Robust Model Predictive Control for Trajectory Tracking by an Unmanned Ground Vehicle - An LMI Approach.

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Abstract

Autonomous vehicles are becoming closer to people's reality. Navigation is a great concern for this robotic system. In this paper, we present a control technique for trajectory tracking by the autonomous vehicle. We designed an augmented vehicle model composed by the bicycle model dynamics and a simplified steering model. For maintaining the vehicle in the desired trajectory, a Robust Model Predictive Control (RMPC) technique was implemented using Linear Matrix Inequalities (LMIs) to control the lateral dynamics of the vehicle. Simulation tests were realized using the Double Lane Change maneuver.

Key words:

Unmanned Ground Vehicle, Robust Model Predictive Control, Trajectory Tracking.

Introduction

The autonomous vehicles, or unmanned ground vehicles, are already a reality in the streets. An automatic driven system is considered the solution for many problems in the cities, as traffic and pollution. In this scenario, the present paper aims to develop a control technique to maintain the vehicle tracking a desired trajectory with minimum distance error. To accomplish this, and regarding the existence of uncertainties between the model and the real vehicle, a Robust Model Predictive Control was implemented and simulated in the *Matlab*. The assumptions, development and results of this technique can be seen in the next section.

Results and Discussion

The concept of the Model Predictive Control is to, using the system's model, make predictions of its future outputs and compare with its desirable states to, via an optimization problem, calculate the optimal input to be applied in the current time. Therefore, a good model of the system is required. With that in mind, we propose an augmented vehicle system, combining the bicycle dynamics with a simplified steering model. For trajectory tracking, usually, the vehicle dynamics is represented by the errors between the vehicle center of gravity (CG) and the trajectory, as shown in the figure below:

Fig 1: Bicycle Dynamics with respect to the trajectory (left) and simplified steering (right).

Using small angles approximation, considering a constant longitudinal velocity and a flat plane, the state space for the system model is derived and shown below:

$$\dot{x}_c = [x_c : \dot{x}_c]^T = [\theta_f \delta_f \delta_r e_{cg} \theta_{cg} \theta_{dr}]^T \dot{x}_c = [V_{cm} \delta_{cm} \theta_{dr} \theta_{dr}]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{C_f + C_r}{mV_{cm}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Taking into account mass uncertainties in a real model, a Robust Model Predictive Control technique is presented. For the cost function, we used other augmented system, consisted of the vehicle system and the trajectory dynamics.

$$J_\infty[k] = \sum_{i=0}^{\infty} \{ \|C_r x[k+i] - C_r x_r[k+i]\|_{Q_1}^2 + \|u[k]\|_R^2 \}$$

$$J_\infty[k] = \sum_{i=0}^{\infty} \{ \|z[k+i]\|_{Q_1}^2 + \|u[k]\|_R^2 \}$$

We considered a polytope with two vertices to represent our uncertain system. Also, we considered an upper bound for the cost function, which derived the LMIs conditions, for an infinite prediction horizon.

$$\begin{bmatrix} Q & \tilde{x}[k|k]^T \\ \tilde{x}[k|k] & 1 \end{bmatrix} \geq 0 \quad \begin{bmatrix} Q & 0 & 0 & (\tilde{A}Q + \tilde{B}Y) \\ * & \gamma I_R & 0 & R^{1/2}Y \\ * & * & \gamma I_{Q_1} & Q_1^{1/2}\tilde{C}Q \\ * & * & * & Q \end{bmatrix} \geq 0$$

The result obtained by the controller when executing a double lane change maneuver can be seen below:

Fig 2: Desired and Realized trajectory by the vehicle.

Conclusions

The paper presented a technique based on LMIs for Robust Model Predictive Control of an autonomous vehicle. The technique presented a good behavior in simulations for different values of mass. To increase the complexity and to become more reliable, future studies should add input saturation.

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