

Multicriteria decision making applied to allocating resources for public services in cities

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ABSTRACT

This work reflects results of research into the use of the modification of the Bellman-Zadeh approach to decision making in a fuzzy environment for analyzing multiobjective problems. Its application agrees with the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions. This is achieved by solving the corresponding *maxmin* problems on the basis of applying an algorithm of a non-local search. The paper results have served for developing an adaptive decision system (ADS). Its use can be directed at analyzing three types of models for multicriteria allocation of resources or their deficits. The paper results are illustrated by formulating and solving problems of multicriteria allocation of resources for public services in cities.

KEYWORDS. Multiobjective models. Allocation of resources. Public services in cities.

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1. Introduction

Diverse kinds of uncertainty are encountered in various problems related to planning and operation of complex systems. The consideration of the uncertainty factor in constructing mathematical models serves as a means for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis. The works [Pedrycz et al. 2011], [Ekel et al. 2020] show the benefits of applying fuzzy set theory [Zadeh 1965] to deal with diverse types of uncertainty. Its use in the problems of optimization character offers advantages of both a fundamental nature (the possibility of obtaining more effective, less "cautious" solutions) and of a computational character [Ekel 2002].

Goal uncertainty is an important type of uncertainty. It is related to the multicriteria character of many decision problems, which can be classified into two major classes.

The first class is associated with the problems in which solution consequences cannot be estimated based on a single criterion. These problems are directed at the analysis of models, including economic and physical factors (when alternatives cannot be reduced to comparable form). The second class is related to problems that, from the substantial point of view, may be solved based on a single criterion or several criteria. However, if the uncertainty of information does not permit one to obtain a convincing solution, it is possible to reduce the problem to multicriteria decision-making: the use of additional criteria can serve as a convincing means to contract the decision uncertainty regions [Ekel et al. 2020].

Therefore, two classes of models (so-called $\langle X, F \rangle$ models as multiobjective models [Hwang and Masud 1979] and $\langle X, R \rangle$ models as multiattribute models [Hwang and Yoon 1981]) may be constructed. Their analysis can be associated with modifying the Bellman-Zadeh approach to decision making in a fuzzy environment and applying techniques of fuzzy preference relations [Pedrycz et al. 2011], [Ekel et al. 2020], respectively. The main substantial subject of the present paper is the analysis of $\langle X, F \rangle$ models with their application to multicriteria allocating resources for public services in cities. The importance of this problem is associated with the following considerations.

The year of 2007 marked a watershed moment in human civilization when, for the first time in history, more than half of the world's population resided in urban areas. By 2023, the planet's urbanization rate had reached 57.25%, totaling over 4.61 billion people living in cities [Ritchie et al. 2024]. Much of this urbanization is occurring in developing regions, where a significant number of people are migrating from rural areas to cities. While this urbanization is also correlated with social, technological, and economic advancements in various nations, it has simultaneously generated a series of challenges and, in particular, such as social inequality [Nijman and Wei 2020], [Fernández et al. 2023].

Analyzing the phenomenon of urban inequality across various global cities, the authors of [van Ham et al. 2021] identified macro-trends which highlight the multidimensional nature of urban inequality, challenging traditional approaches that focus solely on economic inequality based on unidimensional economic indices, with income being the most commonly used metric [Nelson et al. 2023]. There is also a growing recognition that social inequalities are embedded within specific socio-spatial contexts, shaped by a range of geographic, social, economic, and political factors, which have varied impacts on different population groups [Franklin et al. 2022].

Specifically, the literature on inequalities in urban infrastructure access highlights the persistent and multi-dimensional nature of disparities in infrastructure provision and accessibility, and they argue that a universal access to infrastructure is a key development goal to the peripheral and lower-income areas. Finally, the authors of [Sarkar et al. 2024] review spatial inequalities in urban settings and argue that cities, as engines of economic growth, also function as mechanisms of social stratification, with infrastructure playing a crucial role in determining quality of life and opportunities for different social groups.

