

A Decision Support System for Optimizing Crews in Rowing Championships

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ABSTRACT

Due to technological advancements, using computational tools to solve problems is increasingly common. In sports, the scenario is no different. In Master's rowing regattas, allocating athletes in competitions is crucial to the club's performance in championships. We proposed an integer programming formulation for allocating athletes in Masters Rowing Regattas to get the best possible results in each race. We solved this problem using CPLEX to find optimal solutions. We created a support decision tool with a graphical interface that allows users to enter data, obtain solutions, and test several configurations. This tool enables coaches to find solutions in less time and test several parameters much more straightforwardly than manually doing these activities.

KEYWORDS. Integer Programming. Scheduling. Recreation/sports. Rowing.

Paper topics: ESP-PO in Sports. OD-Discrete Optimization

1. Introduction

Due to technological advancements, it is increasingly common to use computational tools to solve problems, and such use is even becoming indispensable in some areas.

In sports, the scenario is no different. As it is a context of increasing competitiveness, it is widespread to use computational applications to assist training programs, monitor the performance of athletes and teams, and even assist referees and judges in supervising competitions.

Rowing Masters Regattas comprises several events with rowers aged at least 27 years over a distance of 1000 meters. Up to eight boats start simultaneously with a few minutes' interval between races to determine the winning teams in up to 13 different age groups, each with up to eight boat classes. Rowing races can have boats with only men or women or crews of both sexes [World Rowing, 2024]. The regattas may last one to four days; therefore, there are hundreds of events, and the clubs have to define the participation of crews for each event. It is not unusual that the clubs have tens of rowers who have to be allocated to the events, respecting the restriction of age, gender, and personal preferences of the athletes. Furthermore, the same athlete can be scheduled to participate in several events, which, in most cases, take place on the same day, making it necessary to observe the athlete's rest and preparation between participation in events. Defining all crews is a challenging and laborious task to do manually, so the clubs perform well.

We conducted a literature review, and we did not find any tool to assist in decision-making regarding the allocation of rowers in masters competitions. We found a few works about using mathematical programming to allocate athletes to competitions, aiming to perform well. In athletics, Bailey and Nowak [Bailey and Nowak, 2018] proposed a mixed integer programming model to solve the problem of assigning a team member to athletics events. The objective is to maximize the number of points in the events or the chance of victory. They also proposed a decision-making assistance software to solve the model using the CPLEX tool. This work has the same restriction as our problem on the maximum number of participations that an athlete can have per championship. Nowak et al. [Nowak et al., 2006] proposed a binary integer model that determines an overall assignment that maximizes the total number of points scored by swimming squads based on the times for swimmers on the team and for the expected opponent. In addition to considering the number of participations, this solution contemplates multiple participations in the same event and the team size. Finally, Simona Mancini [Mancini, 2018] proposed two different integer programming models, based on estimating opponents' performance capability. The first model maximizes the total score obtained by the team, and the second model aims to optimize the placement achieved by the team in the final meeting ranking and the advantage over the first follower in the ranking. It has in common with the rower's problem the restrictions related to the number of participations of each athlete and multiple participations in the same event. However, the proposed models do not consider gender, age, or rest between participations of the same athlete, which are part of the challenges of this problem that will be presented below and, subsequently, the proposed solution.

In this way, we propose an integer programming formulation [Maculan and Fampa, 2006; Wolsey, 1994] for allocating athletes to the Masters Rowing Championships. This model respects the restrictions of each rowing event to maximize winning gold, silver, and bronze medals in championship events. We also developed a tool with a graphical interface that uses the IBM Cplex ILOG CPLEX Optimization Studio [2021] to solve this model and find an optimal solution for the problem. This tool receives the data related to the event and athletes and presents the proposed allocation of athletes. It allows several parameters to be changed, making it possible to test several configurations quickly.

Section 2 describes the problem, and section 3 presents the mathematical model. Section 4 specifies the graphical interface. Section 5 shows the computational experiments and obtained

results. The conclusions are in section 6.

2. Assignment of rowers in master regattas

Masters regatta is a category of the rowing sports competition for athletes aged 27 years or more [Brazilian rowing confederation , 2018]. This sporting competition comprises one or more events that can occur on the same day or different days [World Rowing, 2024].

