

Maximizing the Sortino Ratio: downside risk management and alpha yielding

Leonardo Riegel Sant'Anna

Universidade Federal do Rio Grande do Sul
Rua Washington Luiz, 855, Porto Alegre, Brasil, 90010-460
leonardo.santanna@ufrgs.br

Pablo Cristini Guedes

Universidade Federal do Rio Grande do Sul
Rua Washington Luiz, 855, Porto Alegre, Brasil, 90010-460
pablo.guedes@ufrgs.br

ABSTRACT

This paper presents an empirical analysis of portfolio optimization aimed at maximizing the Sortino ratio. We used a data set comprising weekly returns for 166 stocks traded in the Brazilian stock market from 2014 to 2024, to examine the capacity of this approach to generate excess returns and control downside risk in the long run. The portfolios use two different portfolio rebalancing frequencies (approximately three and six months) and three different target returns (minimum acceptable return thresholds – MAR), as required by the Sortino ratio. We find that especially portfolios using a three-month rebalancing window exhibited superior risk-adjusted performance, demonstrating the expected risk-return trade-off (elevated risk compensated by greater returns) for three target returns. In addition, we explore the sensitivity of portfolio construction to different MARs and check portfolio robustness, offering practitioners guidance on parameter selection that aligns with specific investment objectives in a multidimensional approach.

KEYWORDS. Sortino ratio. Downside deviation risk. Portfolio optimization.

PAPER TOPICS. (1) Other Applications in OR. (2) Non-linear Optimization. (3) Continuous Optimization.

1. Introduction

Portfolio optimization strategies have evolved significantly since Markowitz's seminal work on Modern Portfolio Theory (MPT) introduced the concept of maximizing returns for a given level of risk [Markowitz, 1952]. The concept of diversification initiated a new era for risk management. Subsequently, the trade-off between risk and return became central to asset allocation analysis and fundamentally shaped the definition of various risk metrics. In this context, particularly when seeking excess returns, performance measures play a crucial role in portfolio evaluation, as they quantify the efficiency of the risk-taking process in generating returns over time.

Among numerous performance metrics, the Sortino ratio [Sortino and Van Der Meer, 1991; Sortino and Price, 1994] stands out for its distinctive approach. It defines the relationship between portfolio returns relative to a target return, and an alternative risk assessment method: the downside deviation risk, which refers to the “negative” part of volatility (instead of considering total volatility in the form of standard deviation). The primary objective of the Sortino ratio is to manage downside risk, with return generation emerging as a potential consequential benefit. In this paper, we present an empirical analysis of portfolio optimization aimed at maximizing the Sortino ratio. Using a data set spanning approximately 10 years from the Brazilian stock market, we examine

Sortino ratio maximization to evaluate both its primary purpose (downside risk management) and its secondary potential benefit (generation of excess returns over time) to analyze the capacity of this approach to generate excess returns and control downside risk in the long run.

Although the mean-variance framework revolutionized portfolio construction, its reliance on standard deviation as a risk measure could also be criticized, since it treats upside and downside volatility as equally undesirable [Nawrocki, 1999; Sikalo et al., 2022]. This symmetrical treatment of volatility fails to align with investor preferences, as most investors are primarily concerned with downside risk, i.e. the risk of falling below a certain minimum acceptable return (MAR – a target return chosen by the investor). As argued by Sortino and Van Der Meer [1991], contrary to what a normal distribution would suggest, asset returns typically exhibit skewed rather than symmetric distributions. In response to these empirical realities, researchers have developed various asymmetric risk measures, with the Sortino ratio emerging as particularly significant in both academic literature and investment practice due to, among other factors, its incorporation of an asymmetric risk measure: the downside deviation.

The Sortino ratio represents a variation of the Sharpe ratio by replacing the standard deviation with the downside deviation. Thus, unlike the Sharpe ratio, which penalizes both upside and downside volatility through the use of standard deviation, the Sortino ratio concentrates solely on harmful volatility, the negative returns that investors want to avoid [Farinelli et al., 2008]. By focusing exclusively on returns below a predetermined threshold (MAR, the minimum acceptable return), the Sortino ratio offers a more nuanced measure of risk-adjusted performance [Sortino and Van Der Meer, 1991]. This distinction can be particularly interesting in markets characterized by asymmetric return distributions and during periods of market stress, when traditional risk metrics may underestimate potential losses.

