57º SBPO
SIMPÓSIO BRASILEIRO DE PESQUISA OPERACIONAL
Gramado • RS
DE 5 A 9 DE OUTUBRO DE 2025

OPTIMAL STRATEGY FOR *WOLBACHIA*-BASED BIOCONTROL DEPLOYMENT

Helenice de Oliveira Florentino
IB-Intituto de Biociências, UNESP
helenice.silva@unesp.br

Antone dos Santos Benedito, Claudia Pio Ferreira
IB-Intituto de Biociências, UNESP
antone.santos@unesp.br, claudia.pio@unesp.br

Daiver Cardona-Salgado, Lilian S. Sepulveda-Salcedo
UAO-Universidad Autonoma de Occidente, Cali, Colombia
dcardona@uao.edu.co, lssepulveda@uao.edu.co

Jose Luis Orozco Gonzales, Olga Vasilieva
UNIVALLE- niversidad del Valle, Cali, Colombia
jose.gonzales@correounivalle.edu.co,
olga.vasilieva@correounivalle.edu.co

ABSTRACT

Optimizing Wolbachia-Based Biocontrol Strategies for Arbovirus Control This work proposes two methods to determine optimal strategies for *Wolbachia*-based biocontrol, which involves releasing *Aedes aegypti* mosquitoes infected with *Wolbachia* bacteria to reduce diseases such as dengue, Zika and chikungunya. In this approach, two optimization models were proposed, which incorporate the dynamics of wild and infected mosquito populations, considering biological factors such as imperfect maternal transmission, incomplete cytoplasmic incompatibility, and loss of infection due to thermal stress. The objective is to find strategies that ensure stable establishment of infected mosquitoes while minimizing the release time and quantity. Two solution approaches were proposed: one using Pontryagin's Maximum Principle and the other based on a Genetic Algorithm. The results showed that the metaheuristic approach was more effective in generating cost-efficient and accurate strategies. However, both methods are promising tools for decision making and guiding the development of public health policy.

KEYWORDS. Dengue. Genetic Algorithm. Pontryagin's Maximum Principle.

SA and MH

PALAVRAS CHAVE. Dengue. Algoritmo Genético. Princípio do Máximo de Pontryagin.

SA and MH

https://proceedings.science/p/212243?lang=pt-br

1. Introduction

The intracellular endosymbiotic bacterium *Wolbachia* is not naturally present in wild *Aedes aegypti* populations, but it can be artificially introduced by microinjections into mosquito eggs. Mosquitoes carrying *Wolbachia* will hereafter be referred to as wolbitos. Female wolbitos have a significantly reduced ability to transmit dengue and other arboviral diseases. In addition, they gain reproductive advantages over uninfected mosquitoes through maternal transmission (MT) and the cytoplasmic incompatibility (CI) mechanism. However, these traits may be imperfect depending on environmental factors such as temperature, humidity, and photoperiod, which can influence the success of population replacement or the stable coexistence between wild and wolbitos, (Orozco-Gonzales et al. [2024], Pereira et al. [2018]).

Several theoretical and empirical studies have investigated the use of *Wolbachia* as a biological control strategy for the transmission of arboviruses, often supported by mathematical models based on differential equations to describe the dynamics of the mosquito population and assess the impact on vector-borne infections. However, most classical models assume perfect MT and CI, which limits their applicability under real-world conditions. More recent models have incorporated features such as imperfect transmission, thermal stress-induced loss of infection, and partial incompatibility, thereby extending their relevance to more diverse environmental scenarios, (Benedito et al. [2020]; Farkas et al. [2017]; Ferreira [2020]; Li and Wan [2019]).

In a previous study (Orozco-Gonzales et al. [2024]) we proposed a comprehensive model capable of realistically describing the dynamics of wild and wolbitos populations under varying ecological conditions. Building upon this foundation, a follow-up study was conducted (Orozco-Gonzales et al. [2025]) in which we propose two distinct approaches to optimize discrete and impulsive release strategies of wolbitos, aiming to achieve their long-term establishment with minimal cost and in the shortest possible time. The first approach adapts continuous-time dynamic optimization techniques to construct sequences of periodic impulses with variable release sizes. The second approach, novel in the current literature, formulates the optimization problem using the ϵ-Constraint Method and solves it via a metaheuristic based on a Genetic Algorithm (GA), enabling the determination of practical and optimized release strategies.

Both approaches allow for varying release frequencies and quantities, and are evaluated in terms of their effectiveness and practical feasibility under different climatic conditions and for two distinct *Wolbachia* strains (wMel and wMelPop). The main contribution of this work lies in the development and validation of a discrete and realistic optimization methodology based on genetic algorithms, capable of guiding more efficient and adaptable Wolbito release programs across diverse operational settings.

2. Modeling the Invasion Dynamics of Wolbitos

In the first approach of this research (Orozco-Gonzales et al. [2024]) a mathematical model was proposed that describes the temporal dynamics of two populations of *Aedes aegypti* mosquitoes: the wild population, indicated by $x(t)$ and the wolbitos population, denoted by $y(t)$, where $t \in [0, T]$. The spread of *Wolbachia* is governed by two key mechanisms: maternal transmission (MT) and cytoplasmic incompatibility (CI). In the MT mechanism, infected females transmit bacteria to their offspring with probability $\nu \in [0, 1]$. In the CI mechanism, matings between wolbitos males and wild females produce non-viable offspring with probability $\eta \in [0, 1]$.