Each race is held with a type of boat with 1, 2, 4, or 8 seats. In addition to the number of seats, there are boats where athletes use one or two oars. Another aspect that defines a type of boat is the presence of the coxswain, who, despite being part of the crew, does not row but only coordinates the other athletes. Boat types are represented by a number, which means the size of the crew, by the letter “x,” which indicates that each rower uses two oars, and by the sign “+” or “-”, which specifies that each rower uses one oar with the presence (“+”) or absence of the coxswain (“-”). Although there are coxswains on some boats, their age and gender are not considered in the boat class, and there are no restrictions in allocating a coxswain, so this work does not address their allocation in solving the problem. In addition to the type of boat, the races may require the crew to be exclusively male, exclusively female, or mixed. In the case of mixed races, the crew must have the same number of athletes of each gender. Table 1 shows all boats in the master category [Brazilian rowing confederation , 2018].

Boat type	Gender
1X	Male and Female
2X	Male, Female and Mixed
2-	Male and Female
2+	Male and Female
4X	Male, Female and Mixed
4-	Male and Female
4+	Male and Female
8+	Male, female and Mixed

Table 1: Boat Types on Master Regattas

Class	Average Age
A	27
B	36
C	43
D	50
E	55
F	60
G	65
H	70
I	75
J	80
K	83
L	86
M	89

Table 2: Age Classes

The races also have categories related to the average age of the crew. There are 13 (thirteen) age categories for races called by capital letters A to M, each indicating the minimum average age each boat must have. Table 2 presents the categories and age groups. All athletes have to be older than 27 years old. To calculate the average age of the crew, the age each athlete will reach at the end of the tournament year is considered.

In addition to the rules restricting participation in races, it is important to consider the number of races they would like to participate in when allocating rowers. The coach and athletes define a minimum and maximum value for the number of races of each athlete.

The lower limit must always be met, but any intermediate value between the lower and upper limits is a valid solution.

Another critical aspect of allocating rowers is the interval between participation in races. This time must be considered as the athletes' performance may be negatively affected if they participate in races very close to each other. Furthermore, the athletes must have enough time to prepare for their next competition and go to the race location.

The objective of the problem is to allocate the rowers to crews, maximizing the chance of winning the race. Therefore, we had to define how to evaluate the quality of a team. Each athlete's score is between 0.0 and 1.0, which denotes the probability of the athlete winning the race. Assigning specific weights to each medal won (1.0 for gold, 0.5 for silver, and 0.25 for bronze) allows us to correlate the quality of the boat's team with the boat's expected performance in the race.

So, each athlete's initial score is defined based on their performance and age. The trainer may define the performance score by comparing their racing time with the times achieved by athletes in similar competitions [World Rowing]. As the quality of the crew is an average of the athletes' scores, we modified each athlete's initial score in a crew according to their age difference within the age category of the race in question. A penalty is subtracted from the athlete's score each year their age is above the category. Similarly, the value of the penalty is added if the athlete is younger than the age category. Finally, the average of the adjusted scores for each athlete is calculated, thus obtaining the crew's score. The sum of the scores of all crews is used to compare the quality of different allocation solutions, so that the highest value indicates the best solution.

3. Mathematical programming formulation

In this section, we present a mathematical formulation to model the problem. First, we show the indices and sets used.

- $R = \{1, \dots, r\}$ is the set of all rowers in the competition subdivided into R_m and R_w representing, respectively, the set of male and female rowers, so that $R = R_m \cup R_w$.
- $E = \{1, \dots, e\}$ is the set of races ordered by the times they occur.
- B_j represents the number of boats available for a race $j \in E$.
- a_i represents the athlete's age $i \in R$.
- pg_i is the value of the rower's performance level $i \in R$.
- l_i and u_i indicate the lower and upper limits of the number of races to which the rower $i \in R$ should be allocated.
- η represents the minimum number of events an athlete needs to rest and prepare to compete again.

- m_j and w_j define the number of male and female rowers in a race $j \in E$.
- g_{ij} is the rower's level of performance $i \in R$ in the competition $j \in E$.

Next, we describe the decision and auxiliary variables.