Previous literature has explored the application of the Sortino ratio in portfolio optimization through some approaches. Farinelli et al. [2008] present a problem in which the investor's primary goal is wealth maximization. To investigate this objective, the authors examined a decision framework encompassing eleven performance metrics. Among these metrics is the Sortino-Satchell ratio, a generalization of the Sortino ratio that modifies the risk measurement from downside deviation to the use of lower partial moments, where negative returns serve as a risk proxy. Meade and Beasley [2011] also utilize a modified Sortino ratio for active investing with the enhanced indexation strategy, in which case a market benchmark is the basis to define the target returns (MAR). Through the use of a factor model, the authors devote specific attention to momentum, then employing the Sortino ratio maximization as an objective function to define the portfolio weights. Guastaroba et al. [2020] expand the enhanced indexation based on performance metrics using not only the Sortino ratio but also other performance measures. Both Meade and Beasley [2011] and Guastaroba et al. [2020] highlight interesting results in terms of portfolios that are able to generate excess returns relative to the benchmark, as expected from the enhanced indexation strategy.

However, as mentioned above, past studies have focused on aspects such as comparative analyzes among asset allocation based on the Sortino ratio and other performance metrics. Furthermore, determining appropriate minimum acceptable returns through the use of a specific strategy could potentially constrain robust empirical investigations. In this sense, a comprehensive study that examines the practical implications of Sortino ratio maximization across diverse market conditions, asset classes, and investment horizons adds to the literature by providing insights into the practical efficacy of downside risk optimization under varying market conditions. Our research extends beyond theoretical comparisons to evaluate how Sortino-optimized portfolios perform during different economic cycles, particularly during market downturns when asymmetric risk measures are most valuable. Additionally, we explore the sensitivity of portfolio construction to different minimum

acceptable return thresholds, offering practitioners guidance on parameter selection that aligns with specific investment objectives in a multi-dimensional approach.

In our empirical analysis, we use a dataset comprising weekly returns for up to 166 stocks traded in the Brazilian stock market from July 2014 to December 2024. This time interval allows us to examine the out-of-sample performance of Sortino-optimized portfolios across multiple market regimes, including severe bear markets such as in 2015, the COVID crash in 2020, as well as the solid bull market in 2017-2019 and relatively sideways market conditions in 2022-2024.

We examine three minimum acceptable return (MAR) parameter choices (equivalent to 5%, 10%, and 15% compound annual returns) and two different portfolio rebalancing frequencies (quarterly and semiannual rebalancing). The results demonstrate that Sortino-maximizing portfolios consistently generate excess returns compared to the Ibovespa index – the main Brazilian market benchmark – particularly during bull market intervals. Furthermore, Sortino portfolios exhibited relatively low risk and favorable Sharpe ratios during the 2015 bear market, while experiencing some performance deterioration in drifting market conditions, especially in 2022 and 2023. Although the volatility of Sortino portfolios occasionally exceeds that of the Ibovespa index, these peaks are compensated for in the long run by excess returns, as evidenced by the portfolios' higher cumulative returns and average Sharpe ratios documented in this study.

Next, Section 2 discusses the Sortino ratio, as well as past studies concerning portfolio optimization related to this measure. In addition, Section 3 describes the optimization model and the methodological aspects of the study, Section 4 presents the results of the empirical tests, and Section 5 concludes the article.

2. Literature Review

First, Section 2.1 discusses the main guidelines regarding the Sortino ratio. Then, Section 2.2 contextualizes the past literature related to Sortino ratio and portfolio optimization.

2.1. Sortino Ratio

The Sortino ratio emerges as a compelling performance measure in the context of risk-adjusted performance analysis for portfolios. Introduced by Sortino and Price [1994], it provides a performance analysis by associating risk exclusively with unfavorable outcomes – specifically, returns falling below a minimum accepted return (MAR). This approach considers returns above MAR as “good volatility” that should not count toward the portfolio risk assessment. In this context, only “bad volatility” (returns below MAR) should represent a risk of concern to investors. The Sortino ratio therefore depends on both the investor's specified target return (MAR) and a risk measure capturing only the “downside” portion of portfolio volatility, building on the downside deviation risk concept previously developed in Sortino and Van Der Meer [1991].

To define the Sortino ratio, consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω represents the set of all possible outcomes, $\mathcal{F}$ is a σ -algebra of events, and $\mathbb{P}$ is a probability measure. Let $r_p : \Omega \rightarrow \mathbb{R}$ be a random variable representing the portfolio return, and $r_g \in \mathbb{R}$ be the target return.

The downside deviation, as a measure of downside risk, can be formally defined as the square root of the second-order lower partial moment (LPM) of the return distribution:

$$\sigma_d = \sqrt{LPM_2(r_g)} = \sqrt{\mathbb{E}[(\min(0, r_p - r_g))^2]} \quad (1)$$

where $r_p - r_g$ represents the excess return relative to the target return, and $\mathbb{E}[\cdot]$ denotes the expected value operator, such that σ_d which can be expressed as:

$$\sigma_d = \sqrt{\int_{-\infty}^{r_g} (r_p - r_g)^2 dF(r_p)} \quad (2)$$

where $F(r_p)$ represents the cumulative distribution function of portfolio returns.

