

OPTIMIZATION APPROACHES TO VOLT-VAR CONTROL OF MODERN DISTRIBUTION SYSTEMS

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ABSTRACT

This study evaluates two optimization approaches to Volt-Var control in electric distribution networks with high photovoltaic (PV) penetration: nonlinear programming and upstream-downstream dynamic programming. Volt-Var control is vital for maintaining voltage levels and reducing losses in networks with bidirectional power flow due to distributed renewable generation. Simulations were performed on the IEEE 12-bus system, considering PV penetration levels from 10% to 50%. Both methods effectively reduced technical losses by up to 20% and maintained voltage within regulatory limits set by the Brazilian Electricity Regulatory Agency. Both approaches offer accuracy and scalability for large systems; upstream-downstream dynamic programming is computationally more efficient for the small networks studied, due to its lower complexity. Overall, the study supports the use of optimization in smart grid control and highlights its potential to enhance energy efficiency. These findings can guide future research toward more sustainable and resilient power system solutions.

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KEYWORDS. Volt-Var Control, Dynamic Programming, Nonlinear Optimization.

Upstream-Downstream Dynamic Programming, Tests and Results, Problem Statement, Conclusion, Introduction

1. Introduction

Over the past thirty years, Volt-Var control (VVC) has been widely studied as a viable tool for minimizing losses, adjusting load voltages, and ensuring optimal network performance under specific operating conditions (Hsu and Kuo [1993]; Baran and Hsu [1999]; Federico et al. [2005]; Liu et al. [2009]). Initially, the VVC optimization problem was simplified due to the close relationship between voltage management and the minimization of reactive power flow. In the past, minor adjustments to transformers and regulators, combined with the appropriate switching of independently controlled capacitors, were sufficient to keep voltages within permissible limits, and that was all that needed to be done.

The advance of photovoltaic penetration drastically altered this dynamic, introducing bidirectional flows and overvoltages at specific points in the network (Carvalho et al. [2008]; Baran et al. [2012]). Moreover, the significant increase in distributed generation has considerably heightened the complexity of control problems, consequently demanding more advanced and integrated solutions.

Due to current policies aimed at sustainability and reduced dependence on fossil fuels, the penetration of solar energy sources in electrical energy distribution networks has increased considerably. The Brazilian iNDC (Intended Nationally Determined Contributions) proposes, by 2030, an expansion in the use of renewable energy in the Brazilian energy matrix by approximately 33% (Ministerio das Relações Exteriores [2020]). According to the Ten-Year Expansion Plan, prepared by the Energy Research Company under the guidelines and support of the Energy Planning and Development and Petroleum, Natural Gas and Biofuels Secretariats of the Ministry of Mines and Energy, by the end of 2031, 17% of the Brazilian energy matrix will be composed of distributed micro and mini generation (Empresa de Pesquisa Energetica [2021]).

Given these considerations, this study aims to develop a method to address optimal power flow problems related to Volt-Var control in networks with high photovoltaic penetration. For this purpose, two approaches are tested:

1. Nonlinear Programming (NLP), which employs relaxation methods and linearizations to handle complex problems (Tabares et al. [2016]);
2. UD-DP approach, which seeks to leverage dynamic programming concepts to optimize flows in radial topology networks (Lyra [2019]).

Therefore, this study aims to evaluate the effectiveness of VVC based on the UD-DP technique, in comparison with conventional methods reported in the literature. The analysis assesses result quality, computational performance, and the potential advantages of the UD-DP approach in highly photovoltaic-integrated networks.

2. Problem Statement

The increasing presence of distributed generation, especially photovoltaic (PV) sources, in radial distribution networks has introduced new operational challenges. In response, this work proposes a mathematical modeling framework aimed at optimizing power flows to address these issues. The main objectives are to minimize technical losses and ensure voltage profiles remain within the regulatory limits established by ANEEL.

2.1. Objective Function

The primary objective is to minimize technical losses across all lines of a radial network. In distribution networks, losses are almost entirely attributed to the resistance of conductors, which dissipate a percentage of the transferred energy in the form of heat. Resistance losses are directly proportional to the square of the active power P and reactive power Q flowing through each line.

