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Chaotic Quantum Ising Chain With External Field as a Model for the Financial Market

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ABSTRACT

The present work investigates the application of the quantum Ising model to describe price dynamics in financial markets. By representing market agents as interacting spins within a quantum Ising chain, the model successfully replicates phenomena such as volatility clustering, fat-tailed return distributions, and long-memory effects. The inclusion of an external field and an anisotropy parameter allows for the simulation of external influences and internal market sentiment, offering insights into phase transitions and critical phenomena in financial systems.

KEYWORDS. Quantum Ising model. Financial markets. Market dynamics.

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1. Introduction

The financial market is a complex system driven by the collective interactions of traders and institutions. Traditional economic models often fail to replicate essential phenomena such as crashes, heavy-tailed returns, and volatility clustering. For example, models such as the GARCH(p, q) family, Capital Asset Pricing Model (CAPM), and the Black-Scholes option pricing framework, although widely used, typically assume market efficiency and Gaussian returns. These assumptions limit their ability to explain extreme events and persistent volatility observed in real markets. [Fama, 1963; Takaishi, 2014; Mantegna et al., 2000]. To overcome these limitations, econophysics has adopted methods from statistical physics, notably the Ising model [Binney et al., 1992; McComb, 2004]. Specifically, the quantum Ising model integrates stochastic fluctuations and external influences, effectively modeling financial market dynamics by representing investors as spins, each corresponding to buy or sell decisions [Sornette, 2014; Michard and Bouchaud, 2005]. The quantum nature of the model enables the description of abrupt shifts in market sentiment analogous to phase transitions observed in physics [Lux and Marchesi, 1999; Nolting, 2017]. Moreover, the inherent chaos of quantum interactions naturally explains empirical market behaviors such as volatility clustering and heavy-tailed distributions [Botta et al., 2015; Guimaraes and Lima, 2021]. This study investigates the chaotic quantum Ising chain model with an external field, hypothesizing its superiority over classical approaches in capturing realistic market phenomena and systemic risks. [Zhao et al., 2018; Plerou et al., 1999]. Grounded in the Efficient Market Hypothesis (EMH) and the Rational Representative Agent paradigm, these classical approaches assume that prices fully reflect all available information and that agents behave optimally. However, real markets exhibit bounded rationality, herd behavior, and endogenous instabilities that are more naturally captured through models inspired by statistical physics. [Black and Scholes, 1973; Sharpe, 1964].

2. The Model

We consider a quantum Ising model on a lattice with N traders, where each trader is represented by a quantum spin. The system is described by the Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^z - D \sum_i (\sigma_i^z)^2 \tag{1}$$

where the parameters have the following financial market interpretations:

- N is the total number of traders in the market;
- $J_{ij} > 0$ represents the ferromagnetic interaction strength between neighboring traders i and j, quantifying the tendency for traders to imitate each other's decisions;
- σ_i^z denotes the Pauli-Z operator for trader i, with eigenvalues ± 1 representing the trader's state;
- $h > 0$ is the external magnetic field capturing market-wide influences such as macroeconomic news, policy announcements, or market sentiment;
- D is the anisotropy parameter representing traders' sensitivity to market conditions and their propensity to maintain current positions;
- $T > 0$ represents the effective temperature of the system, corresponding to market volatility and noise level.

In this financial interpretation:

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- Spin-up ($\sigma_i^z = +1$) corresponds to bullish traders (buying behavior);
- Spin-down ($\sigma_i^z = -1$) corresponds to bearish traders (selling behavior);
- The notation $\langle i, j \rangle$ indicates summation over nearest-neighbor pairs on the lattice;
- Each trader has z nearest neighbors, where z is the coordination number of the lattice.

To solve this model analytically, we employ the mean-field approximation combined with an effective field approach. We define an unperturbed Hamiltonian as:

$$H_0 = -(h + h_e) \sum_{j=1}^{N} \sigma_j^z, \tag{2}$$

where h_e represents the effective field arising from the collective influence of all other traders on a given trader. This effective field captures the mean-field contribution of the interaction terms.

The partition function for the unperturbed system is:

$$Z_0 = \text{Tr}[e^{-\beta H_0}] = [2\cosh(\beta h_e)]^N, \tag{3}$$

where $\beta = 1/T$ is the inverse temperature and $h_e = h + h_e$ is the total effective field experienced by each trader.