In view of the above, it can be seen that investments in infrastructure and public services in communities, including low-income communities, are, in addition to being necessary, powerful instruments, not only for improving the quality of life, but also for promoting the growth and social development of these populations. The paper results are useful for achieving the certain UN Sustainable Development Goals [Sustainable Development Goals 2015].

2. Analysis of $\langle X, F \rangle$ models

When analyzing multiobjective models, a vector of objective functions $F(X) = \{F_1(X), \dots, F_q(X)\}$ is considered, and the problem consists in simultaneous minimizing and/or minimizing all objective functions, i.e.,

$$F_p(x) \rightarrow \text{extr}, \quad p = 1, \dots, q \quad (1)$$

where L is a feasible region in $\mathbf{R}^n$.

As the authors of [Ekel et al. 2020] indicate, in solving multicriteria problems, we encounter uncertainty of goals, which is difficult to overcome and deal with because "we simply do not know what we want".

The first step in solving the problem (1) is associated with determining a set of Pareto solutions $\Omega \subseteq L$ [Ehrgott 2005]. This step is useful; however, does not permit one to obtain unique solutions. It is necessary to choose a particular Pareto solution on the basis of information of a decision maker (DM). There are three approaches to using this information [Pedrycz 2011]: *a priori*, *a posteriori*, and adaptive. The most preferable approach is the adaptive one. In its using, the procedure of successive improving the solution quality is realized as a transition from $X_\alpha^0 \in \Omega \subset L$ to $X_{\alpha+1}^0 \in \Omega \subset L$ with considering information I_α of DM (α is a number step of the decision process). The process serves for two adaptation types: computer to preferences of DM and DM to the problem. The first type of adaptation is based on information received from DM. The second type of adaptation is realized as a result of carrying out steps $X_\alpha^0, F(X_\alpha^0) \xrightarrow{I_\alpha} X_{\alpha+1}^0, F(X_{\alpha+1}^0)$, which permit DM to understand the correlation between own needs and possibilities of their satisfaction by the model (1).

Analyzing multicriteria problems, it is necessary to solve some questions related to normalizing criteria, selecting principles of optimality, and considering priorities of criteria. Their solution and, correspondently, development of multiobjective methods is carried out in several directions [Ehrgott 2015], [Ekel et al. 2020]. Without considering them, it is necessary to point out that an important question is the solution quality. It is high if levels of satisfying criteria are equal or close to each other (harmonious solutions), if we do not differentiate the importance of objective functions [Ekel 2002] (the conception of harmonious solutions can be extended for differentiated importance of objective functions [Ekel et al. 2020]). Other directions may lead to solutions with high levels of satisfying some criteria that is reached by low levels of satisfying other criteria). From this point of view, it should be recorded the validity and advisability of the direction related to the principle of guaranteed result.

The lack of clarity in the concept of "optimal solution" is the basic methodological complexity in analyzing multiobjective models. When applying the modification of the Bellman-Zadeh approach to decision making in a fuzzy environment [Pedrycz et al. 2011], [Ekel et al. 2020], this concept is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions. The modification of the Bellman-Zadeh approach [Bellman and 1970] permits one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multicriteria models [Pedrycz et al. 2011], [Ekel et al. 2020]. Finally, its use allows

one to preserve a natural measure of uncertainty in decision-making and to take into account indices, criteria, and constraints of qualitative character.

When applying the Bellman-Zadeh approach, each objective function $F_p(X)$ is replaced by a fuzzy objective function or a fuzzy set

$$A_p = \{X, \mu_{A_p}(X)\}, \quad X \in L, \quad p = 1, \dots, q \quad (2)$$

where $\mu_{A_p}(X)$ is a membership function of A_p [Zadeh 1965].