The objective function is formulated as the aggregate of all technical losses across all lines k within the network:

$$\min \sum_{k \in \mathcal{L}} Losses_k \quad (1)$$

where:

- $\mathcal{L}$: represents the set of all lines in the network.

The technical losses in a line k are calculated as shown in Equation 2:

$$Losses_k = R_k \cdot \left[\frac{(P_k)^2 + (Q_k)^2}{(V_k)^2} \right] \quad (2)$$

where:

- R_k : represents the electrical resistance of line k ;
- P_k : active power flow through line k ;
- Q_k : reactive power flow through line k ;
- V_k : voltage at the sending bus of line k .

Minimizing technical losses is a priority for distribution networks for the following reasons:

1. Cost Reduction: losses represent dissipated energy that is not sold, thereby reducing the revenue of utility companies;
2. Energy Efficiency: more efficient networks are more sustainable and reduce the need for additional generation;
3. Reliability: lower losses result in reduced voltage drops, improving service quality and customer satisfaction.

These benefits are inherently reflected in the objective function, which prioritizes solutions that minimize energy losses during distribution.

2.2. Operational Constraints

The optimization model is governed by constraints that ensure the proposed solution's technical feasibility and operational viability. These constraints encompass voltage limits, power balance equations, and the fundamental electrical laws that dictate the behavior of the network.

2.2.1. Voltage Limits

The voltage levels at the network nodes must be maintained within prescribed limits to ensure the proper performance of electrical equipment and compliance with ANEEL regulations, as indicated in Equation 3:

$$\underline{V} \leq V_k \leq \overline{V} \quad (3)$$

where:

- $\underline{V}$: lower voltage limit at the node;
- $\overline{V}$: upper voltage limit at the node.

2.2.2. Power Balance Equations

The active and reactive power flows at each node must satisfy the respective equations to ensure that all injected, consumed, and lost power is accounted for. Equation 4 represents the active power balance, while Equation 5 represents the reactive power balance.

$$P_k - P_k^{ld} + P_k^{fv} = \sum_{j \in A_k} \left[P_{k+1,j} + R_{k+1,j} \cdot \left(\frac{(P_{k+1,j})^2 + (Q_{k+1,j})^2}{(V_{k+1,j})^2} \right) \right] \quad (4)$$

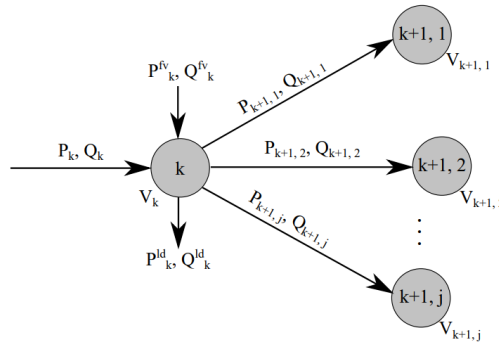
$$Q_k - Q_k^{ld} - Q_k^{fv} = \sum_{j \in A_k} \left[Q_{k+1,j} + X_{k+1,j} \cdot \left(\frac{(P_{k+1,j})^2 + (Q_{k+1,j})^2}{(V_{k+1,j})^2} \right) \right] \quad (5)$$

where:

- P_k^{ld} e Q_k^{ld} : active and reactive power consumed by the load at node k ;
- P_k^{fv} e Q_k^{fv} : active and reactive power provided by photovoltaic distributed generation at node k ;
- A_k : set of nodes connected to node k .

Figure 1 illustrates the power flows at a generic node k .

Figure 1: Active and reactive power flows for a k bus.



For nodes with distributed generation, both active and reactive power can act as sources or consumers of energy, depending on the operational conditions:

1. Active Generation Source: during high solar irradiance periods, photovoltaic inverters inject active power into the grid, contributing to energy flow management;
2. Reactive Power Generation and Consumer: during periods of low generation, such as nighttime, inverters may produce or consume reactive power to maintain voltage levels.