- $x_{ijk} \in \{0, 1\}$, 1 if a rower $i \in R$ is participating in a race $j \in E$ in a boat $k \in B$, and 0 otherwise.
- $y_{jk} \in \{0, 1\}$, 1 if a crew participates in a race $j \in E$ on a boat $k \in B$, and 0 otherwise.
- $b_{jk} \in [0, 1]$ is the average of the crew's adjusted performance score in race $j \in E$ in the boat $k \in B$.
- $avg_{jk} \in R$ is the average age of athletes on a boat $k \in B$ in a race $j \in E$

The aim of solving this problem is to define the crews with the highest scores, in other words, the highest chance of winning the races, following the restrictions described in section 2.

Therefore, we developed the following programming formulation.

$$\max \sum_{j \in E} \sum_{k \in B} b_{jk} \quad (1)$$

Subject to:

$$\sum_{i \in R_m} x_{ijk} = m_j \cdot y_{jk}, \quad \forall j \in E, \forall k \in B \quad (2)$$

$$\sum_{i \in R_w} x_{ijk} = w_j \cdot y_{jk}, \quad \forall j \in E, \forall k \in B \quad (3)$$

$$\sum_{k \in B} x_{ijk} \leq 1, \quad \forall i \in R, \forall j \in E \quad (4)$$

$$l_i \leq \sum_{j \in E} \sum_{k \in B} x_{ijk} \leq u_i, \quad \forall i \in R \quad (5)$$

$$\sum_{i \in R} \frac{a_i \cdot x_{ijk}}{(m_j + w_j)} \geq avg_{jk} \cdot y_{jk}, \quad \forall j \in E, \forall k \in B \quad (6)$$

$$\sum_{q=0}^{\eta} \sum_{k \in B} x_{i(j+q)k} \leq 1, \quad \forall i \in R, \forall j \in \{1, \dots, e - \eta\} \quad (7)$$

$$b_{jk} \leq \sum_{i \in R} \frac{g_{ij} \cdot x_{ijk}}{(m_j + w_j)}, \quad \forall j \in E, \forall k \in B \quad (8)$$

The objective (1) is to maximize the sum of all scores of the boats allocated to races. Constraints (2) guarantee the adequate number of male rowers in the races, and constraints (3) guarantee the number of female athletes. Constraints (4) prevent an athlete from being allocated more than once in the same race. Constraints (5) limit the number of event participation. Constraints (6) guarantee that the average age of the athletes allocated to a boat in a race is equal to or larger than the category age of the race. Constraints (7) guarantee the stipulated rest interval of an athlete between races. Constraints (8) guarantee that the boat's score is correctly set.

4. User interface

The user interface was developed in the C++ programming language using the GTKmm library in version 4.0[Cumming, 2010]. This library is an interface for using GTK together with C++.

The interface initially allows the user to enter the penalty value to be applied to each athlete's score related to the event category to which they are allocated. Furthermore, it is possible to specify the rest interval for athletes between races. The technical team defines these two parameters.

After accepting the initial settings, the user can access a screen with a form for entering the regatta races. In this form, it is possible to add and remove races from the list and enter information such as identifier (ID), date, time, gender, class, places, type of boat, and number of boats available for the race. After, the user can click the "next" button to have the data verified. The software checks the following restrictions: the uniqueness of the ID (race identification number), the gender of the race cannot be mixed if the crew size is equal to 1, the date field cannot be left blank, and the date entered must comply with the format containing the day followed by the character "/" and the month; however, as the date does not include the year, it is considered that February may have 29 days. Still, on the date, the number referring to the month is also checked and cannot be negative or greater than 12. Finally, the time entry field also has restrictions and cannot be left blank. The time format can be hours immediately followed by minutes or separated by a colon (":"). Furthermore, the number of hours and minutes cannot be negative or greater than 23 for hours and 59 for minutes. If the check finds errors in the data, the fields are marked to assist the user in correcting the entry.

Alternatively, the user can load race data from an external file through the application menu. The files should have the extension ". csv", a spreadsheet file with data separated by a particular punctuation. If the program finds an error in the file, it can save a base file in the format that the program accepts. It is also possible to save the race data the user inserted in an external file through an option in the program menu.

A screen is available to enter the athletes' information if the race data is valid. On this page, the user can add and remove athletes from a list and enter information about the athlete's name, minimum participation, maximum participation, gender, age, and performance score. The user can also load a spreadsheet with athletes' data from an external file. Furthermore, saving the athletes' information in an external file is possible. When completing the insertion of athletes, the user must click the next button for the data to be validated. The name cannot be blank in this form, and the minimum number of entries cannot exceed the maximum. Fields with incorrect data are flagged.

