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A variable neighborhood search algorithm for a class of electric vehicle routing problems

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RESUMO

Esta tese apresenta um algoritmo baseado no algoritmo General Variable Neighborhood Search, denominado Flexi-VNS, para tratar de três problemas de roteamento de veículos elétricos: o Problema de Roteamento de Veículos Elétricos clássico (EVRP), o Problema de Roteamento de Localização de Estação de Troca de Bateria com Veículos Elétricos Capacitados (BSS-EV-LRP) e o Problema de Roteamento de Veículos Elétricos com Janelas de Tempo (E-VRPTW). O algoritmo incorpora um método Randomized Variable Neighborhood Descent (RVND) como procedimento de busca local. Avaliamos o desempenho do Flexi-VNS em instâncias de referência dos problemas, comparando seus resultados com os algoritmos existentes na literatura. Os resultados computacionais demonstraram a eficiência do Flexi-VNS. Ele aprimorou várias soluções da literatura e reduziu o número de estações de troca de bateria necessárias, obtendo taxas de sucesso de 90,38%, 60% e 70,65% para instancias do EVRP, BSS-EV-LRP e E-VRPTW, respectivamente.

PALAVRAS CHAVE. Variable Neighborhood Search, Veículos Elétricos, Problemas de Roteamento, Estações de Bateria, Janelas de Tempo.
Tópicos: MH – Metaheurísticas, L&T – Logística e Transportes

ABSTRACT

This thesis presents a flexible General Variable Neighborhood Search based algorithm, named Flexi-VNS, to address three problems in electric vehicle routing: the classical Electric Vehicle Routing Problem (EVRP), the Battery Swap Station Location-routing Problem with Capacitated Electric Vehicles (BSS-EV-LRP), and the Electric Vehicle Routing Problem with Time Windows (E-VRPTW). The Flexi-VNS algorithm incorporates a Randomized Variable Neighborhood Descent (RVND) method as its local search procedure, featuring an intra-RVND mechanism applied exclusively to modified routes upon each solution update. We evaluated the performance of Flexi-VNS on benchmark instances of the problems, comparing its results with existing algorithms from the literature. Computational results demonstrated the efficiency of Flexi-VNS. It improved several best-known solutions and reduced the number of battery swap stations required, getting success rates of 90.38%, 60%, and 70.65% for EVRP, BSS-EV-LRP, and E-VRPTW instances, respectively.

KEYWORDS. Variable Neighborhood Search, Electric Vehicles, Routing Problems, Battery Stations, Time Windows.
Paper topics: MH – Metaheurístics, L&T – Logistics and Transport

[1] Este trabalho apresenta um resumo da tese de doutorado disponível em https://www.repositorio.ufop.br/handle/123456789/18939

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1. Introduction

Governments around the world have been formulating carbon reduction strategies for freight transport. The developing system is usually made by analyzing the Greenhouse Gas GHG/CO2 emissions from transportation operations. Road transport has tended to gain attention. It is the predominant mode related to freight movement and is responsible for the most significant part of emissions of pollutants within nations [McKinnon and Piecyk, 2009]. According to Koç and Karaoglan [2016], the green logistics concept has emerged as an extension of the Vehicle Routing Problem (VRP). Green logistics involves the consideration of alternative fuels for vehicle fleets to reduce carbon dioxide (CO2) emissions. Several studies have proposed methods for the incorporation of these alternative recharge methods into VRPs. Green Vehicle Routing Problem (G-VRP) is a variant of the VRP that merges the environmental and economic costs by implementing effective routes to meet the ecological concerns and the usual objectives of VRP [Rodrigue et al., 2017]. According to Zhang et al. [2018], the VRP focuses on finding the optimal route to minimize transportation costs, and the G-VRP emphasizes economic costs and sustainability issues. The sustainable problems are described as decreasing GHG emissions by reducing the fuel consumption of vehicles and using alternative fuel-powered vehicles that depend on fuel sources such as electricity, natural gas, and hydrogen, among others. In this way, the Electric Vehicle Routing Problem (EVRP) can be classified as a variant of G-VRP. Electric vehicles can decrease transportation costs and pollution effects compared to other vehicles used in the transportation system. However, the limited battery capacity, reduced number of charging or swapping stations, long recharging battery time, and limited range driving make the charging process challenging compared with fossil fuel vehicles [Kucukoglu et al., 2021]. Schneider et al. [2014] defined the EVRP as an extension of the classic VRP. The EVRP considers the vehicle fleet to be composed exclusively of electric cars. The differences between the problems are related to battery constraints and charging operations.

