Numerical Investigation of thin liquid films over a granular bed: Kapitza waves, ripples and dunes formation

João L. S. Almeida

Abstract

A mathematical model of a gravitational flow of a liquid film above a granular bed based on the Shallow Water Equations together with Exner [2] and Meyer-Peter and Muller [4] equation was studied and computationally implemented in a FORTRAN code. The PDE system was solved with the MacCormack finite difference method and the results were compared with the field observations.

Key words: liquid film, roll-waves, numerical investigation

Introduction

When the bed on which a perturbed liquid flows is composed by granular elements, concurrently with the waves propagation phenomenon (roll waves) occurs the erosion and deposition of sediments that can be modelled by means a coupled partial differential equations system.

Results and Discussion

The shallow waters equations in its conservative form are as follows [3]:

$$\frac{\partial h\bar{u}}{\partial t} + \frac{\partial (h\bar{u}^2)}{\partial x} + g \cos \theta \frac{1}{2} \frac{\partial h^2}{\partial x} = -C_f \frac{(\bar{u})^2}{2} + gh \sin \theta \quad (1)$$

$$\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} = 0 \quad (2)$$

Image 1. Mass flow ($h\bar{u}$) profile, with $t = 15$ s.

To complete the system equations of the problem is necessary to add the Exner equation:

$$\frac{\partial \delta}{\partial t} + \frac{1}{p-1} \frac{\partial q_x}{\partial x} = 0 \quad (3)$$

Where δ represents the bed curvature.

The grain discharge along the horizontal axis x is obtained by correlation 4:

$$q_x = A\bar{u}^c \quad (4)$$

$$\frac{\partial q_x}{\partial x} = \frac{q_{sat} - q_x}{L_{sat}} \quad (5)$$

It is a semi-empirical relation, where q_{sat} is the discharge concerning the stationary erosion/deposition state, determined with the Meyer-Peter and Muller correlation

DOI: 10.19146/pibic-2016-51707

[4], and L_{sat} is a characteristic parameter of the phenomenon [4].

Image 2. Bed curvature (δ) profile, with $t = 15$ s.

A and c, are appropriated physical parameters choose according to the problem characteristics, and p are the porosity of the granular bed [2].

Performing the simulation of the system shown above, are obtained the results presented the figures 1 and 2.

Conclusions

The waves profile as seen in the image 1 is similar to the results shown in [1] and [3], what indicates a reasonable consistency between this result and the literature. The erosion and deposition, at first sight, seem to respect the physics in a point; the sediments are accumulating in the regions with low velocities, and occurs an eroding process in the higher flow intensity areas.

Acknowledgements

I am deeply grateful to the professors Ph.D Erick de Mores Franklin and Ph.D William Roberto Wolf by attention and support.

¹ A. N. Edwards and J. M. N. T. Gray, Erosion-deposition waves in shallow granular free-surface flows, School of Mathematics and Manchester Centre for Nonlinear Dynamics, University of Manchester, Manchester, UK, **2014**.

² K. Rehman, J.-H. Lee, H.-R. Cho & Y.-S. Cho, A Coupled Sediment Transport Model for Flow over Movable Bed, *Hanyang University, Department of Civil and Environmental Engineering, Seoul, Korea*. **2014**.

³ Brock, R. R., "Development of Roll Waves in Open Channel", Tese de Doutorado, California Institute of Technology, (1967).

⁴ Franklin, E. M., Numerical Prediction of Bed-Load and Surface Deformation on a Granular Bed Sheared by a Turbulent Boundary-Layer, *J. of the Braz. Soc. of Mech. Sci. & Eng., Itajubá*, v. 34, pp. 1-8 (2011).