The user goes to the allocation page to see if the athletes' data is valid. On this page, it is possible to pin or block an athlete from a race. The user can add as many restrictions as they want, and in each of them, select the race and the athlete, and specify whether they will be fixed or prevented from being allocated to the race in question. Alternatively, this page can be left blank, indicating that athletes will be allocated following only the model restrictions. Then, the user can proceed to the next step by clicking the next button. The information will be validated according to the following competition rules: the same athlete can not be fixed more than once in the same race, the same athlete can not be blocked and fixed in the same race, the minimum and maximum number of participations of each athlete have to be respected, allocations cannot exceed the available places for the athlete's gender in a race. The field with incorrect information is flagged, and when the user hovers the mouse cursor over it, the wrong information is displayed.

After the data on the page for athletes pinned or blocked in races is validated, a summary screen shows the information's status. For each of the previously viewed pages, a status is displayed indicating whether there is incorrect information, whether the data is valid, or whether there are

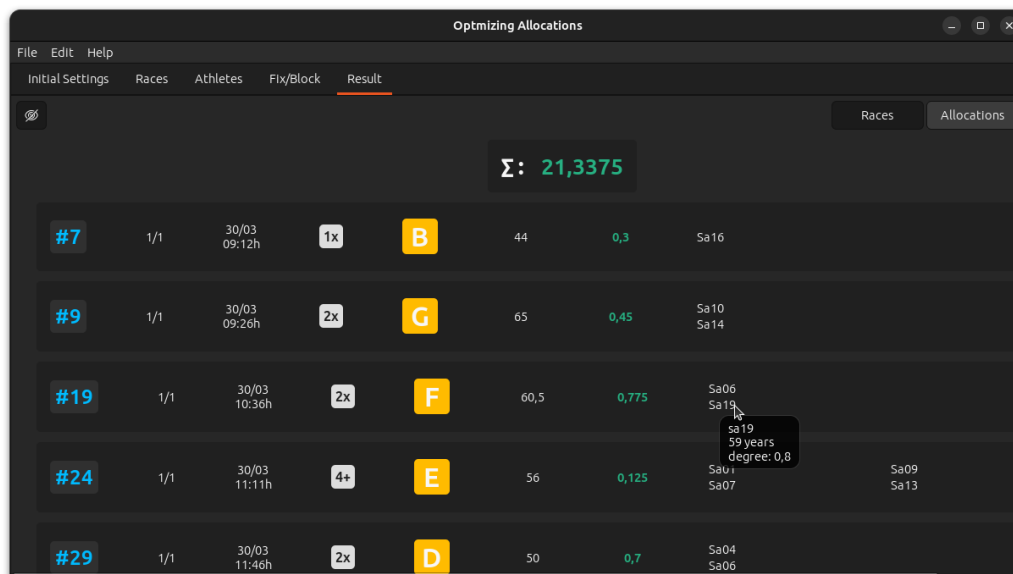


Figure 1: Result screen presented by the graphical user interface

modifications to the data that have not yet been validated. The user can click the button to calculate the result if all the information is valid.

If the solver does not find a result, a warning is displayed so the user can check the data and try again. Otherwise, the results screen is displayed. This screen shows all the races and the allocations in each of them. In the graphical interface, a button allows switching between viewing all races and viewing only races with allocations. The sum of all the boats' performance scores is displayed on the solution screen.

Figure 1 shows an example of the result screen. It is possible to see the sum of the solution's performance levels at the top and some races of the solution. Each line on the solution screen represents an allocation in a race. As the option to only show races with participation is activated, as indicated by the icon in the upper left part of the image, all races have a crew. Each line of the table presents, from left to right, the event identifier, the boat to which the allocation refers, the date and time of the event, the type of boat, the class, the average age of the crew, the average performance degree of the crew and the names of the athletes allocated to the race in question. The figure also illustrates that when hovering the cursor over the athlete's name, information about the athlete's name, age, and level of performance is shown.

Additionally, there is a table in which each line contains race information. This information includes identification of the event, identification of the boat, date and time of the race, class of the race, average age of the crew, performance rating of the crew, and name of the allocated athletes. Hovering the mouse cursor over each element in the results table allows for discriminating this information. In addition to knowing what information each cell deals with when hovering the mouse cursor over the category, the user can see the age range of the category. Furthermore, when hovering the mouse cursor over the athletes' names allocated to the tests, the age and the athlete's performance degree are displayed.