This research addresses three problems. We named the EVRP class, which includes the classical EVRP and its variants, Battery Swap Station Location-routing Problem with Capacitated Electric Vehicles (BSS-EV-LRP), and Electric Vehicle Routing Problem with Time Windows (E-VRPTW). Introduced by Erdoğan and Miller-Hooks [2012], the classical EVRP considers the same goals as VRP, using a fleet of vehicles with alternative fuel types. The authors assumed electric vehicles in these fleets, which, if necessary, must recharge their batteries at recharging stations. The recharging stations are distributed around the customers, and the vehicles can visit them according to their needs. The BSS-EV-LRP is an important EVRP variant. The problem proposes exchanging the batteries of the vehicles. This way, the charging stations used in the EVRP are replaced by swapping stations in this variant. Some works consider the swapping stations as dummy nodes located at the customers, if necessary. Others held the swapping stations around the customers [Yang and Sun, 2015]. The E-VRPTW is another variant that has been extensively researched in the literature. Schneider et al. [2014] introduced the problem by incorporating the recharging plan and the time window restriction on the EVRP. The authors considered the time window constraints essential to get the problem closer to a real-world logistics application.

To solve the three problems, we used an algorithm based on the General Variable Neighborhood Search (GVNS) metaheuristic, named Flexi-VNS. The local search is the main difference between our proposed algorithm and the classical heuristic. Flexi-VNS local search is done through the Randomized Variable Neighborhood Descent (RVND) procedure, a Variable Neighborhood Descent (VND) variant [Mladenović and Hansen, 1997]. To evaluate the performance of the Flexi-VNS, we use instances proposed by Erdoğan and Miller-Hooks [2012] and adapted by Koç and Karaoglan [2016] to test it on the EVRP. The instances created by Augerat et al. [1996], Rochat and Taillard [1995], Rinaldi and Yarrow [1985], and Golden et al. [1998] and adapted by Yang

and Sun [2015] to test our algorithm on the BSS-EV-LRP. Finally, the instances used by Schneider et al. [2014] to test the algorithm on the E-VRPTW. The Flexi-VNS results were compared with the best-known values reported in the literature. The Flexi-VNS algorithm demonstrated efficiency in finding high-quality solutions to the problems. It found almost all optimal solutions from small-scale instances of the EVRP and BSS-EV-LRP and all the optimal solutions of the E-VRPTW instances. We also found high-quality solutions in larger benchmark instances.

2. The Electric Vehicle Routing Problems

The problems researched in this thesis are the EVRP and two of its variants: the BSS-EV-LRP and the E-VRPTW. This section describes the EVRP, the BSS-EV-LRP, and the E-VRPTW and points out the characteristics of each one.

The EVRP is a variant of the classic VRP that includes Electric Vehicles (EVs) and their dependencies. This section presents a standard model of the EVRP. Over the years, variants have been researched, and these variants incorporate several characteristics related to EVs, such as types of charging stations, energy consumption uncertainty, total energy consumed, heterogeneous EVs, and non-linear charging functions. Characteristics of the VRP are incorporated, such as multiple depots, time windows, and pickup and delivery constraints [Qin et al., 2021]. The BSS-EV-LRP is a variant of the EVRP that requires synchronously determining the Battery Swap Station (BSS) locations and the EV routes to minimize the sum of the total travel cost and the cost of constructing the BSSs [Hof et al., 2017]. Accordingly, in the former, there is not only a unit shipping cost associated with any EV route but also a fixed construction cost associated with establishing a station at each candidate BSS. In the BSS-EV-LRP, it is assumed that (a) the BSSs do not have capacity constraints, which means that different EVs can perform a swapping operation at the same time and at the same station if necessary and (b) either a BSS exists at the depot or enough time and swapping ports exist at the depot so that every EV has sufficient charge to start each route with a fully charged battery [Yang and Sun, 2015]. The E-VRPTW is another variant of the EVRP that introduces a time window as an additional constraint to the problem. Another difference in this variant is that the charging stations have their location previously defined. Therefore, there is no cost to build the charging stations [Schneider et al., 2014]. The mixed-integer linear programming formulation for the problems and their details are described in Souza [2024].

