system shown above, are obtained the results presented the figures 1 and 2.

Conclusions

The waves profile as seen in the image 1 is similar to the results shown in [1] and [3], what indicates a reasonable consistency between this result and the literature. The erosion and deposition, at first sight, seem to respect the physics in a point; the sediments are accumulating in the regions with low velocities, and occurs an eroding process in the higher flow intensity areas.

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