A fuzzy solution with applying the fuzzy sets (2) is turned out as a result of $D = \bigcap_{p=1}^q A_p$ with a membership function $\mu_D(X) = \bigwedge_{p=1}^q \mu_{A_p}(X) = \min_{p=1, \dots, q} \mu_{A_p}(X)$. Its use permits one to obtain the solution proving

$$\max \mu_D(X) = \max_{X \in L} \min_{p=1, \dots, q} \mu_{A_p}(X). \quad (3)$$

Thus, the problem (1) is reduced to finding

$$X^0 = \arg \max_{X \in L} \min_{p=1, \dots, q} \mu_{A_p}(X). \quad (4)$$

To get (4), it is necessary to build $\mu_{A_p}(X)$, $p = 1, \dots, q$ reflecting a degree of achieving "own" optima by $F_p(X)$, $X \in L$, $p = 1, \dots, q$. It may be satisfied by using the following correlations:

$$\mu_{A_p}(X) = \left[\frac{F_p(X) - \min_{X \in L} F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \quad (5)$$

for maximized objective functions and

$$\mu_{A_p}(X) = \left[\frac{\max_{X \in L} F_p(X) - F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \quad (6)$$

for minimized objective functions. In (5) and (6), λ_p , $p = 1, \dots, q$ are importance factors for the corresponding objective functions. They are not to be normalized because it is important the relationship between different λ_p but not their absolute values).

The construction of (5) and (6) demands to solve the following problems:

$$F_p(X) \rightarrow \min_{X \in L}, \quad (7)$$

$$F_p(X) \rightarrow \max_{X \in L}, \quad (8)$$

providing $X_p^0 = \arg \min_{X \in L} F_p(X)$ and $X_p^{00} = \arg \max_{X \in L} F_p(X)$, respectively.

Thus, the solution of (1) demands analysis of $2q + 1$ monocriteria problems (7), (8), and (3), respectively.

The solution X^0 is to belong to $\Omega \subseteq L$. Considering this, it is necessary to build

$$\bar{\mu}_D(X) = \bigwedge_{p=1}^q \mu_{A_p}(X) \wedge \mu_\pi(X) = \min \{ \min_{p=1, \dots, q} \mu_{A_p}(X), \mu_\pi(X) \}, \quad (9)$$

where $\mu_\pi(X) = 1$ if $X \in \Omega$ and $\mu_\pi(X) = 0$ if $X \notin \Omega$.

The procedures for solving the problem (3), discussed below, provide the line in obtaining $X^0 \in \Omega \subseteq L$ in accordance with (10). Thus, it can be said about the equivalence of $\bar{\mu}_D(X)$ and $\mu_D(X)$. It permits one to give up the necessity of implementing cumbersome procedures for building the set $\Omega \subseteq L$.

3. Multicriteria allocation of resources

Considering that the present work is dedicated to the problem of resource allocation, it is necessary to identify that methods leading to its solution, based on fundamental allocation principles (proportional allocation, optimal allocation, and principle of inverse priorities) exhibit significant drawbacks [Pedrycz et al. 2011]. They can be eliminated by engaging a multicriteria approach, which permits one to consider and maximize diverse positive consequences or minimize diverse negative consequences of allocating resources or their shortages. Our first results in this area were related to solving a set of problems of allocating power and energy shortages for different levels of territorial, temporal, and situational hierarchies of load management, generated by the impacts of the disaster of Chernobyl [Sklyarov et al. 1987].

Two classes of interrelated problems can be considered:

- multiobjective allocation of resources;
- multiobjective allocation of deficits of resources.

For the first class of problems, the homogeneous, clear, and acceptable for experts, participating in the decision process, formulation of specific objectives, for example, for new business projects can be:

- Predominant provision of financial resources for projects (generally, consumers) with a higher level of innovation.

In the case of multiobjective allocation of deficits of resources, the objectives may be presented in the following form:

- Predominant limitation of financial resources for projects (generally, consumers) with a lower level of innovation.

This form of formulating specific objectives is to be emphasized, given its universal nature.

The satisfaction of objectives in the case of multiobjective allocation of resources is associated with maximizing or minimizing the linear objective functions

$$F_p(X) = \sum_{i=1}^n c_{pi} x_i, \quad p = 1, \dots, q, \quad (10)$$

where $x_i, i = 1, \dots, n$, are variables, which correspond to sought resource volumes intended for the i th project; $c_{pi}, p = 1, \dots, q, i = 1, \dots, n$ are specific indicators, which correspond to the p th specific objective, for the i th project.