3. Solution Approach

Since the mathematical programming-based solvers do not solve large-scale instances of the problems under investigation, we developed a metaheuristic-based algorithm to obtain high-quality solutions. The developed algorithm is a Variable Neighborhood Search (VNS)-based algorithm adapted from the classic VNS by Hansen and Mladenović [2001]. Named Flexi-VNS, this algorithm is flexible enough to solve the three variants of the EVRP and can also easily be adapted to address other alternative variants.

Flexi-VNS explores the solution space of the problems using shaking and local search operators specifically designed to consider the knowledge of the problems. The local search operators were used with the Randomized Variable Neighborhood Descent (RVND) strategy proposed by Souza et al. [2010] and Subramanian et al. [2010]. This strategy has the advantage of not requiring calibration of the order of the neighborhoods, which would be necessary in local search algorithms based on the classic VND. Moreover, not only is it prohibitive to find the best order of neighborhoods in a large set of neighborhoods (10, for the problems considered), but the best order for one instance is not always the best for another instance of the problem. If the solution refined by the local search improves the incumbent solution, the latter is updated and, following the ideas presented in Penna et al. [2019], the routes that have been improved are submitted to a new intensification procedure with intra-route moves.

Following the recommendations of Vidal et al. [2013], we explore the whole solution space of the problems, including infeasible solutions. So, solutions that do not respect the vehicle load or battery capacity are allowed and handled through a dynamic penalty mechanism in the problem evaluation function.

The full description of the proposed algorithm is detailed in Souza [2024].

4. Computational Experiments

This section presents the results and analysis of the proposed solution approach on benchmark instances of the EVRP, BSS-EV-LRP, and E-VRPTW. The algorithm was implemented in C++ and executed on an Intel Core i7-6700HQ CPU @ 2.60 GHz x 8 with 8GB RAM. Subsection 4.1 presents the results obtained by executing our proposed algorithm on the literature instances. Subsection 4.2 shows the results of the statistical analysis comparing the proposed algorithm with other algorithms in the literature.

4.1. Computational Results

This section presents the computational results of the Flexi-VNS algorithm applied to the classical EVRP, the BSS-EV-LRP, and the E-VRPTW. To assess the quality of the proposed algorithm, we compare our computational results for EVRP instances with those reported by Erdoğan and Miller-Hooks [2012], Schneider et al. [2014], and Koç and Karaoglan [2016]. For evaluating the performance of Flexi-VNS on the BSS-EV-LRP instances, we compare our results with those reported by Yang and Sun [2015], Hof et al. [2017], and Affi et al. [2020]. Additionally, to evaluate Flexi-VNS on the E-VRPTW, we compare our results against those presented by Schneider et al. [2014], Goeke and Schneider [2015], Hiermann et al. [2019], and Keskin et al. [2021]. Details about all computational results are available in Souza [2024].

Table 1: Summarized Flexi-VNS Results.

Benchmark Problem	Number of instances	Number of BKS instances Equaled or Improved	Avg Best Solution gap	Avg Solution gap (10 runs)
EVRP	52	47	0.02	0.31
BSS-EV-LRP	30	15	-8.88	-6.92
E-VRPTW	92	65	0.30	0.82

The EVRP instances consist of five benchmarks. They are composed of four scenarios with up to 20 customers and one benchmark with real-world data. The Flexi-VNS algorithm outperformed the other methods used to evaluate our algorithm. Considering the small instances, our approach found all best-known solutions (BKSs) except for three. However, the gap for these instances was less than 0.01%. The computational time of the Flexi-VNS algorithm was always lower than 2 seconds, with an average of 0.86 seconds over all scenarios. The results show that Flexi-VNS has no difficulty solving larger instances based on real-world data. The Flexi-VNS improves 7 of the 12 BKS in the literature, with an average gap of 0.08%. This average gap did not include instance *300c*, because the Flexi-VNS can serve two more customers than the other algorithms; thus, the results are not comparable. Although the computational time spent by the Flexi-VNS algorithm was limited to that of VNS/Tabu Search (TS), it still did not need all the available time to find better-quality solutions.