Environmental stress, especially high temperatures, can cause these mechanisms to be imperfect. This leads to reduced values of ν and η, and possibly to the loss of infection in adult mosquitoes. The latter is modeled by a parameter $\omega \geq 0$, that denotes the per capita loss of Wolbachia infection in adult mosquitoes due to thermal stress.

https://proceedings.science/p/212243?lang=pt-br

In this way, the dynamics of the system are described by the following system of ordinary differential equations (Orozco-Gonzales et al. [2024]):

$$\frac{dx}{dt} = \rho_n x \left(\frac{x + (1-\eta)y}{x+y} + (1-\nu)\rho_w y \right) e^{-\sigma(x+y)} + \omega y - \delta_n x, \tag{1}$$

$$\frac{dy}{dt} = \nu \rho_w y e^{-\sigma(x+y)} - \omega y - \delta_w y + u(t). \tag{2}$$

Where ρ_n, ρ_w and δ_n, δ_w denote the average fecundities and death rates of the wild and wolbitos populations, respectively; $\sigma > 0$ is related to the intraspecific competition; $\nu \in [0,1]$ and $\eta \in [0,1]$ are the MT and IC probabilities, and $\omega \geq 0$ stands for the loss of *Wolbachia* in wolbitos due to thermal stress. Lastly, $u(t)$ is the control variable representing the release rate of wolbitos at time t, where $t \in [0, T]$.

Wild mosquitoes are typically assumed to be more fertile and have lower mortality than infected ones, ie, $\rho_n > \rho_w$ and $\delta_n < \delta_w$.

According to Orozco-Gonzales et al. [2024], the model introduces three reproductive metrics: $Q_x = \rho_n/\delta_n$ (average number of viable offspring per wild mosquito); $Q_y = \nu\rho_w/(\omega+\delta_w)$ (for infected mosquitoes); $Q_{y,x} = \frac{(1-\nu)\rho_w + \omega Q_y}{\delta_n}$ (wild offspring resulting from infected individuals).

A population is viable if its corresponding reproductive number exceeds one, i.e., $Q_x > 1$, $Q_y > 1$, and typically $Q_x > Q_y > 1$.

Orozco-Gonzales et al. [2024] also showed that in the absence of releases (i.e., $u(t) = 0$), the system may exhibit up to four equilibrium points:

$E_0 = (0,0)$: the trivial equilibrium, always unstable;

$E_x = (x^\sharp, 0)$: stable equilibrium representing a purely wild population ($x^\sharp = \dfrac{1}{\sigma}\ln Q_x$);

$E_s = (x_s, y_s)$: stable coexistence of both populations,

$$x_s = \frac{\ln Q_y}{2\eta\sigma}\left((Q_c - 1) - \sqrt{(Q_c-1)^2 - 4\eta\frac{Q_{y,x}}{Q_x}} \right), \quad y_s = \frac{1}{\sigma}\ln Q_y - x_s; \tag{3}$$

$E_u = (x_u, y_u)$: a saddle point separating the basins of attraction of E_x and E_s,

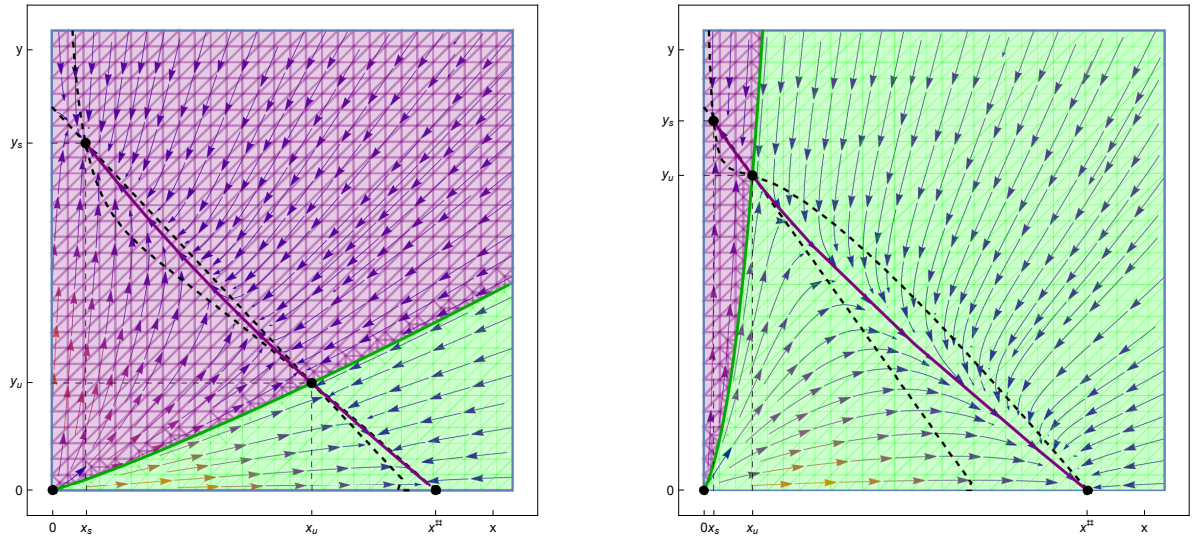
$$x_u = \frac{\ln Q_y}{2\eta\sigma}\left((Q_c - 1) + \sqrt{(Q_c-1)^2 - 4\eta\frac{Q_{y,x}}{Q_x}} \right), \quad y_u = \frac{1}{\sigma}\ln Q_y - x_u. \tag{4}$$