In empirical applications using discrete observations over a time interval $t = 1, 2, \dots, T$, the downside deviation is estimated as:

$$\hat{\sigma}_d = \sqrt{\frac{1}{T} \sum_{t=1}^T (\min(0, r_t^p - r_g))^2} \quad (3)$$

where r_t^p is the portfolio return for the t -th time period as defined below in Equation 4. To define r_t^p , consider a portfolio composed of $i = 1, 2, \dots, N$ assets, with each asset having weight x_i in the portfolio. Let r_{it} represent the return of the i -th stock during the t -th time period. Then, portfolio return is defined as:

$$r_t^p = \sum_{i=1}^N x_i r_{it} \quad (4)$$

Besides downside deviation, the Sortino ratio also utilizes the concept of excess return relative to target return. The expected excess return above the target return r_g is defined as:

$$\mu = \mathbb{E}[r_p - r_g] \quad (5)$$

which in discrete form is estimated as:

$$\hat{\mu}_e = \frac{1}{T} \sum_{t=1}^T (r_t^p - r_g) \quad (6)$$

The Sortino ratio S is then formally defined as the ratio of expected excess return to downside deviation:

$$S = \frac{\mu_e}{\sigma_d} = \frac{\mathbb{E}[r_p - r_g]}{\sqrt{\mathbb{E}[(\min(0, r_p - r_g))^2]}} \quad (7)$$

This formulation positions the Sortino ratio within the broader class of reward-risk measures, while emphasizing its focus on downside risk. It can be generalized to incorporate different risk preferences through the order of the lower partial moment:

$$S_\alpha = \frac{\mathbb{E}[r_p - r_g]}{\sqrt[n]{LPM_n(r_g)}} = \frac{\mathbb{E}[r_p - r_g]}{\sqrt[n]{\mathbb{E}[(\min(0, r_p - r_g))^n]}} \quad (8)$$

where n represents the order of the lower partial moment, with $n = 2$ corresponding to the standard Sortino ratio. Higher values of n place greater emphasis on larger negative deviations, reflecting increased risk aversion. This general framework allows for a comprehensive analysis of downside risk-adjusted performance measures, with the Sortino ratio representing a specific parameterization that balances computational tractability with effective risk assessment.

2.2. Related Work

We find the Sortino ratio to be a topic of research in past studies. Nawrocki [1999] provides a comprehensive historical perspective on downside risk measures, tracing their development from semi-variance concepts to more sophisticated frameworks, as well as the use of such measures in performance metrics like the Sortino ratio. Farinelli et al. [2009] advance the discussion beyond

one-size-fits-all performance measures, proposing an optimal asset allocation system that recognizes the need for custom performance ratios. Among other metrics, the authors use the Sortino-Satchell ratio (a variation of the original Sortino ratio used in our study), and their findings show that optimizing the Sortino-Satchell ratio might be an interesting choice relative to other performance metrics, especially in the case of using a “conservative” value for target returns.

Pedersen and Satchell [2002] offer critical insights into the theoretical foundations of performance measures under asymmetric returns. Their work provides mathematical justification for preferring measures like the Sortino ratio in markets characterized by non-normal distributions, which could be considered particularly relevant for emerging markets like Brazil. This theoretical underpinning helps explain why traditional performance measures may lead to suboptimal decisions in markets with significant skewness and kurtosis.

Meade and Beasley [2011] examine the detection of momentum effects for portfolios based on the enhanced tracking strategy, offering insights into how momentum strategies might be evaluated using different performance metrics. Their work has implications for how the Sortino ratio might capture the effectiveness of momentum-based portfolio strategies, particularly relevant in markets where trend-following approaches are prevalent. In a similar line, Guastaroba et al. [2020] also explores the Sortino ratio as a metric to evaluate enhanced tracking portfolios. However, different from Meade and Beasley [2011] (who utilized the Sortino maximization to define the weights of the stocks previously chosen based on momentum in the portfolio), Guastaroba et al. [2020] only use the Sortino as a measure to analyze the quality of the portfolios obtained based on other optimization models.

More recently, Kroll et al. [2021] discuss coherent portfolio performance metrics and their relationship to investor utility under different assumptions, arguing in favor of the use of such metrics that take into account risk-adjusted performance for portfolio optimization. Then, in a subsequent paper [Kroll et al., 2024], the same authors propose a modified version of the Sortino ratio, which uses an adjusted threshold that addresses some limitations of the original formulation. This modification reflects the ongoing refinement of downside risk measures to better capture investor preferences and market realities.