It is also possible to view the summary of allocation information, in which a user can view the number of times each athlete was allocated to races and compare this value with the minimum and maximum participation displayed.

When obtaining a solution, the user can save the complete result in an external file con-

taining all the races, including those without participation. Furthermore, the user may save the result in a more compact format containing only information from the participation races. Finally, a user can save the list with a summary of the rowers' allocations, containing the minimum and maximum allocations and total participations suggested by the solution. The saved files are spreadsheets with ". csv" format.

In addition to the pages discussed previously, the graphical interface has a menu with commands for saving and loading external files. Some options allow saving base files for creating an external file of events and athletes. Also, an option displays information about the software and its version.

5. Experimental Results

We tested the model with data from athletes from the Piraquê Club races from the South American Master Rowing Championship in 2022 and the Brazilian Master Rowing Championship in 2024. The club's technical team scheduled the first manually; the second was planned using the tool developed in this work.

The athletes' rest interval was five events, and the age penalty was 0.05, as defined by the technical team. The allocations presented below were calculated on the Linux operating system version 23.04 running on a virtual machine with 4 GB and two 3.7 GHz processors using CPLEX version 22.1.1 running in default mode.

The South American Masters Rowing Championship 2022 contained 170 events in three days. The club had to allocate 23 athletes, as displayed in Table 3. Each row of the table shows the athlete's identifier, the minimum and maximum number of races the athlete wants to participate in each day, the gender, age, and degree. The technical team defined each athlete's degree based on the athlete's performance in past competitions. The first test was to run the model, restricting the participation of one boat in each race. In this execution, 4,080 variables and 8,303 restrictions were created, and the software found the solution in 0.28 seconds. The function objective value was 21,8375. The second test was to run the model, allowing the participation of up to two boats in each race. In this execution, 8,160 variables and 8,983 restrictions were created, and the software found the solution in 0.35 seconds. The function objective value was 25,3.

The function objective value is greater when two boats are allowed in each race. However, upon inspection of the solution, the second boat in the same race usually has a degree lower than 0.5. The technical team analyzed these crews and verified they had a low chance of winning a medal. Therefore, one way to use this tool as an efficient decision support tool is to find an initial solution allowing just one boat in each race. Then, the solutions are analysed, and the crews with good quality scores are separated. Next, execute the program again, obliging these crews to be present and allowing the tool to choose the crews for the other races. In this way, we try to satisfy two objectives: achieve good results and have all athletes participating in regattas.

The first solution, restricting one boat in each race, obtained 44 participations, whereas 30 had a degree greater than 0.5, which is a good solution. However, this solution did not meet some of the specific, real particularities of the problem. Some people wanted to row with other particular people. The coach already thought that some events may have two boats, and others do not. The club invited some rowers as guests to row in specific races.

Therefore, a third test was performed with some of these characteristics. All races have a maximum of one boat, except for five races that may have two boats. Three races have one fixed crew, and four have one guest fixed as one crew member. The technical coach team defined this configuration as more similar to the requisites used in the manual scheduling. In this execution, 3,872 variables and 7,658 restrictions were created, and the software found the solution in 0.35 seconds. The objective function value was 23,3.

Athlete	Day 1	Day 2	Day 3	Gender	Age	Degree
1	1 - 3	1 - 3	0 - 2	M	59	0.2
2	1 - 3	1 - 3	0 - 2	M	59	0.8
3	1 - 1	0 - 0	0 - 0	M	67	0.2
4	0 - 0	1 - 1	0 - 0	M	37	0.5
5	1 - 1	0 - 0	0 - 0	M	38	0.6
6	0 - 0	1 - 1	0 - 0	M	37	0.5
7	1 - 3	1 - 3	0 - 2	M	51	0.7
8	1 - 3	1 - 3	0 - 2	M	62	0.8
9	1 - 3	1 - 3	0 - 2	M	48	0.1
10	1 - 3	1 - 3	0 - 2	M	56	0.4
11	1 - 3	1 - 3	0 - 2	M	58	0.3
12	1 - 3	1 - 3	0 - 2	M	63	0.1
13	1 - 1	0 - 0	0 - 0	M	53	0.7
14	1 - 3	1 - 3	0 - 2	M	59	0.1
15	1 - 3	1 - 3	0 - 2	M	67	0.8
16	0 - 0	1 - 1	1 - 1	M	58	0.7
17	1 - 1	0 - 0	0 - 0	W	60	0.8
18	1 - 3	1 - 3	0 - 2	W	44	0.7
19	1 - 3	1 - 3	0 - 2	W	57	0.9
20	1 - 3	1 - 3	0 - 2	W	48	0.5
21	1 - 3	1 - 3	0 - 2	W	59	0.8
22	1 - 3	1 - 3	0 - 2	W	49	0.3
23	1 - 3	1 - 3	0 - 2	W	56	0.6

