
Multi-objective Optimal Test Planning Using Bayesian Reliability Estimation: A Case for Equipment Under Development

Plínio Ramos^a, Rafael Azevedo^a, Ana Beatriz^a, Isis Lins^a, Márcio Moura^a

^aCenter for Risk Analysis, Reliability Engineering and Environmental Modeling, Universidade Federal de Pernambuco, Recife, Brazil

plinio.marcio@ufpe.br, rafael.valenca@ufpe.br, anna.blpereira@ufpe.br, isis.lins@ufpe.br, marcio.cmoura@ufpe.br

ABSTRACT

This paper proposes a multi-objective optimization framework for planning reliability tests in equipment technologies under development. The objective is to select and configure a set of component or subsystem tests such that system-level reliability is demonstrated with a specified confidence, while minimizing both cost and execution time. The formulation integrates test precedence constraints and Bayesian reliability estimation based on test outcomes. The NSGA-II algorithm is adapted to efficiently explore the space of feasible test plans, ensuring scalability and the ability to handle realistic constraints. The proposed framework contributes to the methodological robustness and practical applicability of intelligent test planning under uncertainty.

KEYWORDS. Test planning, optimization, metaheuristics.

Paper topics (EN&OG, OMO, MH)

1. Introduction

The optimal planning of reliability tests is a fundamental challenge in the development of novel equipment technologies, especially when system-level tests are unfeasible due to cost, schedule, or infrastructure constraints [Zheng et al., 2023]. In such contexts, designing a cost-effective and technically robust test plan requires the selection and configuration of a subset of component-level tests that together can ensure the desired system reliability [Kim et al., 2019].

This work presents a multi-objective and stochastic model for optimal test planning, where tests are selected and configured to meet a system-level reliability target at a specified confidence level. The proposed model handles multiple decision variables per test (number of specimens, exposure time, and stress), including precedence constraints, and uses Bayesian inference to dynamically update failure probabilities as tests progress. The approach combines the rigor of statistical estimation with the flexibility of modeling real-world operational constraints. In addition, this formulation gives rise to a multi-objective constrained optimization problem, where trade-offs must be carefully managed among cost, time, and reliability assurance. A particular emphasis is placed on the application of the Non-dominated Sorting Genetic Algorithm II (NSGA-II) [Deb et al., 2002], a metaheuristic designed to efficiently explore large and complex design spaces and identify sets of Pareto-optimal solutions.

2. Methodology

2.2. Mathematical Formulation

The test planning problem is formulated as a constrained multi-objective optimization task, where the goal is to determine the optimal configuration of reliability tests to minimize cost and execution time while ensuring a required level of system reliability. The decision variables of the test planning model are:

- n_i : number of test specimens for test i ;
- t_i : exposure or execution time;
- s_i : stress level applied during testing.

The optimization problem searches for minimizing total testing costs (TC), considering both manufacturing and laboratory components:

$$\min_{n, t, sd} TC = \sum_{i=1}^N C_{A,i} \times n_i + C_{E,i} \times \left(FT_i + \left\lceil \frac{n_i}{Cap_i} \right\rceil \times t_i \right)$$

$$\min_{n, t, sd} ED$$

Subject to:

$$U(n_1, \dots, n_N, t_1, \dots, t_N, s_1, \dots, s_N) \leq U_{target}$$

$$ed_i = sd_i + FT_i + \left\lceil \frac{n_i}{Cap_i} \right\rceil \times t_i, \quad i = 1, \dots, N$$

$$ED = \max\{ed_i\}, \quad i = 1, \dots, N$$

$$sd_i \geq D_i, \quad i = 1, \dots, N$$

$$sd_j \geq sd_i + t_i - M(1 - y_{ij}), \quad i = 1, \dots, N; \quad j = 1, \dots, N$$

$$TC \leq CL$$

$$ED \leq TL$$

$$n_i^l \leq n_i \leq n_i^u, \quad i = 1, \dots, N$$

$$t_i^l \leq t_i \leq t_i^u, \quad i = 1, \dots, N$$

$$y_{ij} \in \{0,1\}, \text{ where: } \begin{cases} y_{ij} = 1, & \text{if test } j \text{ occurs after test } i \\ y_{ij} = 0, & \text{otherwise} \end{cases}$$

Table 1 - Summary of Parameters and Decision Elements in the Optimal Test Planning Model

Parameter	Definition
$C_{A,i}$	Unit cost of test i specimen
$C_{E,i}$	Unit cost of running test i
Cap_i	Capacity of test i
FT_i	Fixed time of test i
D_i	Date available to run test i
k_i	Estimation of the test i acceleration constant
$S_{U,i}$	Use stress level for test i
TL	Time limit
CL	Cost Limit
M	Large constant

The system reliability is computed based on the composition of components and their respective reliabilities, potentially including series and parallel subsystems. While alternative formulations such as semi-infinite linear programming have been proposed in the literature as seen in Feyzioglu et al., [2006] for designing component test plans, these approaches often impose strong assumptions and may become computationally rigid in the presence of practical testing constraints and nonlinear system behavior.