This flexibility is crucial for voltage profile management and plays a significant role in integrating renewable energy sources.

2.2.3. Voltage Drop Equation

As presented in Eq.6, Kirchhoff's Voltage Law describes the relationship between the voltages at adjacent nodes and the power flows along the connecting lines.

$$(V_{k+1,j})^2 = (V_k)^2 - 2 \cdot (R_{k+1,j} \cdot P_{k+1,j} + X_{k+1,j} \cdot Q_{k+1,j}) \quad (6)$$

According to this equation, the voltage at the downstream node $V_{k+1,j}$ is influenced by the active power $P_{k+1,j}$, and reactive power $Q_{k+1,j}$ transmitted through the line. The voltage drop increases proportionally with the resistance $R_{k+1,j}$ and reactance $X_{k+1,j}$ of the line.

Kirchhoff's Voltage Law is fundamental for ensuring the validity of voltage calculations at each node, accurately reflecting the operational conditions of the network.

2.3. Photovoltaic Inverter Modeling

Photovoltaic inverters, modeled here as constant power devices, control both active and reactive power in distributed generation networks. Equation (7) defines the active power injected. In contrast, the capacity to inject or absorb reactive power is constrained by the maximum apparent power rating of the inverter, as expressed in Equation (8).

$$P_{fv} = P_{fv}^{generated} \quad (7)$$

where:

- P_{fv} : active power injected by the inverter into the network;
- $P_{fv}^{generated}$: active power generated by the photovoltaic system.

$$Q_{fv} \leq \sqrt{(S_{fv}^{inv})^2 - (P_{fv})^2} \quad (8)$$

where:

- Q_{fv} : reactive power injected or consumed by the inverter;
- S_{fv}^{inv} : maximum apparent power of the inverter.

3. Nonlinear Programming Approach

Nonlinear programming (NLP) is a classic approach to the challenges presented in the Problem Formulation Section above, i.e., minimizing technical losses and maintaining the voltage profile of the radial distribution system with high PV penetration. Much literature discusses NLP, citing its accuracy in solving the nonlinearities present in electrical systems.

In this context, the problem is formulated through an objective function and a set of nonlinear constraints, which are directly utilized by nonlinear optimization solvers such as IPOPT, KNITRO, or fmincon. Each component of the model plays a specific role in the optimization process, as outlined below:

- **Objective Function:** Represented by the Equation (2), this function defines the criterion to be minimized by the solver. In this case, the technical losses in each distribution line are computed based on the active (P_k) and reactive power flows (Q_k), as well as the voltage values (V_k), which are updated iteratively during the optimization procedure;

- **Voltage Limits:** As defined in Equation (3), these constraints are modeled as inequalities, ensuring compliance with the operational voltage limits established by regulatory bodies, such as ANEEL;
- **Power Balance Equations:** Equations (4) and (5) are modeled as equality constraints, representing the conditions imposed by the principles of energy conservation;
- **Voltage Drop Equation:** Equation (6) is also formulated as an equality constraint, establishing the relationship between the voltages at two consecutive buses and the power flow between them. This equation is essential for accurately representing the electrical behavior of the distribution system;
- **Modeling of Photovoltaic Inverters:** Equations (7) and (8) define the operational constraints of the inverters. The first limits the active power output according to the inverter's rated capacity, while the second restricts the reactive power injection or absorption based on the equipment's apparent power capability.

The solver iteratively searches for a feasible solution that satisfies all constraints while minimizing power losses between substations. Despite the accuracy and detailed representation provided by the nonlinear model, Nonlinear Programming presents limitations related to high computational costs and the potential convergence to conflicting suboptimal solutions. This issue is particularly relevant due to the frequent presence of multiple local minima in such problems.

It is important to note that the problem described above is nonlinear, as it involves products of variables (such as power and voltage) which introduce nonlinearities into the formulation. These characteristics significantly increase the computational effort required and pose challenges to obtaining a globally optimal solution. To address this, the problem is linearized to enhance computational efficiency and increase the likelihood of finding a global optimum. In this work, the linearization approach proposed in Tabares et al. [2016] is adopted, allowing the reformulation of the original nonlinear problem into a tractable form suitable for efficient computational resolution.