According to Orozco-Gonzales et al. [2024], under the conditions $\rho_n > \rho_w$, $\delta_n < \delta_w$, $Q_x > Q_y > 1$, and

$$Q_c := \frac{Q_{y,x} + Q_y + \eta Q_x}{Q_x} > 1 \quad \text{and} \quad Q_y - Q_{y,x} - 2\sqrt{\frac{Q_{y,x}}{Q_x - Q_y}} > 0,$$

two positive equilibria, E_s and E_u, may coexist. This gives rise to a pitchfork-type bifurcation. As illustrated in Figure 1 for the *wMel* and *wMelPop* strains and taking into account all the conditions discussed, the phase portrait presents a saddle point E_u, whose unstable manifold connects the wild-only equilibrium E_x to the coexistence equilibrium E_s. A curve separates the phase space into two attraction basins: one leading to E_x and the other to E_s. The nullclines (dashed lines) and the stable manifold of the origin E_0 further define the dynamics of the system.

According Orozco-Gonzales et al. [2024], in the idealized case where $\nu = 1$, $\omega = 0$, and η is sufficiently high, complete replacement of the wild population by infected individuals is possible.



(a) *w*Mel strain (b) *w*MelPop strain

Figure 1: Phase portraits of the system with $u(t) = 0$. Source: Orozco-Gonzales et al. [2025].

From a control standpoint, if the initial infected population $y(0)$ is large enough to be within the basin of attraction of E_s, the infection can establish successfully. For strains like *wMel*, a small release may suffice. For more virulent strains as *wMelPop*, either a single massive release or a periodic release strategy may be required.

The first approach of this research focused on establishing thresholds and analyzing the dynamic behavior required for a successful invasion of the wolbitos. The second approach aims to design optimal release strategies that minimize both the total number of wolbitos released and the duration of biocontrol application that allows the establishment of conditions for the system to reach the desired equilibrium between the wild and Wolbito populations, ensuring a successful transition of the system from a wild-dominated state to one in which the Wolbachia infection persists. In the next section, the optimization models proposed to achieve the objectives of this second approach will be presented.

3. Optimization Models

The objective is to drive the system from the initial state $\mathbf{E}_x = (x^\sharp, 0)$ to the region of attraction of the stable equilibrium $\mathbf{E}_s = (x_s, y_s)$, while minimizing both the total number of wolbitos released and the intervention time. This ensures the stability and coexistence of the wild and Wolbito populations. In this context, it is sufficient to drive the system to any point within the attraction region shown in Figure 1, (purple-colored region), and the system will naturally tend toward the equilibrium $\mathbf{E}_s$. To ensure this, we propose the following condition for the system:

$$x(T) < x_u \text{ and } y(T) > y_u, \tag{5}$$

where T is the final time of biocontrol application.

The decision variable $u(t)$ (control variable), representing the number of wolbitos released at time t, is constrained by

$$0 \leq u(t) \leq K, \tag{6}$$

where K represents the maximum number of wolbitos that can be released in each time $t, t \in [0, T]$) (i.e., daily production capacity).

Therefore, we aim to find a control strategy $u^* = (u^*(0), u^*(1), \ldots, u^*(T^*))$, that simultaneously satisfies the following three objectives:

(i) drive the system from the initial state $\mathbf{E}_x$ to the basin of attraction of the stable equilibrium $\mathbf{E}_s$ in finite time (It is sufficient to satisfy condition (5));

(ii) minimize the duration of the control intervention (i.e., $T^* = min\{T\}$);

(iii) minimize the total number of released wolbitos ($min \sum_{t=0}^{T} u(t)$).

To find a control strategy (u^*) that satisfies the objectives (i), (ii) and (iii), we propose two approaches to determine optimal (or suboptimal) strategies for implementing *Wolbachia*-based biocontrol. The first is a more classical approach, which determines an **impulsive control strategy** ($u_1^* = (u_1^*(0), u_1^*(1), \ldots, u_1^*(T^*))$) based on an optimal continuous solution ($u_c^* = u_c^*(t)$, defined on the interval $[0, T^*]$), to a continuous optimal control problem. The second is a less conventional approach for this research area, where a **discrete control strategy** ($u_2^* = (u_2^*(0), u_2^*(1), \ldots, u_2^*(T^*))$) is directly obtained by solving a proposed discrete optimization model. These approaches are presented in the next sections.

3.1. Suboptimal Impulsive Control

The objective (i) can be achieved by selecting the target end-point $(x(T), y(T))$ within the "secure region" in the attraction basin of $\mathbf{E}_s$, satisfying condition (5), specifically

$$x(T) = x_u - 1. \tag{7}$$

The objective (ii) involves regulating $y(t)$ through $u(t)$, as detailed in Orozco-Gonzales et al. [2025]. The third objective (iii) minimizes the total number of released wolbitos, expressed in continuous form as

$$\int_0^T u(t)\, dt, \tag{8}$$

with normalization of $u(t)$ by a constant K, to ensure the terms in the objective functional are dimensionless.

In this way, an objective functional can be defined as:

$$\mathcal{J}(u, T) = \int_0^T \left(\mathcal{P} + \frac{1}{2} \left(\frac{u(t)}{K} \right)^2 \right) dt, \tag{9}$$

where the first term normalizes the treatment duration, and the second term normalizes the control effort.