Overall, past literature has explored the Sortino ratio in various ways over time. Nonetheless, we notice a gap in the literature concerning a more comprehensive analysis of Sortino ratio optimization across diverse market conditions. The existing literature has established theoretical advantages in using the Sortino ratio based on its consideration of downside deviation risk, but empirical validation over a substantial time period, encompassing a large dataset of assets combined with different parameterizations, would make a valuable contribution to both academic understanding and practical portfolio management.

3. Problem Formulation

First, the optimization model is presented in Section 3.1. Then, Section 3.2 describes the key methodological aspects of the empirical study.

3.1. Optimization Model

In this study, we propose an optimization model that seeks to maximize the Sortino ratio, with additional constraints on portfolio cardinality and maximum portfolio weights.

The Sortino ratio maximization inherently takes the form of a fractional programming problem, where we seek to maximize a ratio of functions. Let $\mathbf{x} \in \mathbb{R}^N$ represent the vector of portfolio weights, with x_i as variable decision that denotes the weight allocated to the i -asset. The general problem can be expressed as:

$$\max_{\mathbf{x}} \frac{f(\mathbf{x})}{g(\mathbf{x})} \quad (9)$$

where: the expected excess return function is defined as $f(\mathbf{x}) = \mathbb{E}[r_p(\mathbf{x}) - r_g]$, the downside deviation function is defined as $g(\mathbf{x}) = \sqrt{\mathbb{E}[(\min(0, r_p(\mathbf{x}) - r_g))^2]}$, the portfolio return as a function of weights is defined as $r_p(\mathbf{x}) = \sum_{i=1}^N x_i r_i$, and r_g is the target return (MAR).

Thus, we can define the empirical formulation of the model as follows. We discretize the problem using historical return data. For a set of assets $i = 1, 2, \dots, N$ over time periods $t = 1, 2, \dots, T$, the optimization problem becomes:

$$\max_{\mathbf{x}} \frac{\hat{\mu}_e}{\hat{\sigma}_d} = \frac{\frac{1}{T} \sum_{t=1}^T (\sum_{i=1}^N x_i r_{it} - r_g)}{\sqrt{\frac{1}{T} \sum_{t=1}^T (\min(0, \sum_{i=1}^N x_i r_{it} - r_g))^2}} \quad (10)$$

The set of constraints contains standard restrictions for portfolio optimization problems as follows:

$$\sum_{i=1}^N x_i = 1 \quad (11)$$

$$x_{min} \leq x_i \leq x_{max} \quad \forall i \in \{1, \dots, N\} \quad (12)$$

$$x_i \leq z_i \quad \forall i \in \{1, \dots, N\} \quad (13)$$

$$\sum_{i=1}^N z_i \leq K \quad (14)$$

$$z_i \in \{0, 1\} \quad \forall i \in \{1, \dots, N\} \quad (15)$$

where Equation 11 is the budget constraint to ensure full allocation of portfolio wealth, Equation 12 imposes lower and upper limits to the weight of each stock in the portfolio to restrict concentration and ensure diversification, and Equations 13-15 limit the number of assets in the portfolio (maximum number defined by K), with binary variable z_i taking the value 1 if the i -th stock included in the portfolio and 0 (zero) otherwise.

3.2. Methodological Procedures

The optimization problem combines several challenging characteristics: it has a non-linear fractional objective function and includes binary variables for the cardinality constraint. It is a mixed-integer non-linear programming problem (MINLP) that is NP-hard, and we solve it through a direct non-linear optimization using AMPL programming language and solver Gurobi [Gurobi Optimization, LLC, 2024].

Our database comprises up to 166 Brazilian stocks with weekly returns from July 6, 2014, to December 15, 2024, as detailed in Section 4.1. As a result, for this time frame, the data have a total of 546 time data points (546 weeks).

We constructed Sortino-optimized portfolios using a rolling window approach with a 52-period in-sample (approximately one year) for optimization. Additionally, for out-of-sample evaluation, we employed two rebalancing frequencies: quarterly (13-week periods) and semi-annual (26-week periods) rebalancing intervals. This yielded 38 total portfolios with quarterly rebalancing and 19 portfolios with semi-annual rebalancing. For instance, the first portfolio was optimized using data from July 6, 2014, to June 28, 2015 (52 weeks), then evaluated over either the next 13 weeks (July 5 to September 27, 2015) for quarterly updates or 26 weeks (July 5 to December 27, 2015) for semi-annual updates. The second quarterly portfolio used data from October 5, 2014, to September 27, 2015, with out-of-sample evaluation from October 4 to December 27, 2015, and so on.