2.3. Metaheuristic Optimization Strategy

Given the combinatorial nature of the multi-objective optimization problem and the presence of nonlinear constraints, we adopt metaheuristics as an efficient alternative to exact methods. Specifically, the NSGA-II is applied to identify Pareto-optimal test plans that balance: a) Total cost; and b) Execution time.

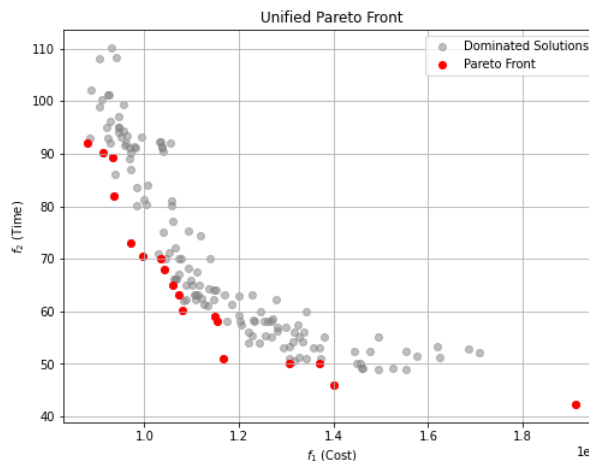
This method enables the exploration of large solution spaces, even when analytical evaluation of reliability or feasibility is computationally intensive. A comparative benchmark with an exhaustive enumeration approach revealed that while over 360,000 test configurations were generated in a simple scenario, only a handful (e.g., 8) were non-dominated. NSGA-II recovered these solutions with significantly lower computational overhead.

3. Results and Discussion

The NSGA-II algorithm was applied to an instance involving five candidate tests, each characterized by decision variables for the number of specimens and exposure time. The system under analysis included fifteen basic failure events, modeled using a hierarchical logic structure with series and parallel configurations.

A reliability threshold of 0.9 was imposed with a 60% confidence level. Execution capacities and cost profiles were defined for each test, incorporating realistic resource constraints. A maximum total budget of R\$ 2,000,000 was also imposed, limiting feasible solutions to those that comply with both reliability and cost constraints. The NSGA-II algorithm was configured with a population size of 100 and 200 generations. The algorithm produced a well-defined Pareto frontier, illustrating the trade-off between total cost and execution duration (Figure 1).

Figure 1 - Pareto Front Obtained via NSGA-II for the Optimal Test Planning Problem



This result demonstrates the capability of metaheuristic optimization in solving complex test planning problems and provides a benchmark for comparison with exhaustive methods in future research.

4. Conclusion

This work presents a structured model for optimal test planning, integrating mathematical modeling, operational constraints, and intelligent search strategies to support reliability demonstration of emerging equipment technologies. The proposed framework is flexible and scalable, capable of adapting to different system architectures, test cost structures, and reliability requirements. Future work will focus on: Expanding the model to account for uncertainties during testing (e.g., failure during tests); exploring other metaheuristics; applying the model in industrial case studies involving oil & gas technologies.

References

- Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2), 182–197. <https://doi.org/10.1109/4235.996017>
- Feyzioglu, O., Kuban Altinel, İ., & Özekici, S. (2006). The design of optimum component test plans for system reliability. *Computational Statistics & Data Analysis*, 50(11), 3099–3112. <https://doi.org/10.1016/j.csda.2005.06.015>
- Kim, S. J., Mun, B. M., & Bae, S. J. (2019). A cost-driven reliability demonstration plan based on accelerated degradation tests. *Reliability Engineering and System Safety*, 183, 226–239. <https://doi.org/10.1016/j.ress.2018.11.017>
- Zheng, H., Yang, J., & Zhao, Y. (2023). Reliability demonstration test plan for degraded products subject to Gamma process with unit heterogeneity. *Reliability Engineering & System Safety*, 240, 109617. <https://doi.org/10.1016/j.ress.2023.109617>