At the same time, the satisfaction of objectives in the case of multiobjective allocation of deficits of resources is associated with maximizing or minimizing the linear objective functions

$$F_p(\Delta X) = \sum_{i=1}^n c_{pi} \Delta x_i, \quad p = 1, \dots, q, \quad (11)$$

where $\Delta x_i, i = 1, \dots, n$ are variables, which correspond to the sought limitations of resource volumes intended for the i th project.

The objective functions (10) and (11) are not the only kind of objective functions, which can be used in multiobjective allocation of resources or their shortages. In particular, in [Sklyarov et al. 1987], other types of objective functions are considered; sometimes they can better reflect the essence of specific objectives.

Three models for allocating resources or their shortages are discussed in [Ramalho et al. 2019]. In particular, the first model "*Allocation of available resource*" is the following.

We are given demands $D_i, i = 1, \dots, n$ of the consumers and a total available resource $R < \sum_{i=1}^n D_i$. Then, the problem exhibits the following structure:

$$F_p(X) \Rightarrow \max_{X \in L} \text{ or } \min_{X \in L}, \quad p = 1, \dots, q \quad (12)$$

subject to the constraints

$$0 \leq x_i \leq D_i, \quad i = 1, \dots, n \quad (13)$$

and

$$\sum_{i=1}^n x_i = R. \quad (14)$$

To describe a general scheme of solving the problems formalized within the framework of (12)-(14), as well as other two types of the problems, it is advisable to introduce into consideration a vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of importance factors. In particular, if the solution X_α^0 with the values $\mu_{A_p}(X_\alpha^0), p = 1, \dots, q$ is not satisfactory, DM has to have the possibility to correct it, passing to $X_{\alpha+1}^0$ with changing the importance of one or more objective functions.

The general scheme of solving the problem (12)-(14), implemented within an adaptive decision system (ADS), is associated with the following sequence of blocks:

1. Solution of the problems (7) and (8) to get $X_p^0, p = 1, \dots, q$ and $X_p^{00}, p = 1, \dots, q$, respectively.
2. Construction of the membership functions defined by (5) or (6).
3. Construction of an initial vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of the importance factors.
4. Search for the solution X_α^0 defined by (4).
5. Analysis of the current solution X_α^0 . If DM is satisfied by the solution, then go to Block 7; otherwise go to Block 6, taking $\alpha := \alpha + 1$.
6. Correction of the vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of the importance factors, then go to Block 4.
7. Calculations are completed because the solution X^0 is obtained.

The main functions of a calculating kernel of ADS are associated with solving the problems (7) and (8) and obtaining X^0 in accordance with (4). The solution of (7) and (8) creates no difficulties. The maximization of (3) is based on a non-local search that is a modification of the "long valley" method [Raskin 1976].

Computational experiments show that variables of (3) can be divided into two groups: inessential and essential. The change of inessential variables leads to essential variations of (3). The change of essential variables leads to inessential variations of (3). Thus, a structure of (3) may be considered as a multidimensional "long valley". If we use direct search methods [Rao 1996], this circumstance requires the ascent from different initial points X_p^0 (Pareto points), if we minimize $F_p(X)$, or X_p^{00} (Pareto points), if we maximize $F_p(X)$, to find the most convincing solution X^0 . This explains the use of a non-local search, which can be presented as follows:

Step 1. The sequence $\{X^{(l)}\}, l = 1, \dots, q$ is built from the points X_p^0 (if we minimize $F_p(X)$) or X_p^{00} (if we maximize $F_p(X)$), obtained as a result of executing Block 1 of the general scheme. The property of the sequence is $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(l)}) \geq \min_{1 \leq p \leq q} \mu_{A_p}(X^{(l+1)}), l = 1, \dots, q - 1$.

Step 2. The local search for X^0 is carried out from $X^{(l)}$ ($l = 1$). As a result, we obtain a point $X^{(1)0}$ with $\mu_{A_p}(X^{(1)0})$, $p = 1, \dots, q$.

Step 3. The local search for X^0 is carried out from $X^{(l+1)}$. As a result, we obtain a point $X^{(l+1)0}$ with $\mu_{A_p}(X^{(l+1)0})$, $p = 1, \dots, q$.