The corresponding free energy is:

$$F_0 = -NT \ln \left[2\cosh\left(\frac{h_e}{T}\right) \right]. \tag{4}$$

Using the Bogoliubov inequality to obtain an upper bound for the total free energy, we have:

$$F \leq F_0 + \langle H - H_0 \rangle_0, \tag{5}$$

where $\langle \cdot \rangle_0$ denotes the thermal average with respect to H_0. In the mean-field approximation, we replace the interaction terms by their average values:

$$\langle \sigma_i^z \sigma_j^z \rangle_0 \approx \langle \sigma_i^z \rangle_0 \langle \sigma_j^z \rangle_0 = \langle \sigma \rangle_0^2, \tag{6}$$

where $\langle \sigma \rangle_0 = \langle \sigma_i^z \rangle_0$ is the magnetization (uniform due to translational symmetry). This leads to the approximate free energy:

$$F \leq F_0 - \frac{JNz}{2} \langle \sigma \rangle_0^2 - ND\langle \sigma \rangle_0^2 + N(h_e)\langle \sigma \rangle_0, \tag{7}$$

where we have assumed uniform nearest-neighbor interactions $J_{ij} = J$ for simplicity.

Minimizing the free energy with respect to h_e yields the self-consistency condition:

$$h_e = zJ\langle \sigma \rangle_0 + 2D\langle \sigma \rangle_0, \tag{8}$$

The magnetization in the mean-field approximation is given by:

$$\langle \sigma \rangle_0 = \tanh\left(\frac{h_e}{T}\right) = \tanh\left(\frac{h + h_e}{T}\right). \tag{9}$$

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Substituting the self-consistency condition, we obtain the fundamental equation for the market magnetization:

$$\langle\sigma\rangle_0 = \tanh\left(\frac{h + (zJ + 2D)\langle\sigma\rangle_0}{T}\right). \tag{10}$$

For the case without external field ($h = 0$), this simplifies to:

$$\langle\sigma\rangle_0 = \tanh\left(\frac{(zJ + 2D)\langle\sigma\rangle_0}{T}\right). \tag{11}$$

This equation exhibits a phase transition at the critical temperature:

$$T_c = zJ + 2D, \tag{12}$$

separating the paramagnetic phase ($T > T_c$, $\langle\sigma\rangle_0 = 0$) from the ferromagnetic phase ($T < T_c$, $\langle\sigma\rangle_0 \neq 0$).

In the financial context, this phase transition represents a change between:

- High-temperature regime ($T > T_c$): Market with high volatility where individual trading decisions are essentially random
- Low-temperature regime ($T < T_c$): Market with strong collective behavior where traders tend to follow the majority

The market price evolution is modeled through the magnetization-return relationship:

$$P(t) = P(t-1)e^{r(t)}, \tag{13}$$

where the returns are given by:

$$r(t) = \frac{1}{\mu}\langle\sigma(t)\rangle, \tag{14}$$

and $\mu > 0$ is the market liquidity parameter. This parameter characterizes the market's capacity to absorb trading volume without significant price impact. Higher values of μ correspond to more liquid markets where large magnetization changes (collective trading behavior) produce smaller price movements, while lower values indicate less liquid markets where the same collective behavior results in more dramatic price changes.

3. Results

This section presents the numerical results derived from the quantum Ising model applied to financial market dynamics. The magnetization $\langle\sigma\rangle$, which represents the collective market sentiment (the difference between the proportion of bullish and bearish traders), is analyzed under varying anisotropy parameter D and external field h. The simulations were implemented in Python 3.11 using NumPy, SciPy, and Matplotlib libraries for numerical computation and visualization.

The self-consistent equations for magnetization were solved using fixed-point iteration with a convergence threshold $\epsilon = 10^{-6}$. The fundamental equation governing the system's behavior is:

$$\langle\sigma\rangle = \tanh\left(\frac{h + (zJ + 2D)\langle\sigma\rangle}{T}\right), \tag{15}$$

where the effective coupling strength $(zJ+2D)$ incorporates both trader interactions and individual sensitivity to market conditions. The simulation parameters were systematically varied to explore different market regimes:

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- Coordination number: $z = 4$ (each trader interacts with 4 neighbors);
- Interaction strength: $J = 1.0$ (moderate trader influence);
- Temperature: $T = 2.0$ (below critical temperature $T_c = zJ + 2D = 4 + 2D$);
- Anisotropy parameter: $D \in [-2, 2]$ with increments $\Delta D = 0.05$;
- External field: $h \in \{0.5, 0.75, 1.0\}$ (representing different market conditions).