For the portfolio parameters in Section 3.1, first we choose three values for the target return r_g required in Equation 6. We selected three fixed weekly returns that are equivalent to 5%, 10%, and 15% annual compound returns (0.0939%, 0.1835%, and 0.2691% weekly). Although alternative approaches exist, such as using market benchmarks or dynamic targets that adjust to market conditions, we chose fixed targets independent of market trends for this study, leaving these alternatives for future research.

In addition, we set $K = 30$ stocks (approximately 20% of the total available stocks) as the maximum portfolio size in Constraint 14. Furthermore, in Constraint 12 we established $x_{min} = 0$ to prohibit short positions and $x_{max} = 25\%$ to ensure adequate diversification. This maximum weight constraint was implemented after preliminary tests revealed that some stocks frequently received weights exceeding 50% in several portfolios. Thus, the choice to limit the stock weights to 25% for every portfolio.

4. Empirical Results

First, we discuss aspects concerning the database in Section 4.1. Then, Section 4.2 introduces the general results, Section 4.3 analyzes our findings in a risk-adjusted context, and Section 4.4 investigates the robustness of the portfolios among the different choices for the target returns.

4.1. Database

As mentioned in 3.2, the database of stocks selected for this research contains approximately 10 years (from July 2014 to December 2024), with weekly returns for a total of 166 stocks.

First, we selected all stock components from two primary Brazilian market benchmarks in February 28th, 2025: the Ibovespa index (currently composed by 87 stocks) and the SMLL Small Caps index (121 stocks), yielding an initial dataset of 208 stocks.

Using *yfinance* Python library¹, we collected weekly stock prices from July 2014 to February 2025, obtaining data for 168 stocks out of the 208 initially selected. Then, we excluded two stocks with price data only after December 2024, resulting in a final sample of 166 stocks. Our sample begins with 107 stocks in 2014, with additional stocks incorporated into the optimization as they became publicly traded.

4.2. General Results

To evaluate portfolio performance out-of-sample, we consider net returns, i.e., returns that are already discounted from transaction and management costs. To do so, first we define the portfolio turnover as $turnover_p = \sum_{i=1}^N (|x_{i,p} - x_{i,p-1}|)$, where $x_{i,p}$ is the weight of the i -th stock in the current portfolio, and $x_{i,p-1}$ is the weight of the i -th stock in the previous portfolio.

Then, net returns are defined as $r_{t^*}^{net} = (1 - c \times turnover_p)(1 + r_t^p) - 1$ [Thornton and Valente, 2012], where t^* is every moment in time a portfolio is updated, and c represents transaction and management costs².

Table 1 presents descriptive statistics for each portfolio along with the Ibovespa index (IBOV), Brazil's main market benchmark. Portfolios are labeled according to rebalancing frequency and target return: “3m-5” indicates quarterly rebalancing (roughly 3 months) with 5% compound annual target return, while “6m-5” indicates semi-annual rebalancing (6 months) with the same target return. Similarly, portfolios with target returns 10% and 15% are labeled as “3m-10”, “6m-10”, “3m-15”, and “6m-15”, respectively. It is relevant to note that the Sortino ratio is not in any way related to the market benchmark (IBOV). Therefore, to evaluate the performance of the Sortino-optimized portfolios, the Ibovespa index is used just as a benchmark for comparison.

¹*yfinance* is an open-source package that is part of the project on PyPI (the Python Package Index), the official repository for Python packages (<https://pypi.org/project/yfinance/>). This package uses Yahoo Finance's publicly available APIs to retrieve market data such as series of stock prices.

²In this study, we use c equal to 15 basis points.

Table 1: Descriptive statistics¹

Descriptive statistics	IBOV	Portfolios					
		3m-5	6m-5	3m-10	6m-10	3m-15	6m-15
Average weekly return	0.22%	0.34%	0.41%	0.38%	0.41%	0.45%	0.44%
Compound annual growth rate (CAGR)	9.2%	16.1%	19.5%	18.3%	19.3%	22.4%	21.3%
Total return	130.7%	315.8%	442.9%	395.7%	437.7%	583.2%	526.9%
Average annual risk (std deviation)	20.9%	21.8%	23.2%	22.5%	23.6%	23.6%	24.8%
Sharpe Index (annual average)	0.02	0.63	0.76	0.68	0.70	0.88	0.79
Average number of stocks per portfolio	–	11.8	11.8	11.3	11.0	11.3	11.1
Average stock weight across portfolios	–	8.44%	8.48%	8.82%	9.88%	8.86%	9.92%
Average maximum weight per portfolio	–	23.64%	23.50%	23.66%	23.33%	24.01%	23.76%

¹ Average weekly return refers to the average return per week; Compound annual growth rate (CAGR) refers to the yearly compound return; Total return refers to the return cumulative return for the entire period; Average annual risk refers to the standard deviation of the weekly returns; Sharpe Index (annual average) refers to the average of the Sharpe Index considering yearly risk and return.

