

RELIABILITY INFERENCE VIA QUANTUM BAYESIAN NETWORKS WITH EXPONENTIAL DISTRIBUTION MODELING

Lavínia Maria Mendes Araújo^{a,b}

lavinia.mendes@ufpe.br

Isis Didier Lins^{a,b}

isis.lins@ufpe.br

Enrique Lopez Droguett^c

eald@g.ucla.edu

Márcio José das Chagas Moura^{a,b}

marcio.cmoura@ufpe.br

^aCentro de Estudos e Ensaio em Risco e Modelagem Ambiental (CEERMA)

^bUniversidade Federal de Pernambuco (UFPE)

^cUniversity of California, Los Angeles (UCLA)

ABSTRACT

This study explores the application of Quantum Bayesian Networks (QBNs) to reliability analysis, focusing on systems with variables modeled using exponential distribution. A four-node Bayesian structure is proposed, integrating both discrete and continuous variables. The continuous node is encoded using quantum amplitude encoding, while probabilistic dependencies are implemented using controlled quantum gates. Failure probability is estimated via Iterative Amplitude Estimation (IAE) and compared to a classical Monte Carlo simulation. Results show that the quantum method provides more precise estimates with narrower credible intervals, despite a slight difference in the mean values. The findings highlight the potential of QBNs to enhance reliability assessments by capturing complex conditional structures with high precision and fewer computational resources. This proof-of-concept contributes to advancing quantum methods within the domain of operations research and reliability engineering.

KEYWORDS. Quantum Bayesian Networks, Reliability Assessment, Inference.

Paper topics: SE-QPO; EST&AM.

RESUMO

Este estudo explora a aplicação de Redes Bayesianas Quânticas (QBNs) na análise de confiabilidade, com foco em sistemas com variável modelada por distribuição exponencial. É proposta uma estrutura bayesiana com quatro nós, integrando variáveis discretas e contínuas. O nó contínuo é codificado utilizando codificação de amplitude quântica, enquanto as dependências probabilísticas são implementadas por meio de portas quânticas controladas. A probabilidade de falha é estimada por meio do Iterative Amplitude Estimation (IAE) e comparada a uma simulação clássica de Monte Carlo. Os resultados mostram que o método quântico fornece estimativas mais precisas, com intervalos de confiança mais estreitos, apesar de uma leve diferença nos valores médios. Os achados destacam o potencial das QBNs para aprimorar avaliações de confiabilidade, ao capturar estruturas condicionais complexas com alta precisão e menor custo computacional. Este estudo de prova de conceito contribui para o avanço de métodos quânticos no domínio da pesquisa operacional e da engenharia de confiabilidade.

PALAVRAS-CHAVE. Redes Bayesianas Quânticas, Confiabilidade, Inferência.

Tópicos: E-QPO; EST&AM.

1. Introduction

Reliability engineering increasingly demands methods capable of accurately representing uncertainty, especially in continuous distribution scenarios. One of the most common continuous distributions in reliability analysis is the Exponential, which is widely used to model the time to failure of components with a constant failure rate. This is particularly relevant in systems such as electronic devices, mechanical components under steady stress, or industrial systems operating under stable environmental conditions, where failures occur randomly over time and no aging or self-improvement effects are expected unless external interventions (e.g., design changes or corrective actions) take place.

Bayesian Networks (BNs) are versatile tools for representing probabilistic dependencies and updating beliefs considering new evidence. However, classical inference methods, such as Monte Carlo simulation, face computational limitations in complex systems. In this context, quantum computing emerges as a promising alternative, particularly through Quantum Bayesian Networks (QBNs) [Borujeni et al. 2021]. This work aims to explore the application of QBNs for continuous distributions, focusing on the Exponential distribution. By discretizing the probability distribution and implementing quantum circuits, we estimate the failure probability of a hypothetical system and compare the results with a classical Monte Carlo simulation. This study focuses on a specific Bayesian network composed of four nodes: a continuous variable representing the system's degradation level at a given moment, modeled as an exponentially distributed latent state, and three binary nodes representing environmental conditions, maintenance quality, and equipment failure, respectively. The exponential distribution is discretized and encoded in a quantum circuit, and the failure probability is inferred using Iterative Amplitude Estimation (IAE).