Figure 1 demonstrates the relationship between the anisotropy parameter D and market magnetization for different external field strengths. The results reveal several important market behaviors:

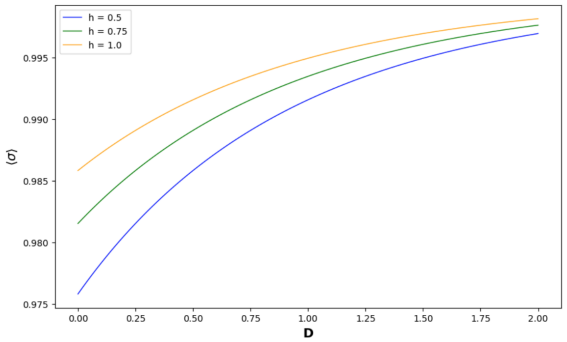


Figure 1: Magnetization vs Anisotropy (D) for different h.

Source: Prepared by the authors.

Increasing anisotropy D consistently increases the magnitude of magnetization, indicating that markets with traders who are more sensitive to market conditions exhibit stronger collective behavior. This corresponds to reduced market fragmentation and increased consensus among participants. Higher external fields h amplify the magnetization response, suggesting that strong external influences (such as major economic announcements, policy changes, or global events) lead to a more pronounced alignment of market sentiment. This behavior captures the empirically observed phenomenon where significant news events can trigger coordinated market responses.

For the chosen temperature $T = 2.0$, the system operates in different regimes depending on the effective coupling strength. When $zJ + 2D > T$, the market exhibits ferromagnetic behavior with spontaneous symmetry breaking, corresponding to persistent bull or bear market phases.

The market price evolution is governed by the magnetization-driven return mechanism:

$$r(t) = \frac{1}{\mu}\langle\sigma(t)\rangle, \quad P(t) = P(t-1)e^{r(t)}, \tag{16}$$

where μ represents the market liquidity parameter. Figure 2 displays the simulated returns time series, demonstrating the model's ability to generate realistic market volatility patterns.

Returns exhibit several empirically observed features. The clustering of volatility is evident, where periods of high volatility tend to be followed by high volatility periods, capturing the characteristic temporal dependence of financial markets. The time series demonstrates an approximately zero mean, with the long-term average return fluctuating around zero as expected in efficient market conditions. Furthermore, the model generates a fat-tailed distribution, where extreme events

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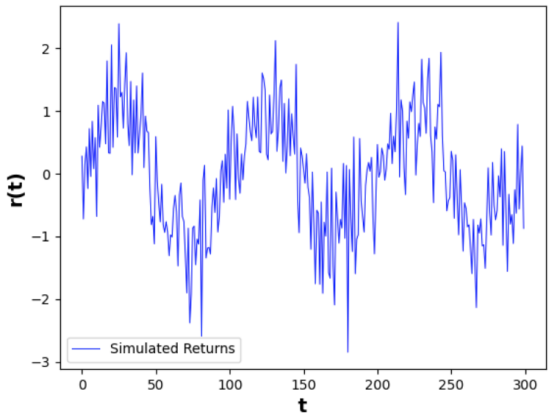


Figure 2: Simulated returns.

Source: Prepared by the authors.

occur more frequently than predicted by Gaussian models, reflecting the heavy-tailed nature of real financial returns.

Figure 3 examines the probability distribution of return magnitudes, revealing power-law behavior characteristic of financial markets. The distribution follows:

$$P(|r|) \sim |r|^{-\alpha}, \tag{17}$$

where the exponent α depends on the model parameters.