Regarding BSS-EV-LRP instances, the Flexi-VNS performed well. The algorithm found three BKS and improved the results in three instances with an average gap of 0.38% on the adapted instances of Augerat et al. [1996]. The comparison results of Rochat and Taillard [1995] instances the Flexi-VNS obtained five new solutions, with an average gap of 0.24%. Concerning the number

of BSSs, the Flexi-VNS solutions used BSSs in only two instances. Regarding the running time, we use the adjusted time as a stopping criterion in our algorithm. However, when executed without this limitation, the Flexi-VNS improved 8 out of 12 instances. The values of these improved solutions are presented in the Appendix of the thesis. On the other hand, the computational results about Rinaldi and Yarrow [1985] instances show that our algorithm improved the solution values of all these literature instances, with an average gap of -26.62%, and had the lowest running time. Concerning the adapted Golden et al. [1998] instances, the Flexi-VNS average gap was -9.51%. The Flexi-VNS was faster on average compared to the adjusted time than the others. Regarding the number of constructed BSSs, Flexi-VNS strongly reduced this number compared to other algorithms.

The Flexi-VNS algorithm successfully identified all the BKS for small-size benchmark instances across three classes reported by Schneider et al. [2014]. Our proposed VNS algorithm demonstrated excellent performance, matching all BKS solutions reported in the literature and achieving good CPU times. Schneider et al. [2014] also proposed instances with 100 customers in the same three classes: C, R, and RC. In class C instances, Flexi-VNS identified 15 out of 17 best-known solutions reported in the literature, with an average gap of 0.15%. In class R, Flexi-VNS improved the results for two instances and matched the BKS for the other three, with an average gap of 0.80%. In class RC, our algorithm improved five instances and equaled the BKS for the other four, with an average gap of 0.83%. Overall, Flexi-VNS performed best in classes C and RC; it successfully matched 22 BKS and improved 9 out of 56 BKS values reported in the literature.

4.2. Statistical Analysis

We performed separate statistical analyses to evaluate whether there is a statistically significant difference between our algorithm (Flexi-VNS) and the best algorithms from the literature for each problem. For the EVRP, the VNS/TS authors did not provide average results, so we executed statistical tests using only the gaps calculated from the best solutions. Since the data from these tests did not follow a normal distribution, we employed the non-parametric U-test. The test results indicated no significant difference between the algorithms, with a p-value of 0.15. For the BSS-EV-LRP, where the best algorithm from the literature is Adaptive Variable Neighborhood Search (AVNS), we analyzed the Flexi-VNS gaps, considering both our algorithm's best gaps and the average gaps. The data also did not follow a normal distribution, so we used the non-parametric U-test. The results showed a p-value of 0.63 for comparing the best gaps and a p-value of 0.08 for comparing the average gaps. As the p-values were greater than 0.05, this statistical test did not indicate a statistically significant difference between the AVNS and Flexi-VNS algorithms, considering a 95% confidence level. For the E-VRPTW, where the best algorithm from the literature is Adaptive Large Neighborhood Search (ALNS), the statistical analysis used the Flexi-VNS gaps, considering our algorithm's best gaps. This analysis focused on data from larger-scale instances. The data also did not follow a normal distribution, so we used the non-parametric U-test. The results showed a p-value of 0.16. As the p-value was higher than 0.05, this statistical test also did not indicate a statistically significant difference between the Flexi-VNS and ALNS algorithms, considering a 95% confidence level.