Let the decision variables be $u = u(t)$ (the control variable), and T (the final time of control application), and let $x = x(t)$ and $y = y(t)$ be the state variables of the system (1)-(2). The following optimal control problem (OCP) is proposed:

$$\min_{u,T} \quad \mathcal{J}(u, T) \tag{10}$$

$$\text{subject to} \quad \text{Equation } (1), \tag{11}$$

$$\text{Equation } (2), \tag{12}$$

$$u(t) \in [0, K], 0 \leq T \leq \infty. \tag{13}$$

The solution to the OCP exists for sufficiently large K, given the convexity of the objective functional and linearity in the control (Orozco-Gonzales et al. [2025]). A continuous optimal

control can be determined using the Pontryagin's Maximum Principle, yielding:

$$u_c^*(t) = \max\left(0, \min\left(K, \frac{K^2}{2}\lambda_2(t)\right)\right), \tag{14}$$

where $\lambda_2(t)$ is the adjoint variable associated with $y(t)$.

Let $T^* > 0$ be the optimal final time and $u_c^* = u_c^*(t),\ t \in [0, T^*]$, the optimal control obtained by solving the OCP numerically. Although u_c^* provides an ideal continuous release profile, its direct implementation is impractical. To derive a practically implementable suboptimal strategy based on discrete impulses, we can apply discretization techniques to the continuous solution in such a way that the resulting impulsive solution is feasible for the original problem (i.e., it satisfies conditions (5) and 6). In this work, we adopted a hybrid methodology that integrates two distinct approaches: the first, introduced in (Bliman et al. [2024]), focuses on the optimal deployment of the Sterile Insect Technique (SIT); the second, proposed in (Vicencio et al. [2023]), addresses the replacement of wild mosquito populations by Wolbachia-infected individuals without cost optimization. By combining these frameworks, we constructed a suboptimal release protocol denoted by u_1^*, which consists of a sequence of periodic impulses with varying magnitudes and frequencies, designed to approximate the continuous optimal control strategy while remaining feasible for real-world implementation. A full study of this approach is presented in Orozco-Gonzales et al. [2025].

3.2. Discrete Control

We propose a optimization model to determine a near-optimal sequence of discrete daily releases $u_2^* = \big(u_2^*(0), u_2^*(1), \ldots, u_2^*(T^*)\big)$ over a minimal time horizon $[0, T^*]$ (objective (ii)) such that the state variables satisfy the terminal conditions (5) (objective (i)), with each release constrained by (6). The total release size $\sum_{t=0}^{T} u(t)$ is minimized (Objective (iii)) while ensuring the system evolves to the stable coexistence equilibrium $\mathbf{E}_s$.

This constitutes a constrained multi-objective optimization problem: minimizing the intervention time and the total released insects. To explore the trade-offs between these objectives, we apply a metaheuristic approach based on Genetic Algorithms (GA), which iteratively tests combinations of release sequences to find a near-optimal solution.

Let $S_u = \big(x(0), x(1), \ldots, x(T), y(0), y(1), \ldots, y(T)\big)$ be a solution to the system (1)-(2) with a given sequence of decisions $u = \big(u(0), u(1), \ldots, u(T)\big)$, and let S denote the set of all solutions S_u obtained for all feasible u. Here, $u(t)$ and $t \in \{0, 1, \ldots T\}$ may only take integer values. Knowing values for ϵ, x_u, y_u, $x(0)$, $y(0)$, and K, we now propose an optimization model with decision variable $u(t)$ and state variables $x(t), y(t)$:

$$\min\ J(u, T) = \sum_{t=0}^{T} u(t) \tag{15}$$

$$\text{subject to:}\quad T \leq \epsilon \tag{16}$$

$$S_u \in S \tag{17}$$

$$x(T) < x_u \tag{18}$$

$$y(T) > y_u \tag{19}$$

$$0 \leq u(t) \leq K, \quad t = 1, \ldots, T \tag{20}$$

To solve the ϵ-constraint subproblems, for a fixed value of ϵ, we propose a metaheuristic based on a Genetic Algorithm (GA). Each GA solution (individual i) $u^i = \big[u^i(0), \ldots, u^i(T)\big]$ with $0 \leq u^i(t) \leq K$, represents a control strategy that generates the solution $S_{u^i} = \big(x(0), x(1), \ldots,$

https://proceedings.science/p/212243?lang=pt-br

$x(T), y(0), y(1), \ldots, y(T)\big)$ to the system (1)-(2) . The quality of the solution u^i can be evaluated using the fitness function:

$$F(u^i) = \frac{1}{J(u^i) + pKT \cdot \mathbf{I}_c(u^i)}, \quad \mathbf{I}_c(u^i) = \begin{cases} 0, & \text{if } x(T) < x_u,\ y(T) > y_u \\ 1, & \text{otherwise} \end{cases} \tag{21}$$

where $I_c(u^i)$ is a binary indicator function that penalizes the violation of the constraints (18) and (19), and p is the length of the interval between consecutive releases ($p = 1$ for daily, $p = 7$ for weekly, and $p = 14$ for fortnightly releases, etc.).

The GA operates with standard evolutionary operators:

Selection: Individuals with higher fitness are more likely to reproduce (The selection process was carried out using the Tournament Method).