All Sortino portfolios outperformed the IBOV benchmark in terms of average weekly returns, compound annual returns, and total returns. Generally, higher target returns (from 5% to 15%) corresponded to higher actual returns in most portfolios. One exception occurred between portfolios 6m-5 and 6m-10, where the higher target return portfolio (6m-10) yielded slightly lower returns (19.%) than the lower target return portfolio (6m-5) at 19.5%, despite having higher average annual risk (23.6% versus 23.2%).

Comparing portfolios with identical targets but different rebalancing intervals (3 vs. 6 months) yields mixed results regarding risk-return profiles. For portfolios with targets 5% and 10%, longer rebalancing periods (6m) resulted in higher returns, risk, and average annual Sharpe ratios. In contrast, when comparing the 15% target portfolios, the 3-month rebalancing window demonstrated superior performance, with higher returns (CAGR of 22.4% for 3m-15 vs. 21.3% for 6m-15) and lower risk (23.6% for 3m-15 vs. 24.8% for 6m-15), leading to a more favorable annual average Sharpe ratio (0.88 vs. 0.79).

Notably, when comparing all portfolios with IBOV, the annual risk across all portfolios ranged from 21.8% (3m-5) to 24.8% (6m-15), only marginally higher than IBOV's annual risk of 20.9%. Meanwhile, CAGR for most portfolios (excluding 3m-5) exceeded at least twice that of the IBOV benchmark.

This risk-return profile resulted in consistently higher Sharpe ratios across all portfolios compared to the market benchmark, demonstrating that Sortino-maximizing portfolios can generate excess returns while maintaining appropriate risk levels. This is particularly noteworthy considering that these portfolios comprise only 11.5 stocks on average, as shown in Table 1. The average weight of the stocks in these portfolios is approximately 9%, with maximum weights approaching 24% on average. Interestingly, many portfolios include at least one stock close to the maximum allowable weight of 25%, reflecting the constraint imposed in our methodology (Section 3.2), and none of the portfolios used a number of stocks up to 30, which was defined as the maximum number of stocks per portfolio (parameter K for Equation 14).

4.3. Risk-Return Analysis

Tables 2, 3, and 4 present the annual cumulative returns, risk levels, and Sharpe ratios, respectively. These tables utilize color coding where darker green indicates superior performance and darker red indicates inferior performance. For returns (Table 2) and Sharpe ratios (Table 4), higher values represent better performance, while for risk (Table 3), lower values are preferable.

Together, these tables provide a comprehensive visualization of the risk-return profiles across all portfolios.

The analysis of the compound returns in Table 2 reveals that the highest returns occurred during 2017-2019. In 2020, quarterly rebalanced portfolios suffered significant losses, while semi-annually rebalanced portfolios experienced milder negative returns, though all portfolios underperformed the IBOV benchmark. The pattern reversed in 2021, with semi-annual portfolios achieving positive returns while quarterly portfolios underperformed, slightly trailing the IBOV. Despite these periodic fluctuations, all portfolios delivered at least double the index return on average across the entire out-of-sample period.

Table 2: Compound returns per year¹

Year	IBOV	Portfolios					
		3m-5	6m-5	3m-10	6m-10	3m-15	6m-15
2015	-18.5%	-3.9%	-8.5%	-3.9%	-9.2%	-5.8%	-7.2%
2016	31.6%	27.5%	31.2%	27.2%	26.7%	24.0%	25.5%
2017	31.9%	120.2%	129.3%	128.9%	135.9%	177.4%	181.0%
2018	15.0%	27.0%	39.0%	29.4%	40.7%	31.3%	40.2%
2019	32.6%	67.5%	73.2%	68.9%	69.7%	84.8%	72.4%
2020	1.1%	-38.9%	-7.6%	-33.2%	-0.7%	-33.5%	-3.5%
2021	-11.0%	14.2%	-15.0%	13.1%	-16.1%	14.3%	-14.4%
2022	4.6%	-9.5%	-9.2%	-10.3%	-10.9%	-9.0%	-12.0%
2023	22.3%	11.9%	2.0%	16.2%	1.4%	21.1%	-1.5%
2024	-7.1%	2.6%	12.6%	2.8%	10.1%	3.7%	10.7%
Average	10.3%	21.8%	24.7%	23.9%	24.8%	30.8%	29.1%

¹ Table is color-coded: **the darker the green, the better the result**, i.e. *larger* returns; **the darker the red, the worse the result**, i.e. *smaller* returns. Lighter shades of both colors indicate intermediate performance.