Table 3: Athletes performing on South American Master Rowing Championship

The technical team analyzed these results. The manual scheduling defined 35 participations, and the tool got 47, allowing more rowers to participate in races. The tool solution defined crews for 31 races as the manual, with 26 having similar qualities and five different ones. The tool defined 16 new participations and did not find four participations defined manually.

The technical team defined the club's performance by giving each medal a weight of 1, 0.5, and 0.25. The club won 12 gold medals, eight silver medals, and four bronze medals in this competition, for a total of 17. The objective function value of the manual scheduling was 15.3, close to 17. This result shows that the definition of the objective function makes its value related to performance. Table 4 contains the function objective value obtained by the three test configurations and manual scheduling.

Type of Scheduling	Funtion objective value
Tool with one boat	21,8375
Tool with two boats	25.3
Tool with one and two boats	23.3
Manual	15.6

Table 4: Results using the tool and manual results for the South American Master Rowing Championship

The three values obtained by the tool are greater than those obtained by manual scheduling. As we have already discussed, the results obtained by the tool using one and two boats are more realistic. Therefore, the club had better results using this setup.

The technical team used the tool to define the scheduling for the Brazilian Masters Rowing Championship with 170 races. The club opted to enroll 22 rowers optimally.

They quickly ran several configurations, changing the preferences and weights of the athletes and fixing some boats. In addition, some athletes decided not to participate close to the competition. Using the tool allowed the technical team to reconfigure the crews very quickly.

The function objective value obtained by the tool was 10.2. The club won six gold medals, six silver medals, and three bronze medals in this competition. Using the same weights of 1, 0.5, and 0.25 for gold, silver, and bronze medals, we have a value of 9.75, which is quite close to the weight of 10.2 obtained using the tool. This result confirms that the defined way to calculate the objective value leads to the results obtained. Also, the technical team may use the actual results to tune the degree of each athlete for future events, allowing even better predictions of expected results.

It is important to highlight that using the software greatly facilitated the possibility of generating variations in athlete allocation results, speeding up the change of input information. Before, the technical team manually prepared the list of participations in the Piraguê Regatta Club on a spreadsheet. This task was time-consuming and exhausting due to the number of events, athletes, and restrictions. Another advantage of using the software is the possibility of saving different solutions, allowing comparison with other crew formations. These facilities, combined with the feature of fixing crews and the quick calculation of the result, make the software a powerful tool for helping technical teams. Another advantage is related to the display of results, since on a single screen, it is possible to view the crews and obtain detailed information on each athlete and the championship races, making it much easier to judge the quality of the result and check the chosen allocations. In this way, we believe this tool considerably sped up the formation of crews to participate in championships, whether using the software's suggested solution entirely or even as a starting point.

6. Conclusions

The proposed model could develop a solution with optimal allocations respecting the problem restrictions. It proved to be an efficient tool to assist coaches of all experience levels in allocating athletes to competitions. The tool allows agility in forming crews that can be saved in external files to compare and collect solutions. The speed with which the solution is found enables the trainer to carry out experiments supported by the ease of modifying the input data that the graphical interface guarantees.

This tool may be easily improved by changing restrictions and the objective function. For example, an athlete's score is linearly penalized according to their age, and another type of function may be used. Also, with a few changes, it is possible to adapt the model and software to find solutions for other types of competitions with similar aspects to rowing competitions.

All the advantages the software provides are powerful tools to assist coaches and save considerable time, commonly used when listing athletes' race allocations. This activity is decisive in the result of the club's participation in championships.

The software is available upon request.

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