Step 4. Analysis is realized:

- a) if $X^{(1)0} \neq X^{(l+1)0}$, then go to Step 5;
- b) if $X^{(1)0} = X^{(l+1)0}$ for $l \neq q - 1$, then go to Step 3, taking $l := l + 1$;
- c) if $X^{(1)0} = X^{(l+1)0} = X^{(q)0}$, then go to Step 8, taking $X^0 = X^{(1)0}$.

Step 5. A line between points $X^{(t)0}$ and $X^{(t+1)0}$ is "built" to generate points $X_s^{(t,t+1)}$, $s = 1, 2, 3$ (see Figure 1). Among these points (if they are acceptable from the point of view of (13), a point $X^{(t,t+1)0} = \arg \max_t \min_{1 \leq p \leq q} \mu_{A_p}(X_s^{(t,t+1)})$ is selected to define a direction for a future search.

Step 6. The next local search for X^0 is carried out from $X^{(t,t+1)0}$. As a result, we obtain a point $X^{(t+2)0}$ (see Figure 1).

Step 7. Analysis is executed: if three "last" points $X^{(t)0}$, $X^{(t+1)0}$, and $X^{(t+2)0}$ differ on $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t)0})$, $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+1)0})$, and $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)0})$ less than the accuracy desired, then go to Step 8, taking $X^0 = \arg \max [\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)}), \min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)}), \min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)})]$; otherwise go to Step 5, taking $X^{(t)0} := X^{(t,t+1)0}$ and $X^{(t+1)0} := X^{(t+2)0}$.

Step 8. Calculations are completed because the solution $X^0 \in \Omega \subseteq L$ is obtained.

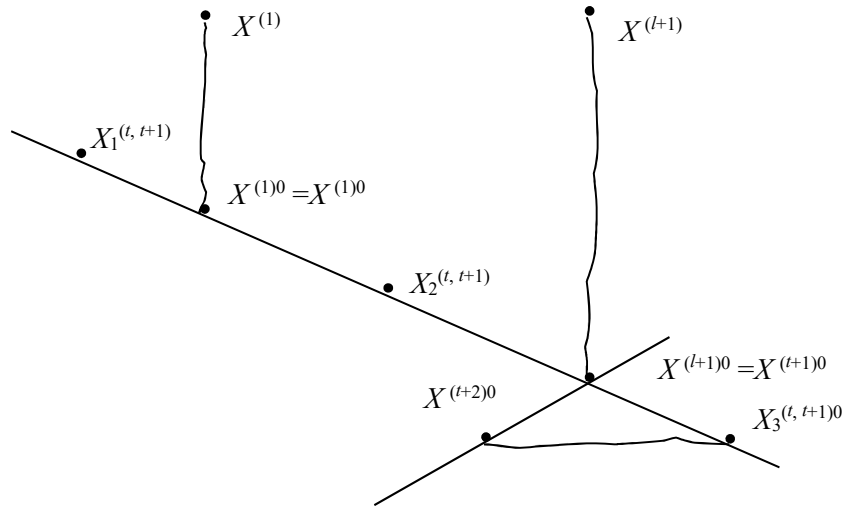


Figure 1. Non-local search for X^0

The execution of Steps 2, 3, and 6 of the algorithm is possible on the basis of any search method (in particular, a modification of the univariate method [method] was implemented within the framework of the ADS). If $X^{(m)}$ is a current point, the transition to $X^{(m+1)}$ is expedient if

$$(\forall p = 1, \dots, q) : \mu_{A_p}(X^{(m+1)}) \geq \min_{1 \leq p \leq q} \mu_{A_p}(X^{(m)}). \quad (15)$$

In contrast, if

$$(\exists p = 1, \dots, q) : \mu_{A_p}(X^{(m+1)}) < \min_{1 \leq p \leq q} \mu_{A_p}(X^{(m)}), \quad (16)$$

the transition to $X^{(m+1)}$ is not expedient from the point of view of maximizing (3). This way of evaluating the expediency of the transition to the next point $X^{(m+1)}$ leads to the solution (4) that is Pareto, if all inexpedient transitions are rejected.