Crossover: Two parents are recombined to generate new individuals (Figure 2).

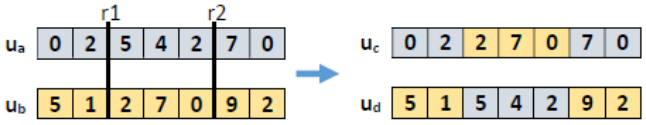


Figure 2: Two-point crossover between GA-individuals. Source: Orozco-Gonzales et al. [2025].

Mutation: Random changes are introduced to maintain diversity (Figure 3). A low mutation rate (0.05) was used to maintain solution quality and prevent the disruption of promising individuals.

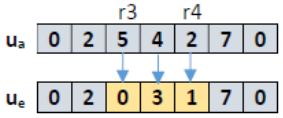


Figure 3: Mutation on selected genes of a GA-individual. Source: Orozco-Gonzales et al. [2025].

An elite set ($\{u^i_{best}\}$) of top-performing individuals is preserved across generations to improve convergence. The GA iterates until a stopping criterion is met, such as maximum generations, fitness stagnation, or convergence. The best solution from the elite set is selected as the final solution to the problem (15)-(20), i.e. $u_2^* = u_{best}$, where u_{best} is the best individual in the final elite set.

This approach is flexible for different release frequencies p (daily, weekly, fortnightly), and the release size adapts accordingly.

4. Numerical results

To validate the proposed optimization models, we performed computational experiments to determine control strategies with different release intervals (daily, weekly and fortnightly), using two *Wolbachia* strains (*w*Mel or *w*MelPop). These two strains exhibit contrasting characteristics regarding their ability to block viruses and their impact on the individual fitness of mosquitoes. The computational implementation was developed in MATLAB (Version R2023a). The simulations were executed on a computer equipped with an Intel Core i7-3770 processor (3.40 GHz), 12 GB of RAM, running a 64-bit Windows operating system. To solve the optimal control problem (10)-(13), we employed the advanced solver GPOPS-II.

Tables 1, 2, and 3 present the parameters to be used for solving the proposed models. We assign (in equation (9) the higher priority to the time optimization and assume in the sequel that $\mathcal{P} = \frac{10^6}{K^2}$.

Table 1: Model parameters, values used in numerical simulations, and references, Orozco-Gonzales et al. [2025]

Description	**Value / Notes**
Initial wild mosquito population $x^\sharp$	7030 individuals (avg. per hectare)
Saddle-point unstable equilibrium $E_u = (x_u, y_u)$	wMel: $(4592,\ 1793)$ wMelPop: $(1050,\ 3778)$
Upper daily release limit K for optimal control	wMel: 750 individuals wMelPop: 1000 individuals

Table 2: Values of the model's parameters used in numerical simulations, Orozco-Gonzales et al. [2025]

Description	**Assumed value**
Fecundity rate of uninfected insects	$\rho_n = 4.55$ days^{-1}
Fecundity rate of *w*Mel-infected insects	$\rho_w = 0.9 \times \rho_n = 4.095$ days^{-1}
Fecundity rate of *w*MelPop-infected insects	$\rho_w = 0.5 \times \rho_n = 2.275$ days^{-1}
Natural mortality rate of uninfected insects	$\delta_n = 1/28 = 0.0357$ days^{-1}
Natural mortality rate of *w*Mel-infected insects	$\delta_w = \delta_n/0.9 = 0.0397$ days^{-1}
Natural mortality rate of *w*MelPop-infected insects	$\delta_w = \delta_n/0.5 = 0.0714$ days^{-1}
Feature related to mosquito competition	$\sigma = 0.1/140$ individual^{-1}
MT probability in *w*Mel-infected insects	$\nu = 0.95$
MT probability in *w*MelPop-infected insects	$\nu = 0.99$
CI probability in *w*Mel-infected insects	$\eta = 0.98$
CI probability in *w*MelPop-infected insects	$\eta = 0.95$
Loss rate of *w*Mel infection	$\omega = 0.001$ days^{-1}
Loss rate of *w*MelPop infection	$\omega = 0.00015$ days^{-1}

Table 3: GA-related parameters, Orozco-Gonzales et al. [2025]

Parameter	Description	*w*Mel strain			*w*MelPop strain		
		day	week	fortnight	day	week	fortnight
N	population size	100	100	100	100	100	100
G	number of generations	100	100	100	100	100	100
pK	control upper bound	750	7×750	14×750	1000	7×1000	14×1000

Selected results from this research are presented and discussed below. Due to space constraints, graphical results are provided only for the case of daily applications. However, further discussions and additional results can be found in Orozco-Gonzales et al. [2025].

- *Impulsive Control*

The continuous optimization problem (10)-(13) was solved using Pontryagin's Maximum Principle. Based on the resulting continuous optimal solutions, impulsive control strategies were determined using the technique discussed in the Section 3.1. Table 4 summarizes the results of the suboptimal impulsive release strategies implemented at daily, weekly, and fortnightly intervals, using

mosquitoes infected with the *w*Mel and *w*MelPop Wolbachia strains. The impulsive release strategies are more realistic than the continuous release strategies, since in practice, the continuous release of wolbitos is unfeasible.