This study also fits into the broader scope of operations research, as it explores probabilistic inference and decision modeling under uncertainty, core themes in the field. The use of QBNs for reliability estimation aligns with classical operations research problems involving system performance evaluation, risk assessment, and optimization under uncertainty. Moreover, the application of quantum algorithms such as IAE, which is conceptually connected to Grover's search algorithm, underscores the potential of quantum-enhanced methods to improve computational efficiency in solving operations research problems involving large probabilistic models. The methodology applies to reliability and risk assessment problems where both discrete and continuous variables need to be represented. Typical examples include fault tree and event tree analysis, probabilistic safety assessment, degradation modeling, and system reliability studies combining binary failure states with lifetime or environmental distributions.

2. Fundamentals of Quantum Computing and Quantum Bayesian Networks

Quantum computing leverages principles such as superposition, entanglement, and interference to process information in fundamentally different ways than classical computing. Qubits can represent both 0 and 1 simultaneously, allowing quantum circuits to explore multiple probabilistic paths in parallel [Lins et al. 2025]. This quantum parallelism is particularly beneficial for tasks such as sampling and inference.

QBNs extend the classical BN framework into the quantum domain by encoding the joint probability distributions of the network in quantum states. These are implemented through unitary operations that prepare quantum states representing the desired probability distributions. One powerful approach to performing inference in this context is Quantum Amplitude Estimation (QAE), which allows the estimation of the amplitude (i.e., probability) associated with a particular measurement outcome. An important variant of QAE is IAE, which avoids the need for quantum phase estimation and reduces circuit depth. For further theoretical and implementation details, see [Borujeni et al. 2021; Nielsen & Chuang 2010]

3. Experiments

3.1 Framework for Quantum Bayesian Network considering the Exponential distribution

The methodology adopted in this study follows a structured process composed of the following steps:

(1) Modeling the Bayesian Network: Definition of the nodes and their probabilistic dependencies. The Bayesian network used consists of four nodes ([Figure 1](#)):

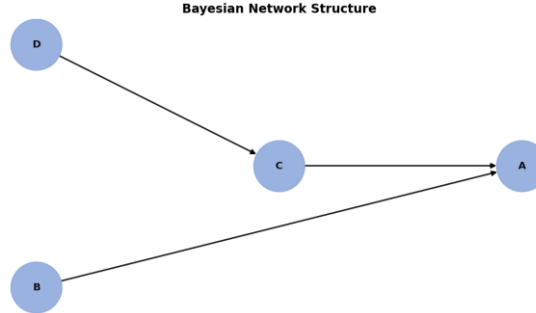


Figure 1: Bayesian Network Structure.

- A: equipment failure, dependent on B and C, with a conditional distribution defined for each binary combination: $P(A = 1 | B = 0, C = 0) = 0.1$; $P(A = 1 | B = 0, C = 1) = 0.3$; $P(A = 1 | B = 1, C = 0) = 0.6$; $P(A = 1 | B = 1, C = 1) = 0.9$.
- B: maintenance quality, with $P(B = 1) = 0.6$;
- C: local environmental stressors experienced by the equipment. The latent degradation state D can create or intensify these stressors, with $P(C = 1 | D) = 0.2 + 0.7D$;
- D: latent degradation state (continuous variable with exponential distribution of rate parameter $\lambda = 2$).

(2) Discretization of the Continuous Distribution: The Exponential distribution was discretized into 64 equally spaced points over the interval $[0,1]$. We started with 8 points and progressively refined the grid; the results reported use 64 points, which proved adequate for our tests. A finer discretization is desirable and will be pursued in future work. Each discretized value corresponds to a probability mass derived from the exponential probability density function. The square roots of these probabilities were used to construct the amplitude vector, which was then normalized to ensure a valid quantum state.