4. Application examples

To illustrate the results presented above, a series of calculations were conducted on the allocation of resources for the implementation of public services aimed at improving the quality of life of population of different regions of the city of Maringa, PR, Brazil.

For this purpose, data published in the 2020 IBGE Census [IBGE 2010], were used. In particular, ten regions and three public services, namely Public Lighting, Sewage Collection System, and Water Distribution Network, were selected to conduct the resource allocation study. As for the costs of implementing the selected services, they were estimated on the basis of an informal survey conducted among suppliers and government agencies.

Tables 1 and 2 present the data used for the calculations. Table 1 shows the unit values, per residence, referring to each service, and Table 2 lists the total number of residences, the number of residences served by the service, their percentage and the cost for the entire population of the region to be served.

The data presented in Tables 1 and 2 allow one to construct the objective functions and constraints of the model (12)-(14).

According to Table 2, it would be needed R\$ 956,000.00 to serve 100% of the residences in all services. However, it is assumed that the available amount of resources is R\$ 700,000.00, that is, 73.2% of the required amount.

Table 1 – Unit cost for each selected public service

Service	Unit Cost (R\$)
Public Lighting	1,500.00
Sewage Collection System	3,000.00
Water Distribution Network	2,000.00

The allocation objectives are as follows:

1. Predominant provision of financial resources for regions with a lower percentage of residences served by the public lighting.
2. Predominant provision of financial resources for regions with a lower percentage of residences served by the sewage collection system.
3. Predominant provision of financial resources for regions with a lower percentage of residences served by the water distribution network.

Based on the above definitions, the following objective functions to be minimized can be written:

$$F_p(X) = \sum_{i=1}^{10} c_{pi}x_i, \quad p = 1, 2, 3,$$

where $i = 1, 2, \dots, 10$ corresponds to the region index.

Table 2 – Service characteristics of each selected region

Region	Residences	Public Lighting			Sewage Collection System			Water Distribution Network			Total Cost R\$10 ³
		Served Residences	Cost (R\$)	%	Served Residences	Cost (R\$)	%	Served Residences	Cost (R\$)	%	
1	243	203	60000	83.5	223	60000	91.8	243	0	100.0	120
2	252	242	15000	96.0	245	21000	97.2	243	18000	96.4	54
3	191	187	6000	97.9	157	102000	82.2	188	6000	98.4	114
4	190	187	4500	98.4	189	3000	99.5	184	12000	96.8	19.5
5	185	182	4500	98.4	182	9000	98.4	182	6000	98.4	19.5
6	254	199	82500	78.3	209	135000	82.3	209	90000	82.3	307.5
7	178	159	28500	89.3	175	9000	98.3	178	0	100.0	37.5
8	197	173	36000	87.8	191	18000	97.0	193	8000	98.0	62
9	226	209	25500	92.5	209	51000	92.5	210	32000	92.9	108.5
10	256	235	31500	91.8	234	66000	91.4	248	16000	96.9	113.5
											956

The objective functions, to be minimized, with the initial data in Table 2, are:

$$F_1(X) = 0.8354 x_1 + 0.9603 x_2 + 0.9791 x_3 + 0.9842 x_4 + 0.9838 x_5 + 0.7835 x_6 + 0.8933 x_7 + 0.8782 x_8 + 0.9248 x_9 + 0.9180 x_{10};$$

$$F_2(X) = 0.9177 x_1 + 0.9722 x_2 + 0.8220 x_3 + 0.9947 x_4 + 0.9838 x_5 + 0.8228 x_6 + 0.9831 x_7 + 0.9695 x_8 + 0.9248 x_9 + 0.9141 x_{10};$$

$$F_3(X) = 1.00 x_1 + 0.9643 x_2 + 0.9843 x_3 + 0.9684 x_4 + 0.9838 x_5 + 0.8228 x_6 + 1.00 x_7 + 0.9797 x_8 + 0.9292 x_9 + 0.9688 x_{10}.$$

The constraints are represented by the regional demand values presented in Table 3.