Table 4: Key indicators of the suboptimal impulsive release strategies, Orozco-Gonzales et al. [2025]

Indicators	*w*Mel strain			*w*MelPop strain		
	day	week	fortnight	day	week	fortnight
Number of releases	$\hat{T}^* = 14$	$\hat{T}^*_7 = 2$	$\hat{T}^*_{14} = 1$	$\hat{T}^* = 65$	$\hat{T}^*_7 = 9$	$\hat{T}^*_{14} = 5$
Overall size of releases	5966	5966	5966	33169	35574	41804

Figures 4 and 5 show, respectively, the suboptimal impulsive control strategies for releasing mosquitoes infected with the *w*Mel and *w*MelPop strains (left column), as well as the corresponding population dynamics of wild mosquitoes $x(t)$ and *Wolbachia*-infected mosquitoes $y(t)$ (right column). The horizontal lines indicate the coordinates of the saddle-point unstable equilibrium $\mathbf{E}_u = (x_u, y_u)$, where the blue line denote x_u, and the red lines indicate y_u. The dashed vertical line in the right-hand column of Figures 4 and 5 represents the time at which condition (5) is met.

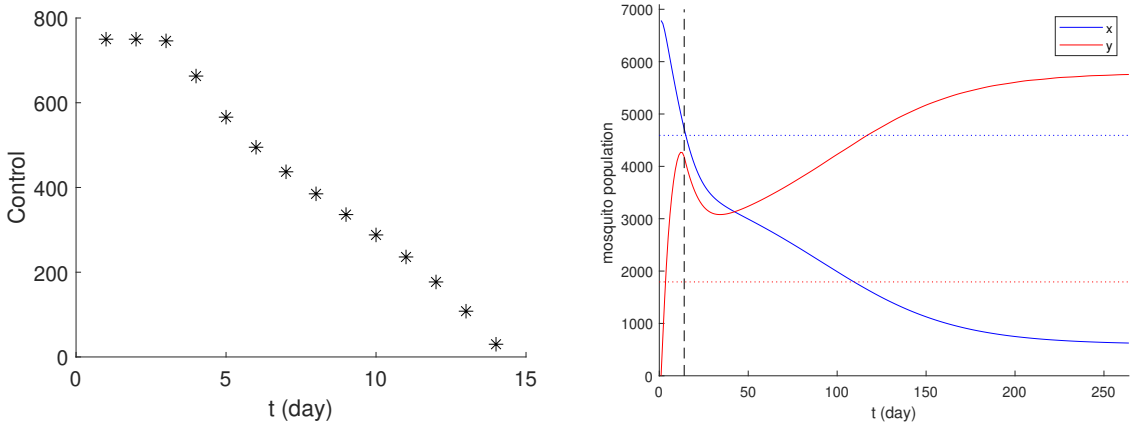


Figure 4: Suboptimal impulsive control strategies for releasing *w*Mel-carrying mosquitoes *(left column)* and the corresponding evolution of wild mosquitos $x(t)$ and *w*Mel-infected mosquitoes $y(t)$ *(right column)*, where the vertical line marks the time of fulfillment of the condition, 5. Source: Orozco-Gonzales et al. [2025]

For daily releases of mosquitoes carrying the *w*Mel strain, we obtain $\hat{T}^* = 14$ days, whereas for mosquitoes carrying the *w*MelPop strain, we obtain $\hat{T}^* = 65$ days. It can be observed that the number of *w*Mel-infected wolbitos required for the implementation of biocontrol corresponds, on average, to only 16% of the amount needed when using the *w*MelPop strain. Furthermore, the duration of biocontrol application with the *w*Mel strain is, on average, 21% of the time required when employing the *w*MelPop strain. In this context, establishing the *w*MelPop strain in a population is a slower and more costly process compared to the *w*Mel strain.

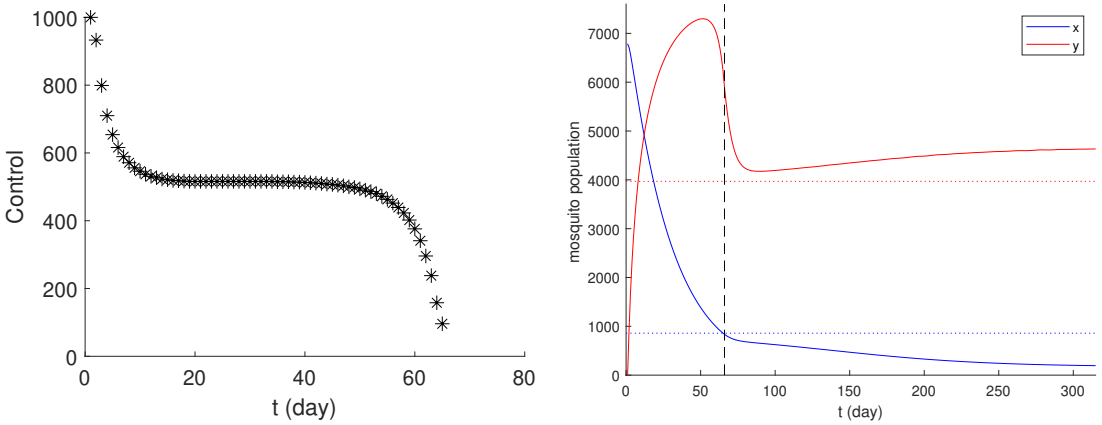


Figure 5: Daily release strategies for releasing *w*MelPop-carrying mosquitoes *(left)* and the corresponding evolution of wild mosquitoes $x(t)$ and *w*MelPop-infected mosquitoes $y(t)$ *(right)*, where the vertical line marks the time of fulfillment of the condition (5). Source: Orozco-Gonzales et al. [2025].