(3) Quantum State Preparation: To implement this encoding, the amplitude vector was passed to Qiskit's StatePreparation class, which automatically constructs a quantum circuit capable of preparing the target state from the initial state $|0\rangle^{\otimes n}$. This ensures an exact encoding of the discretized exponential distribution. Since 64 values were used, this required 6 qubits for node D, as $\log_2(64) = 6$;

(4) Circuit Construction: Implementing the logical structure of the Bayesian network using quantum gates (CRY and MCRY) for nodes C and A, and a single RY gate for B. The quantum circuit additionally includes 3 more qubits, one each for nodes C, B, and A, bringing the total to 9 qubits. Three extra qubits were added to encode the binary nodes C, B, and A, so their conditional dependencies could be implemented with CRY/MCRY gates and the outcome $P(A=1)$ extracted from a dedicated qubit. Node C was implemented using a controlled RY (CRY) gate for each discretized value of D. This involved iterating over the 64 possible states of D and applying the corresponding CRY rotation on qubit C. Node B, being independent, was implemented with a single RY gate using the probability $P(B = 1)$. Node A depends on both B and C and was implemented using multi-controlled RY (MCRY) gates. For each combination of B and C, the circuit applies the corresponding rotation conditioned on their joint state. To control specifically for combinations like $(B = 0, C = 1)$, the qubits were temporarily flipped with X gates before and after applying MCRY, ensuring correct behavior for each logical condition. This circuit architecture was designed following the approach of [Borujeni et al. 2021];

(5) Inference via IAE: Once the complete quantum circuit was constructed, inference was carried out using IAE. The following parameters were adopted: error tolerance (ϵ) = 10^{-4} , confidence level with $\alpha = 0.05$, and 100,000 shots per execution. The inference was performed on a noiseless quantum simulator using Qiskit's Sampler, allowing idealized results for comparison. The target qubit was that of node A, representing whether a failure occurred. The result from IAE

was then compared to a classical Monte Carlo simulation performed with 100,000 samples for validation and benchmarking, implemented in Python using the standard random module and NumPy. Each replication sampled the variables following the same discretization and conditional structure as the quantum model, and $P(A=1)$ was estimated by the sample mean with a normal 95% credible interval.

3.2. Results

Experiments were implemented in Python using. Circuits were executed on a noiseless simulator (no real quantum hardware): AerSimulator for count sampling. Runs were performed on a local Windows 64-bit machine with 8 GB RAM. To better assess the behavior of the estimators, an analytical solution of the Bayesian network was computed, yielding an exact failure probability of approximately 0.51367. This reference value serves as a ground truth to evaluate the bias and precision of both quantum and classical approaches. The results obtained from both inference methods are presented in Table 1, where we can see the estimated probability of $P(A = 1|B, C)$:

Method	Estimated Probability	95% Cred. Int.	StandardError
Quantum (IAE)	0.5283	[0.5283, 0.5284]	2.84e-05
Monte Carlo	0.5132	[0.5101, 0.5163]	1.58e-03

Table 1: Comparison of IAE and Monte Carlo Methods.

The quantum method provided a slightly higher probability estimate than the classical approach (0.5283 vs. 0.5132), accompanied by a substantially narrower credible interval and lower standard error. While the classical result is consistent with expectations for large sample simulations, its broader interval and higher uncertainty illustrate the sampling variability inherent to the method.

In contrast, the quantum IAE method achieved precision with fewer computational resources due to amplitude amplification mechanisms inherent to quantum inference. From a reliability standpoint, the more concentrated distribution around the quantum estimate suggests greater sensitivity in capturing failure pathways in the modeled system, particularly where rare event combinations influence the result. The observed difference of approximately 0.0151 in probability may reflect the nuanced handling of conditional structures in the quantum encoding.

These findings underscore the potential of QBNs for reliability modeling, not only for improving estimation efficiency but also for revealing system behaviors that might be diluted in classical Monte Carlo simulations. Further investigation is warranted to verify whether this difference stems from systematic modeling bias, numerical discretization, or circuit behavior.

4. Conclusions

This study demonstrated the feasibility of encoding exponential distributions in QBNs for reliability applications. The use of StatePreparation enabled accurate representation of a continuous variable, and the IAE-based estimate proved to be efficient. However, a slight overestimation was observed when compared to the analytical reference value. Future studies should consider the following: comparison with other distributions (e.g., Weibull, Lognormal); sensitivity testing regarding the number of bins and qubits; complexity analysis of the generated circuits; and implementation and testing under realistic quantum noise models.

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