Furthermore, in Table 3 we see that the highest risk values occurred during 2017-2018 and 2020-2021. Notably, in 2015 and 2016, portfolios exhibited approximately half the risk of the IBOV during what was a persistent bear market in Brazilian equities, demonstrating effective risk containment despite higher portfolio concentration. Risk was elevated across all portfolios and the IBOV in 2020, while 2021 stands out as a period when portfolios showed increasing risk relative to the benchmark. In general, risk levels across all portfolios ranged from 21.8% (3m-5) to 24.8% (6m-15), representing less than a 4 percentage point difference from the IBOV's average risk of 20.9%.

Finally, Table 4 integrates risk and return metrics through Sharpe ratios. Year-by-year comparison against the IBOV reveals inferior risk-adjusted performance primarily for semi-annually updated portfolios (6 months) in 2020, and underperformance across all portfolios in 2022 and 2023. The results in 2015 were mixed, while the portfolios maintained a superior performance relative to the market benchmark during all other years.

Therefore, we conclude from Table 4 that Sortino portfolios exhibited an aggressive risk profile during the strong bull market of 2016-2019, experienced challenges during the early 2020 downturn and produced mixed results from 2021-2024. This latter period was characterized by a sideways long-term market pattern with intermittent periods of significant volatility in both directions. Overall, portfolios exhibited slightly higher risk but comparable to IBOV, whereas portfolio returns exceeded at least twice the market benchmark, demonstrating the expected risk-return trade-

Table 3: Risk (standard deviation) per year¹

Year	IBOV	Portfolios					
		3m-5	6m-5	3m-10	6m-10	3m-15	6m-15
2015	22.9%	13.4%	12.9%	14.6%	14.2%	15.8%	16.5%
2016	27.2%	14.2%	14.2%	14.8%	15.1%	14.7%	15.5%
2017	17.0%	24.1%	24.4%	24.3%	25.3%	26.5%	29.3%
2018	19.9%	27.2%	27.7%	27.7%	28.4%	29.6%	29.6%
2019	16.8%	17.3%	15.6%	19.3%	16.2%	21.7%	17.1%
2020	38.7%	36.1%	46.3%	34.9%	42.8%	35.2%	43.3%
2021	18.2%	27.6%	36.6%	27.8%	36.7%	27.9%	36.6%
2022	23.3%	23.0%	22.9%	25.4%	24.1%	25.7%	24.6%
2023	14.7%	17.5%	16.2%	18.3%	16.8%	20.2%	18.7%
2024	10.4%	17.6%	15.5%	18.1%	16.5%	18.5%	16.7%
Average	20.9%	21.8%	23.2%	22.5%	23.6%	23.6%	24.8%

¹ Table is is color-coded: **the darker the green, the better the result**, i.e. *smaller* risk; **the darker the red, the worse the result**, i.e. *larger* risk. Lighter shades of both colors indicate intermediate performance.

Table 4: Sharpe Index per year¹

Year	r_f ²	IBOV	Portfolios					
			3m-5	6m-5	3m-10	6m-10	3m-15	6m-15
2015	13.3%	-1.39	-1.28	-1.69	-1.17	-1.58	-1.21	-1.24
2016	14.0%	0.65	0.95	1.21	0.89	0.84	0.68	0.74
2017	10.0%	1.29	4.57	4.89	4.90	4.99	6.31	5.85
2018	6.4%	0.43	0.76	1.17	0.83	1.21	0.84	1.14
2019	5.9%	1.59	3.56	4.30	3.27	3.93	3.64	3.88
2020	2.8%	-0.04	-1.15	-0.22	-1.03	-0.08	-1.03	-0.15
2021	4.4%	-0.85	0.35	-0.53	0.31	-0.56	0.35	-0.52
2022	12.4%	-0.33	-0.95	-0.94	-0.89	-0.96	-0.83	-0.99
2023	13.0%	0.63	-0.06	-0.68	0.18	-0.69	0.40	-0.77
2024	10.9%	-1.74	-0.47	0.11	-0.45	-0.05	-0.39	-0.01
Average	-	0.02	0.63	0.76	0.68	0.70	0.88	0.79

¹ Table is is color-coded: **the darker the green, the better the result**, i.e. *larger* Sharpe Index; **the darker the red, the worse the result**, i.e. *smaller* Sharpe Index. Lighter shades of both colors indicate intermediate performance.