Despite not demonstrating a statistically significant difference between the EVRP, BSS-EV-LRP, and E-VRPTW algorithms, the Flexi-VNS algorithm showed superior performance in several aspects. The Flexi-VNS algorithm reached the optimal values for almost all small-scale instances of the EVRP and discovered new optimal solutions for most large-scale instances. It generally achieved these results in less computational time. We also found at least one new best solution for each set of BSS-EV-LRP instances, which required less computational time for most instances. It also provided new results that were not previously reported in the literature. For the E-VRPTW, we improved nine of the BKS from the literature and equaled the other 22 BKS values.

Overall, the Flexi-VNS algorithm has demonstrated its effectiveness across different problem sets. It has proven to be particularly efficient in improving or matching the best-known solutions while requiring less computational time. These results validate the potential of Flexi-VNS as a robust and efficient algorithm for solving complex electric vehicle routing problems.

5. Conclusion

In this study, we explore several optimization problems related to EVs to define a distinct class of the EVRP. We focus on three problems: the classical EVRP, the BSS-EV-LRP, and the E-VRPTW. While the classical EVRP is a well-established problem, its variants have not been extensively studied. These variants add complexity by requiring the determination of battery swap station locations and vehicle routes simultaneously, under battery range constraints and customer time windows. The BSS-EV-LRP and E-VRPTW are particularly relevant for electric car-sharing and taxi services. To address these problems, we propose a GVNS-based algorithm named Flexi-VNS. This approach incorporates a EVRP method as the local search procedure, which includes an intra-RVND process applied exclusively to modified routes. Our goal was to compare the Flexi-VNS performance against the best algorithms from existing literature for each problem. Thus, we first evaluated Flexi-VNS performance using 52 EVRP instances, consisting of 40 small-scale and 12 large-scale real-world instances. Additionally, we tested the Flexi-VNS on 44 BSS-EV-LRP instances and 92 E-VRPTW instances, which include 36 small-scale and 56 larger-scale instances. We conducted separate statistical analyses to determine if there was a significant difference between the performance of Flexi-VNS and the best literature algorithms for each problem (EVRP, BSS-EV-LRP, and E-VRPTW). Using the non-parametric U-test at a 95% confidence level, we found no statistically significant difference. In computational tests on small-scale EVRP benchmark instances, our proposed solution approach demonstrated superior performance compared to exact algorithms from the literature. Flexi-VNS found all the optimal solutions known in the literature in significantly shorter running times. Furthermore, Flexi-VNS outperformed another VNS-based algorithm from the literature, achieving all best-known solutions except in three cases, where the deviation was less than 0.01%. For large-scale real-world-based instances, we improved 7 out of the 12 BKSs with an average gap of 0.08%. On the BSS-EV-LRP benchmark instances, our algorithm equaled or improved the values of 15 previous BKSs provided by SIGALNS, AVNS, and SGVNS algorithms. Furthermore, Flexi-VNS reduced the number of constructed BSSs compared to results from the literature. Additionally, Flexi-VNS provided solutions for 14 BSS-EV-LRP instances, that had not been previously solved by others' works. Regarding the E-VRPTW benchmark instances, Flexi-VNS found all the optimal solutions already known in the literature. For larger-scale instances, our algorithm improved 9 BKSs and equaled 22 other BKS values from the literature. In summary, Flexi-VNS was able to equal or improve the BKS values for 47 out of 52 instances available for the EVRP, 18 out of 30 instances with available results in the literature for BSS-EV-LRP, and 65 out of 92 instances of E-VRPTW. This corresponds to success rates of 90.38% for EVRP instances, 75% for BSS-EV-LRP instances, and 70.65% for E-VRPTW instances. In conclusion, the experiments and analyses indicate that the Flexi-VNS algorithm is a robust approach for solving various EVs routing problems. Although the U-test did not show a significant statistical difference compared to existing algorithms, the detailed component and neighborhood impact studies provide valuable insights into the algorithm's efficacy and optimization potential.

6. Products

The thesis generated two publications: an article in a Qualis A1 journal [Souza et al., 2024a] and the Roberto Dieguez Galvão Award for the best article in English at the LVI Brazilian Symposium on Operational Research [Souza et al., 2024b].

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