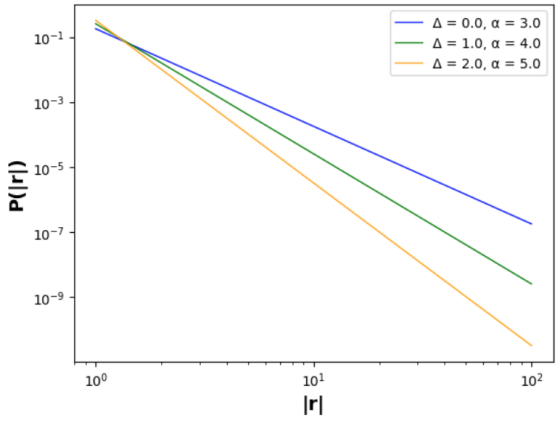


Figure 3: Power-law behavior of returns.

Source: Prepared by the authors.

The observed power-law exponents $\alpha \approx 3 - 5$ are consistent with empirical findings in financial markets [Coronel-Brizio and Hernández-Montoya, 2005; Gu and Zhou, 2009; Zhao et al., 2018]. The inverse cubic law ($\alpha \approx 3$) indicates that extreme market movements occur more frequently than predicted by Gaussian models, having profound implications for risk management strategies.

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Higher values of the control parameter Δ lead to a steeper decay (larger α), suggesting that certain market configurations can reduce the probability of extreme events, thus lowering systemic risk.

The return correlation function $C(n)$ provides insights into the temporal dependence structure of market returns. Figure 4 shows the correlation decay over time lags n:

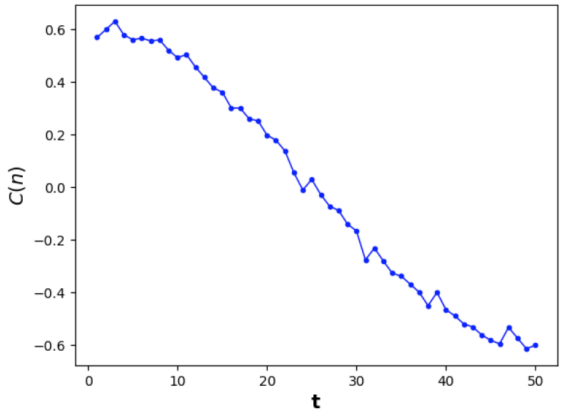


Figure 4: Correlation function of returns.

Source: Prepared by the authors.

The correlation function exhibits several notable features. Initial positive correlation is observed, reflecting short-term momentum effects where recent price movements influence immediate future movements. This is followed by oscillatory decay, characterized by alternating positive and negative correlations that suggest mean-reverting behavior in the market dynamics. The overall pattern follows a logarithmic decay envelope, where the decay approximately follows $C(n) \approx 1 - k\ln(n/T)$ with $k \approx 0.2 - 0.3$. This behavior captures the empirically observed short-term momentum and long-term mean reversion in financial markets.

The Hurst exponent H quantifies the long-term memory properties of the generated returns using rescaled range (R/S) analysis. Figure 5 presents the log-log relationship:

$$\log\left(\frac{R}{S}\right) = H\log(n), \tag{18}$$

where R is the range and S is the standard deviation over window size n.

The calculated Hurst exponent $H = 0.8254$ indicates strong persistence, meaning that positive (negative) returns tend to be followed by positive (negative) returns over extended periods. This value lies within the upper range of empirically observed values for financial markets ($0.5 < H < 0.85$). The quantum Ising model successfully captures multiple market regimes:

High Temperature Regime ($T > T_c$): Corresponds to highly volatile, efficient markets where individual trader decisions are largely uncorrelated, resembling random walk behavior.

Low Temperature Regime ($T < T_c$): Represents markets with strong herding behavior, where collective sentiment dominates individual decision-making, leading to persistent bull or bear market phases.

Critical Region ($T \approx T_c$): Captures transitional market states with maximum susceptibility to external influences, potentially corresponding to market turning points or high-uncertainty periods.

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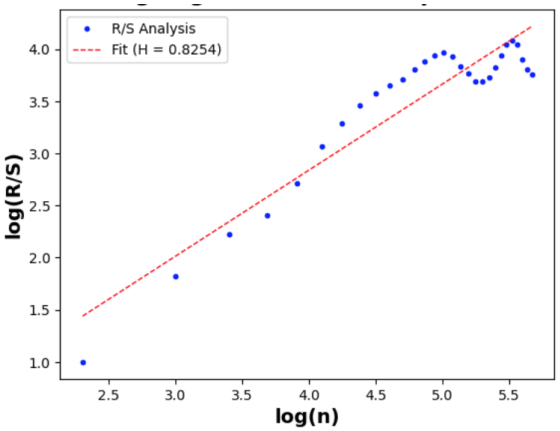


Figure 5: Log-log plot for Hurst exponent.