² Risk-free rate r_f is the yearly compound return of the monthly Selic rate, which is the reference interest rate for the Brazilian economy, equivalent to the federal funds rate in the USA.

off (elevated risk compensated by greater long-term returns).

4.4. Sensitivity Analysis for Portfolio Stability Across MAR Values

To assess the sensitivity of our portfolio optimization results to the choice for the target returns (MARs), we implemented a formal statistical framework using a multivariate distance test with permutation analysis. This approach allows us to quantitatively evaluate whether portfolios optimized with different MAR values exhibit statistically significant differences in their compositions.

The multivariate distance test measures the similarity between portfolios optimized across

different MAR values by computing the Euclidean distance between their weight vectors. For portfolios with N assets, each portfolio is represented as a point in a N -dimensional space, with the coordinates determined by the weight assigned to each asset. The Euclidean distance between two portfolios p_1 and p_2 is calculated as:

$$d(p_1, p_2) = \sqrt{\sum_{i=1}^N (x_{1,i} - x_{2,i})^2} \quad (16)$$

where $x_{1,i}$ and $x_{2,i}$ represent the weights of the i -th stock in portfolios p_1 and p_2 , respectively.

For each pair of MAR values (5% vs. 10%, 10% vs. 15%, and 5% vs. 15%), we computed the pairwise distances between all portfolios optimized with the respective MAR values. The mean distance served as our test statistic, capturing the average dissimilarity between portfolios optimized with different MAR parameters.

To determine statistical significance, we implemented a permutation test with 1,000 iterations. In each iteration, we randomly reassigned portfolios to MAR groups while maintaining the original group sizes, and then recalculated the mean distance. The p-value was computed as the proportion of permuted distances greater than or equal to the observed mean distance. This non-parametric approach allows us to test the null hypothesis that portfolio compositions are independent of the MAR parameter without making assumptions about the underlying distribution of portfolio weights.

Our analysis focused on the three MAR values used throughout this study: 5%, 10%, and 15%. For each MAR value, we included all 38 portfolios generated through our rolling window approach with quarterly rebalancing. The test was carried out on the full set of 166 assets in our universe, with each portfolio represented as a vector of weights across these assets.

The significance level for the hypothesis testing was set at $\alpha = 0.05$, with the null hypothesis that there is no significant difference between portfolio compositions optimized with different MAR values. Rejection of this null hypothesis would indicate that the choice of MAR substantially affects the resulting portfolio structure.

The results presented in Table 5 revealed no statistically significant differences in portfolio composition when comparing portfolios optimized with MAR values of 5%, 10%, and 15% (all p-values = 1.000). The mean Euclidean distances between the portfolio weight vectors across different MAR values remained consistent (approximately 0.51), indicating solid stability in asset allocation regardless of the target return threshold.

Table 5: Multivariate distance test across portfolios for different target returns (MAR)

Comparison between target returns (MAR)	Mean distance between portfolios	p-value	Statistically significant difference
5% vs. 10%	0.5021	1.000	False
10% vs. 15%	0.5118	1.000	False
5% vs. 15%	0.5105	1.000	False

This finding is particularly valuable for practitioners, as it suggests that the Sortino optimization in our empirical study yields robustness in portfolio allocations, since they are not overly sensitive to the target returns (MAR). Thus, the specific choice of MAR within this range tends to not critically impact the portfolio structure, allowing flexibility in setting target returns based on individual investor preferences without substantially altering the optimal investment strategy.

5. Conclusions

This study contributes to the literature on portfolio optimization by providing empirical evidence on the efficacy of Sortino ratio maximization in the Brazilian equity market from 2014 to 2024. Our findings demonstrate that portfolios constructed using this approach consistently presented excess return while maintaining risk under control if we take the market benchmark Ibovespa as a reference, particularly during bull markets. With risk levels only marginally higher than the benchmark, Sortino-optimized portfolios achieved superior risk-adjusted returns, as measured by the Sharpe ratio.

The performance of all portfolios varied with both the rebalancing frequency and the target return parameters. Semi-annually rebalanced portfolios demonstrated more resilient performance during market disruptions, particularly evident in their less severe losses during the 2020 COVID crash and subsequent stronger recovery in 2021, while quarterly rebalanced portfolios exhibited higher volatility during these market transitions. Regarding target returns, our analysis revealed that higher MAR values generally corresponded to increased actual returns, although this relationship was not strictly monotonic in all cases. This non-linearity highlights the complex interaction between risk preferences, portfolio construction constraints, and market conditions. Future research could explore adaptive approaches that transition between optimization methodologies based on prevailing market conditions, potentially offering even more robust performance across diverse investment environments.

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