Besides, we have to take into account that $\sum_{i=1}^{n=10} x_i = 700,000.00$.

The use of ADS has permitted us to obtain the results given in Table 4.

The results of Table 4 provide Satisfaction Levels of the objective functions given in Table 5. Thus, the obtained solution is completely harmonious (for the case of the equal levels of importance of the objective functions). However, the results of the paper permit one to provide the harmonious solutions in the case of different importance of the objective functions.

The proposed results allows one to attribute importance factors to the objective functions, that reflect the knowledge, experience and/or specific interest, of the people involved in the decision process.

To illustrate this, calculations were performed by attributing importance factors to each objective function individually. The obtained results are presented in Table 6 for the case of the importance factors 1, 1, and 1 (case 1). The case 2 corresponds to the importance factors 2, 1, and 1. The case 3 is related to 1, 2, and 1. Finally, the case 3 is associated with the importance factors 1, 1, and 2. The Satisfaction Levels of the objective functions for all considered cases are given in Table 7.

Taking into account that $0.926^2 \approx 0.924^2 \approx 0.928^2 \approx 0.854$, all solutions in Table 6 are harmonious.

Table 3 – Constraints

Region	Demand
1	$x_1 \leq D_1 = 120,000.00$
2	$x_2 \leq D_2 = 54,000.00$
3	$x_3 \leq D_3 = 114,000.00$
4	$x_4 \leq D_4 = 19,500.00$
5	$x_5 \leq D_5 = 19,500.00$
6	$x_6 \leq D_6 = 307,500.00$
7	$x_7 \leq D_7 = 37,500.00$
8	$x_8 \leq D_8 = 62,000.00$
9	$x_9 \leq D_9 = 108,500.00$
10	$x_{10} \leq D_{10} = 113,500.00$

Table 4 – Resource amounts to be allocated to each region

Region	Available Amounts
1	$x_1 = 120.000$
2	$x_2 = 0.00$
3	$x_3 = 75,180.28$
4	$x_4 = 0.00$
5	$x_5 = 0.00$
6	$x_6 = 307,500.00$
7	$x_7 = 0.00$
8	$x_8 = 29,033.59$
9	$x_9 = 54,786.13$
10	$x_{10} = 113,500.00$

Table 5 - Satisfaction Levels

Objetive Function	$F_1(x)$	$F_2(x)$	$F_3(x)$
Level	0.854	0.854	0.854

Table 6 – Solutions for different importance factors

Case	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	120,000.00	0	75,180.28	0	0	307,500.00	0	29,033.59	54,786.13	113,500.00
2	120,000.00	0	75,269.82	0	0	307,500.00	0	33,414.61	50,315.57	113,500.00
3	120,000.00	0	75,032.98	0	0	307,500.00	0	29,040.87	54,926.15	113,500.00
4	120,000.00	0	72,716.09	0	0	307,500.00	0	24,618.16	61,665.74	113,500.00

Table 7 – Satisfaction Levels of the objective functions referring to cases 1, 2, 3, and 4.

Objetive Function	$F_1(x)$	$F_2(x)$	$F_3(x)$
Case 1	0.854	0.854	0.854
Case 2	0.926	0.849	0.849
Case 3	0.854	0.924	0.854
Case 4	0.853	0.853	0.928

5. Conclusions

When using the modification of the Bellman-Zadeh approach to decision making in a fuzzy environment, the concept of "optimal solution" is defined with reasonable validity: the

maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions on the basis of analyzing associated *maxmin* models. The modification of the Bellman-Zadeh approach has served as a basis for solving the problem of multiobjective allocation of resources or their deficits and developing an adaptive interactive decision system (ADS). The calculating kernel of ADS is based on procedures of a non-local search to solve *maxmin* problems. The peculiarities of the modification allow one to obtain solutions that are, indeed, Pareto. The results of some computing experiments have been presented to demonstrate the efficiency of the modification of the Belman-Zadeh approach for multicriteria decision making as applied to solving the important problem of allocating resources for public services in cities. The paper results can serve for reducing inequalities in the allocation of public goods in cities.

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