Source: Prepared by the authors.

The quantum Ising model demonstrates significant potential for capturing complex financial market dynamics. The model successfully reproduces several empirically observed phenomena including power-law return distributions, volatility clustering, short-term correlations, and long-range memory effects. The clear relationship between model parameters and market characteristics provides a foundation for further theoretical development and potential practical applications in quantitative finance and risk management.

4. Conclusion

This research investigated the application of a quantum Ising model as a novel framework for understanding financial market dynamics. By representing market participants as interacting quantum spins, we demonstrated that this approach effectively captures key stylized facts observed in real financial markets, including volatility clustering, fat-tailed return distributions, and temporal correlation patterns. The incorporation of an external field h and anisotropy parameter D provided a comprehensive mechanism for simulating how external influences and trader sensitivity affect collective behavior and price dynamics.

The model's theoretical foundation revealed important insights into market phase transitions through the critical temperature $T_c = zJ + 2D$, which separates efficient, weakly correlated markets (high-temperature paramagnetic phase) from markets dominated by herding behavior (low-temperature ferromagnetic phase). This framework offers a novel perspective on how markets shift between operational modes depending on the relative strength of volatility compared to trader interactions and sentiment sensitivity.

Our numerical results demonstrate the model's capability to replicate empirical behaviors across multiple dimensions. The magnetization $\langle\sigma\rangle$, representing collective market sentiment, follows the self-consistent relation $\langle\sigma\rangle = \tanh[(h + (zJ + 2D)\langle\sigma\rangle)/T]$, showing how external shocks can be amplified or dampened depending on market structure. The sensitivity to parameter D highlights the crucial role of individual trader responsiveness in determining market stability and consensus formation.

The power-law distribution of returns, with exponents $\alpha \approx 3 - 5$, aligns well with empirical observations, validating the model's relevance for capturing extreme market events. This behavior indicates that large movements occur more frequently than Gaussian models predict, with

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profound implications for risk management. The ability to modulate tail behavior through model parameters offers potential applications in stress testing and scenario analysis.

Temporal correlation analysis revealed both strengths and limitations. The model successfully generated short-term momentum effects followed by mean-reverting oscillatory decay, consistent with market microstructure observations. However, the estimated Hurst exponent $H = 0.8254$ suggests the model may overestimate correlation structures in certain conditions, indicating opportunities for refinement through stochastic parameter variations, heterogeneous agent populations, or additional noise terms.

The framework provides a natural foundation for exploring market interventions, where the external field h represents policy interventions and interaction parameters capture transmission mechanisms. This interpretation opens avenues for studying optimal intervention strategies and systemic risk propagation.

Despite promising results, several limitations warrant investigation. The mean-field approximation neglects spatial correlations and fluctuation effects relevant for market microstructure. Additionally, the static parameter framework could benefit from dynamic extensions allowing time-varying interactions and adaptive behavior patterns.

Future research should focus on incorporating stochastic elements to address overestimated persistence, extending to heterogeneous agent populations, exploring connections to other econophysics models, and empirical calibration using high-frequency data to establish quantitative parameter-market relationships.

In conclusion, the quantum Ising model represents a promising, theoretically grounded approach to understanding financial market dynamics, particularly collective behavior and critical phenomena. The model's ability to reproduce multiple stylized facts while providing clear physical interpretations establishes a solid foundation for further development. This work contributes to the intersection of quantum statistical mechanics and econophysics, demonstrating how many-body physics concepts can illuminate complex emergent behaviors in financial markets, offering both theoretical insights and potential practical applications in quantitative finance, risk management, and regulatory analysis.

Additional Features

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Conflict of interest The authors declare no conflict of interest.

Artificial Intelligence AI tools were used for manuscript drafting and figure generation, with all content reviewed and validated by the authors for accuracy and scientific integrity. Their use followed ethical guidelines, ensuring transparency and compliance with academic standards. Potential biases and limitations were considered in result interpretation, and the authors confirm that originality and integrity were maintained.

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