4. Upstream-Downstream Dynamic Programming (UD-DP) Approach

UD-DP generalizes the Hamilton-Jacobi-Bellman equations to optimize flows in radial distribution networks (Fardin [2023]). This approach enables the application of dynamic programming concepts (Bertsekas [2005]) to address optimization problems in modern distribution networks.

The recursive equation that defines the dynamic optimization in UD-DP is given by:

$$F_k(x_k) = \min_{u_k \in \mathcal{U}_k} [e_k(x_k, u_k) + F_{k+1}(x_{k+1})], \quad \forall k \in \mathcal{R} \quad (9)$$

where:

- $F_k(x_k)$: accumulated cost function from state x_k to the feeder leaves;
- u_k : control variable associated with the state x_k ;
- $e_k(x_k, u_k)$: elementary cost associated with state x_k and control action u_k ;
- $F_{k+1}(x_{k+1})$: accumulated cost function for the subsequent states in the feeder.

This equation reflects the essence of the dynamic programming method: the optimal accumulated cost at each stage k is calculated as the local minimum value of e_k , added to the accrued cost of the subsequent stages F_{k+1} .

The update equation describes the relationship between successive states:

$$x_{k+1} = f_k(x_k, u_k) \quad (10)$$

where:

- x_{k+1} : updated system state, calculated based on the current state x_k and the control action u_k ;
- f_k : state transition function that describes how control actions influence network conditions.

This relationship ensures that the optimization accounts for the system dynamics, including the interactions between voltages, power flows, and reactive power along the feeders.

At branching points, where multiple feeders are coupled to a single node, UD-DP performs an internal flow optimization, described by:

$$F_{k+1}(x_{k+1}) = \min_{\forall x_{kl}, kl \in A_k} \left[\sum_{kl \in A_k} F_{kl}(x_{kl}) \right], \quad x_{kl} \in \mathcal{X}_{kl} \quad (11)$$

where:

- A_k : set of arcs connected to node k ;
- $F_{kl}(x_{kl})$: accumulated cost function on each arc kl ;
- x_{kl} : state variables associated with each arc kl .

Through which the accumulated cost F_{k+1} is optimized for all arcs branching from node k , based on the specific conditions of each network segment.

The solution to these recursive equations yields the optimal policies and the corresponding optimal flow distributions:

$$\pi^* = \{\mu_k^*, \forall k\}, \quad \mu_k^*(x_k) = (u_k^*, \mathcal{O}_k(x_k, u_k^*)) \quad (12)$$

where:

- π^* : set of optimal policies that define the best control actions u_k^* for each state x_k ;
- $\mathcal{O}_k(x_k, u_k^*)$: optimal flow distributions associated with each state x_k and control action u_k^* .

The solution to these recursive equations across all possible policies is fundamental for determining the optimal configuration of control devices that would ensure the efficient and safe operation of the network.

4.1. Mathematical Modeling of VVC with UD-DP

The proposed optimization problem is formulated as follows:

$$\begin{aligned}
 \text{Min}_{u_0, u_1, \dots, u_{n-1}} \quad & \left[\sum_{k=0}^{n-1} e_k(x_k, u_k) + F_{k+1}(x_n) \right] \\
 x_{k+1} \quad &= f_k(x_k, u_k) \\
 x_k \quad &\in X_k \\
 u_k \quad &\in U_k
 \end{aligned} \tag{13}$$

The dynamic programming solution defines the following elements:

- Each node in the analyzed network corresponds to a stage k ;
- The state variable is modeled as a one-dimensional vector x_k , representing the state of each node k :

$$[x_k] = [Q_k] \tag{14}$$

- The decision (or control) variable u_k is modeled as a vector of control variables in the optimization problem:

$$u_k = [u_k^1] = [Q_k^{fv}] \tag{15}$$

- The elementary cost function $e_k(x_k, u_k)$ represents the technical losses at node k :

$$e_k(x_k, u_k) = R_k \cdot \left[\frac{(P_k)^2 + (x_k)^2}{(V_k)^2} \right] \tag{16}$$

The backward process proposed below starts from the farthest node in the network. It iteratively computes the optimized values for the state variables (Equation 14) and control variables (Equation 15) at the system buses. This is achieved using the recursive Hamilton-Jacobi-Bellman equations (Equation 14).