- *Discrete Control*

To determine the discrete biocontrol strategy, the optimization problem (15)–(20) is solved using MATLAB (version R2023a) to implement the Genetic Algorithm (GA) described in Section 3.2. The GA-related parameters are summarized in Table 3, while the parameter values associated with mosquito dynamics are listed in Table 2.

The value of T is a multiple of $p = 1, 7, 14$, and in the sequel, we denote by T_p^* the best time rendered by Genetic Algorithm, with release strategies with spacing p. The objective function $J(u, T)$ expresses the overall number of wolbitos to be released during the intervention (T). Figures 6 and 7 display the graphs of the best GA-based control strategies (u_2^*) *(left columns)* for daly case and the associated with populational dynamics of wild and wolbitos mosquitoes *(right columns)* for the *w*Mel and *w*MelPop *Wolbachia* strains, respectively.

The best time $T_p^*, p = 1, 7, 14$ and the total number of wolbitos needed for the successful intervention, $J\left(u_2^*, T_p^*\right)$ are given in Table 5. In this table and also in Figures 6 and 7 can also be observed that the number of *w*Mel-infected *Wolbachia* mosquitoes required for the implementation of biocontrol corresponds, on average, to only 19% of the quantity needed when using the *w*MelPop strain. Furthermore, the duration of biocontrol application with the *w*Mel strain is, on average, only 20% of the time required when employing the *w*MelPop strain. Once again, there is evidence that establishing the *w*MelPop strain in a population is a slower and more costly process compared to the *w*Mel strain.

Table 5: Key indicators of the release strategies yielded by Genetic Algorithm, Orozco-Gonzales et al. [2025]

Indicators	*w*Mel strain			*w*MelPop strain		
	day	week	fortnight	day	week	fortnight
Number of releases, T_p^*	11	2	1	60	9	5
Overall size of releases $J\left(u_2^*, T_p^*\right)$	5436	5226	4956	24481	27259	31323

Unlike the suboptimal impulsive strategies, which typically suggest a gradual reduction in release intensity, the GA-based strategies vary the release quantities in a non-uniform way over

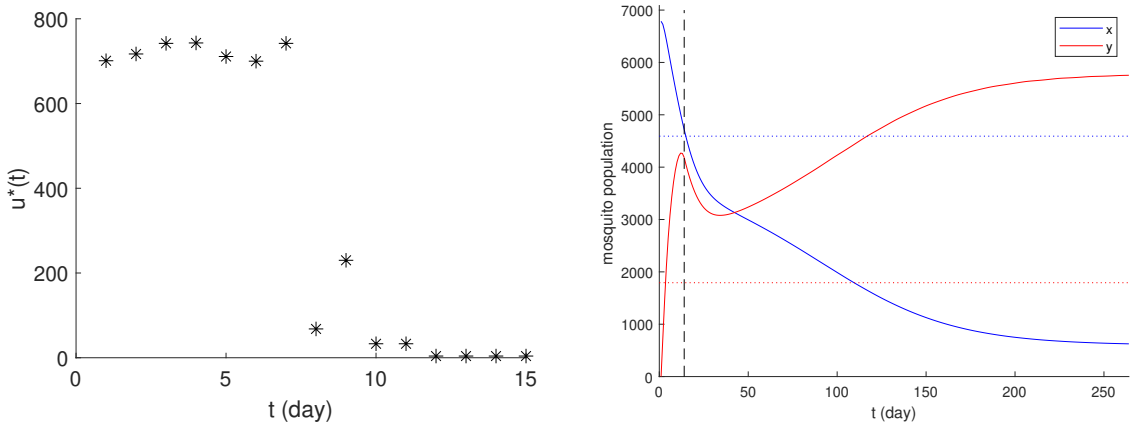


Figure 6: GA-optimal strategies for releasing *w*Mel-carrying mosquitoes *(left column)* and the corresponding evolution of wild mosquitos $x(t)$ and *w*Mel-infected mosquitoes $y(t)$ *(right column)*, where the vertical line marks the time of fulfillment of the conditions (5). Source: Orozco-Gonzales et al. [2025].

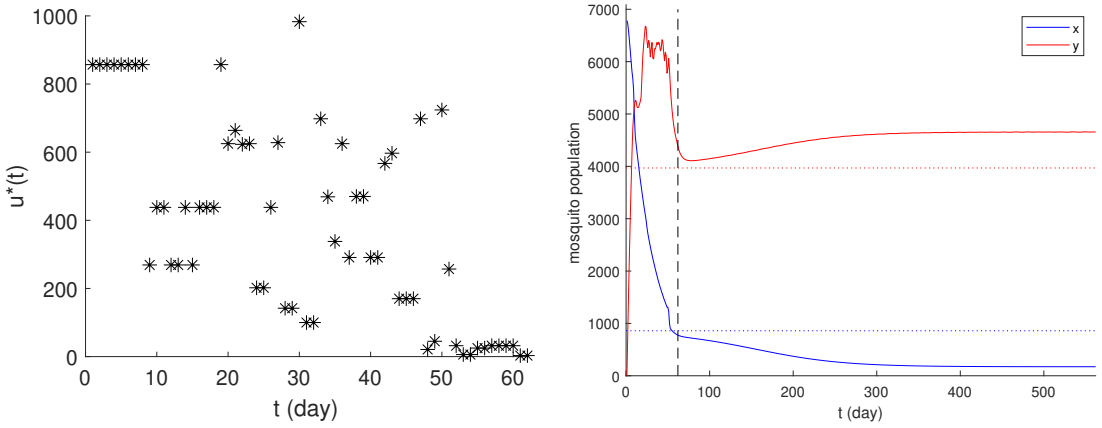


Figure 7: GA-optimal strategies for releasing *w*MelPop-carrying mosquitoes *(left)* and the corresponding dynamics of wild mosquitoes $x(t)$ and *w*MelPop-infected mosquitoes $y(t)$ *(right)*. The vertical line indicates the time at which the condition (5) is fulfilled. Source: Orozco-Gonzales et al. [2025].

time. This behavior reflects the algorithm's ability to explore a broader solution space, but it also requires more careful planning in practice, since each release must be preceded by adequate mass-rearing of the wolbitos. On the other hand, when releases are less frequent ($p = 7$ or $p = 14$), the resulting strategies are smoother and more regular, making them more suitable for advanced planning. Additionally, the GA routine includes the flexibility to optimize not only the amount but also the exact timing of each release within the p-day interval, which enhances the feasibility of implementing sparser release schedules in real-world settings.

5. Conclusion

The numerical results demonstrate that both optimization approaches (based on the Pontryagin Maximum Principle and the Genetic Algorithm (GA)) are effective for designing *Wolbachia*-based biocontrol strategies. However, the GA-based approach consistently outperforms in terms of reducing the total number of *Wolbachia*-infected mosquitoes required for successful intervention, particularly under daily release regimes. Additionally, it offers greater flexibility by directly optimizing over discrete control sequences and accommodating different release frequencies.

Given these advantages, we conclude that the GA-based strategy is more suitable for the

practical planning of biocontrol interventions. This approach is computationally adaptable to different ecological settings, release schedules, and population structures, and may be further integrated into epidemiological models to assess its public health impact. Future work may explore the extension of the current framework to spatially structured or stage-structured mosquito populations, and to the evaluation of epidemiological outcomes, such as the number of infections averted under various release scenarios.

Furthermore, comparing the two *Wolbachia* strains, the *w*Mel strain clearly demonstrates greater operational advantages. It requires, on average, fewer infected mosquitoes and a shorter intervention time to achieve population replacement. These characteristics translate into lower costs, reduced logistical complexity, and potentially fewer unintended ecological impacts. Although the *w*MelPop strain can achieve faster suppression due to its higher pathogenicity, it demands a significantly greater release effort, making *w*Mel the more feasible option for large-scale or resource-constrained applications.

Acknowledgments

The authors thank the Brazilian foundation FAPESP (Grants 2020/10964-0 and 2021/03039-1), CNPq (Grants 306518/2022-8 and 304007/2023-4), CAPES (Finance Code 001 and 88881.878875/2023-01), and the Colombian Ministry of Science, Technology, and Innovation — *MINCIENCIAS* (Contract No. 80740-224-2021) for financial support.

References

Benedito, A. S., Ferreira, C. P., and Adimy, M. (2020). Modeling the dynamics of *Wolbachia*-infected and uninfected *Aedes aegypti* populations by delay differential equations. *Mathematical Modelling of Natural Phenomena*, 15:76.

Bliman, P.-A., Cardona-Salgado, D., Dumont, Y., and Vasilieva, O. (2024). Optimal control approach for implementation of sterile insect techniques. *Journal of Mathematical Sciences*, 279 (5):607–622.

Farkas, J., Gourley, S., Liu, R., and Yakubu, A. A. (2017). Modelling *Wolbachia* infection in a sex-structured mosquito population carrying West Nile virus. *Journal of Mathematical Biology*, 75:621–647.

Ferreira, C. (2020). *Aedes aegypti* and *Wolbachia* interaction: population persistence in an environment changing. *Theoretical Ecology*, 13:137–148.

Li, D. and Wan, H. (2019). The threshold infection level for *Wolbachia* invasion in a two-sex mosquito population model. *Bulletin of Mathematical Biology*, 81:2596–2624.

Orozco-Gonzales, J. L., Santos, A. B., Cardona-Salgado, D., Ferreira, C. P., Florentino, H. O., Sepulveda-Salcedo, L. S., and Vasilieva, O. (2024). Comparing the long-term persistence of different wolbachia strains after the release of bacteria-carrying mosquitoes. *Mathematical biosciences*, 372:109190.

Orozco-Gonzales, J. L., Santos, A. B., Florentino, H. O., Ferreira, C. P., Cardona-Salgado, D., Sepulveda-Salcedo, L. S., and Vasilieva, O. (2025). Optimization approaches to wolbachia-based biocontrol. *Applied Mathematical Modelling*, 137:115663.

Pereira, T., Rocha, M., Sucupira, P., Carvalho, F., and Moreira, L. (2018). *Wolbachia* significantly impacts the vector competence of *Aedes aegypti* for Mayaro virus. *Scientific reports*, 8:6889.

Vicencio, D., Vasilieva, O., and Gajardo, P. (2023). Monotonicity properties arising in a simple model of *Wolbachia* invasion for wild mosquito populations. *Mathematical Biosciences and Engineering*, 20:1148–1175.