In summary, the dynamic programming-based optimization model proposed in this work can be expressed as:

$$\begin{aligned}
 &\text{minimize: (16)} \\
 &\text{subject to: (3) and (4) to (6)}
 \end{aligned}$$

This optimization follows an iterative process proposed by Gonzalez et al. [2012], in which four steps are executed:

1. Solve the optimization problem by minimizing Equation 16, with initial values set to 1 p.u.;
2. Compute the voltage at each bus in the system as given by Equation 6;

3. Re-solve the optimization problem, incorporating the updated voltage values from Step 2;
4. Compare the results. If the solutions differ, return to Step 2; otherwise, the optimal solution has been found.

By iteratively updating the voltage values and recalculating the optimization problem, the process converges to an optimal solution, ensuring stability upon reaching a consistent result.

5. Tests and Results

As an illustrative example, the proposed method was tested on a 12-node test system, based on the system described in Mishra et al. [2014]. The nominal voltage at the substation is 11 kV. The proposed model was implemented in the mathematical modeling language AMPL and solved using the commercial solver IPOPT for the nonlinear problem and CBC for the linearized version. Additionally, it was implemented in Python and solved using principles of dynamic programming.

The photovoltaic distributed generation (DG) units were randomly allocated, and the inverter sizing was determined according to Equation 17:

$$P_{fv}^{nom} = \frac{E_{UC}}{\int_0^{t_{max}} G_{fv}^{pu}(t) \cdot dt} \quad (17)$$

where:

- P_{fv}^{nom} is the nominal power of the photovoltaic system (kWp);
- E_{UC} represents the energy consumption of the load (kWh);
- $G_{fv}^{pu}(t)$ denotes the normalized photovoltaic energy generation at instant t .

This sizing approach ensures net-zero energy consumption over 24 hours.

Different levels of photovoltaic penetration (ranging from 5% to 50%) were evaluated according to Equation 18:

$$NP_{fv} = \frac{N_{fv}}{NB_{el}} \cdot 100\% \quad (18)$$

were:

- NP_{fv} is defined as penetration level;
- N_{fv} is the number of installed photovoltaic systems;
- NB_{el} is the number of buses eligible for DG installation.

Only residential loads were considered in the current analysis, based on the profile shown in Figure 2. For the simulations, the power demand at time t was calculated by multiplying the nominal power by the normalized load curve at that specific instant. Subsequently, the active power injected into the grid at time t was determined using the normalized photovoltaic generation curve for that period. All loads are modeled as constant power with identical profiles.

The average power tends to vary with the number of photovoltaic systems as the load demand fluctuates and the corresponding sizes of inverters change.

Figure 2: Normalized PV generation and power consumption data profiles.

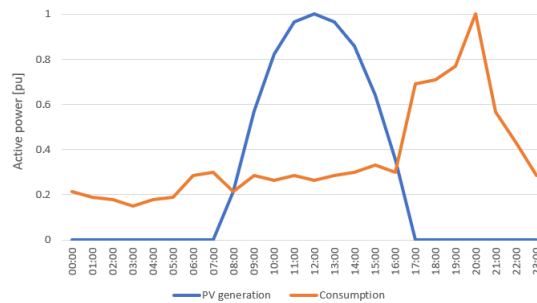
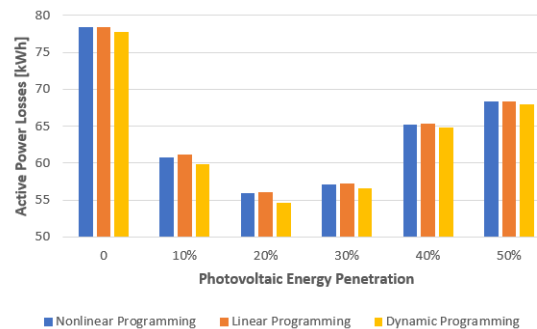


Figure 3 shows the loss reduction effect with PV penetration. In the initial stage, distributed generation reduces losses, as power is generated locally, close to the demand. However, beyond a certain penetration threshold, losses increase due to bidirectional power flows, which result in higher currents and resistive losses in the system. The amount of load that can be connected to a network without violating its technical standards is network-specific and depends on structural features and load distribution. This is referred to as the hosting capacity.

Figure 3: Loss levels considering different photovoltaic penetrations.

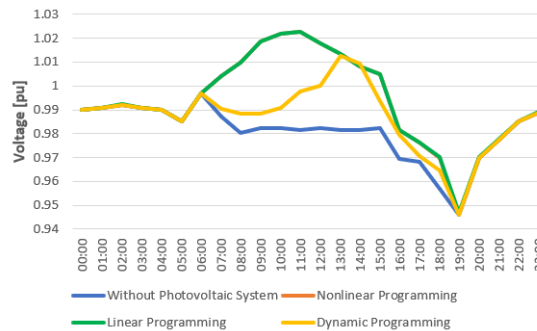


As shown in Figure 3, the results obtained using nonlinear programming, linear programming solvers, and the program developed in dynamic programming were very close. Solvers are expected to yield better results as more extensive networks with more controllable equipment are used. This is because, in nonlinear/linear programming, active power injection is treated as a continuous variable, allowing the problem to find the optimal injection value within the permitted range. In contrast, this variable is discretized in dynamic programming due to the nature of the calculations. As a result, nonlinear/linear programming is more likely to achieve a better solution. However, this does not imply that the result obtained through dynamic programming could not be satisfactory; it simply means it may not be the best possible. The advantage of dynamic programming lies in its processing time. However, this aspect will not be analyzed here since the network is small, and there are no significant differences in processing time.

Figure 4 shows the voltage profile of a specific bus over a day. In this case, the system is analyzed with 50% photovoltaic penetration, and the chosen bus (bus 10) has a photovoltaic generator. When there is no photovoltaic generation, the voltages remain the same regardless of the presence of photovoltaic systems or control mechanisms. However, there is a noticeable theoretical difference in the voltage profiles during a sunny day. For the system without distributed generation, the voltage remains relatively constant throughout the day, with a slight drop in the early evening

until around 8:00 pm (peak period, when consumption naturally increases). When distributed generation is introduced into the grid, an increase in voltage is expected, particularly around midday when generation peaks. However, since the analyzed system is a small network, the voltage increase is minimal and does not exceed the limits established by Brazilian regulations. It is noteworthy that dynamic programming maintained lower voltage levels than nonlinear/linear programming.

Figure 4: Voltage profile for 10 bar throughout the day for a system with 50% photovoltaic penetration.



6. Conclusion

This study provided a comparison between two methods for Volt-Var control in distribution networks with high photovoltaic penetration. Although the system analyzed in this study is simple, the preliminary results provided insights into the performance and applicability of each methodology.

Upstream-Downstream Dynamic Programming delivered good optimization results and showed preliminary indications of superior time efficiency, based on the simulations conducted in this study. Also, it can guarantee optimal solutions for non-convex problems and can be generalized to address VVC problems with random inputs. Its computation efficiency makes it suitable for real-time applications, and for application in networks with stringent operational constraints.

On the other hand, nonlinear programming is characterized by its flexibility and for its capacity to explore continuous solution spaces, which can be advantageous when resource allocation is critical. However, its high cost and suboptimal convergence in complex systems suggest that it can have limitations in addressing VVC problems for large networks. However, complementary strategies, such as hybridization, can help to address larger problems.

In the next stages of the research, further analysis will be carried out with larger and more complex distribution networks, to address the scalability of Upstream-Downstream Dynamic Programming and determine whether its computational advantages remain significant under more demanding conditions. Additionally, the study will be extended to include other distributed generation sources, more detailed load profiles, and stricter regulations.

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