

A Multi-Period Optimization Model For Bus Fleet Allocation: Balancing Operational Costs, Social Discomfort, And Environmental Impact

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ABSTRACT

This paper presents a multi-period mixed-integer linear programming model for optimizing bus fleet allocation and frequency setting in urban transit systems. The model simultaneously minimizes operational costs, passenger discomfort (from overcrowding and waiting), and carbon emissions while adhering to operational constraints. Unlike traditional single-period approaches, our formulation captures temporal demand variations throughout the day, enabling more responsive service planning. Computational experiments using adapted real-world data were used to create 8 different scenarios, which test the sensitivity of the model. Results reveal capacity thresholds beyond which service quality deteriorates, illustrating trade-offs between passenger experience and resource utilization. Furthermore, the model successfully balances service quality during peak hours while maintaining efficiency during off-peak periods. This approach represents an advance in sustainable transit planning by integrating social and environmental considerations with operational metrics, supporting transport authorities in developing more equitable, efficient, and environmentally responsible service strategies.

KEYWORDS. Public transportation optimization, Frequency-based allocation, Sustainable transit planning

Paper topics: L&T – Logistics and Transportation. OD – Discrete Optimization. OMO - Multiobjective Optimization.

1. Introduction

The optimization of public transportation systems has become an increasingly critical issue due to the rapid urbanization observed in large cities. With projections indicating that 70% of the global population will reside in urban areas by 2050 [Ceder, 2021], the demand for public transport is set to surge, particularly during peak hours. This surge leads to overcrowding, longer waiting times, and a decline in service quality. Rising demand exacerbates passenger crowding and contributes to environmental concerns, including elevated CO₂ emissions. The transportation sector is a key target for climate change mitigation [IPCC, 2023], and enhancing public transport efficiency is essential to reducing private vehicle usage, which is stationary 95% of the time and inefficient in terms of space and capacity [Ceder, 2021; Jing et al., 2022].

In parallel, social factors, such as the discomfort experienced by passengers during peak periods, have become significant determinants of public transportation service quality. Research indicates that passenger satisfaction is heavily influenced by factors such as comfort, reliability, and accessibility, with overcrowding being one of the most detrimental elements to the user experience [Sogbe et al., 2025]. These challenges highlight the need for innovative solutions, such as multi-period mathematical models that account for demand peaks and social discomfort, to optimize public transportation systems in terms of both operational efficiency and passenger well-being.

In this context, this paper presents a multiperiod mixed-integer linear programming (MILP) model for bus fleet allocation and frequency setting. The objective is to jointly minimize operational costs, passenger discomfort, and carbon emissions. The proposed model addresses critical aspects of sustainable public transportation planning and supports decision-making processes aligned with environmental goals and service quality standards. Next Section presents a review of related literature. Section 3 defines the problem context. Section 4 introduces the mathematical model. Finally, Section 5 outlines the experimental setup, while Section 6 presents the results and provides concluding remarks, including directions for future research.

2. Related Work

In recent years, the optimization of public transportation systems has addressed multiple dimensions, including operational costs, waiting times, vehicle capacity, service frequency, and passenger comfort. However, few studies have integrated these aspects within a multiperiod framework capable of capturing temporal demand fluctuations and supporting adaptive decision-making.

Several contributions focus on specific facets of the problem. Arbex e da Cunha [2015] address network design and frequency setting using a genetic algorithm to balance operational and user-related costs; Duran-Micco et al. [2020] incorporate CO₂ emissions and fleet heterogeneity into frequency optimization; Jiang et al. [2014] and Pathak et al. [2020] focus on headway adjustments to minimize passenger waiting times and discomfort due to overcrowding; Martínez et al. [2014] use mixed-integer linear programming to improve system efficiency by reducing travel times. Other studies adopt multi-objective approaches: Tekin et al. [2018], Cats e Glück [2019], and Jha et al. [2019] optimize different combinations of cost, emissions, and travel time; Jiang et al. [2022] and Caetano et al. [2025] propose integrated frequency and scheduling models, yet without explicitly accounting for peak-period demand variation.

While these models provide important contributions, most do not jointly address time-varying demand, environmental impact, and social discomfort within a unified decision-support structure. This study addresses this gap by proposing a multiperiod optimization model that dynamically allocates bus frequencies and fleet deployment according to temporal demand patterns. The formulation minimizes operational costs and passenger travel time while internalizing the effects of overcrowding—particularly during peak hours—and CO₂ emissions, offering a comprehensive and adaptive approach to urban transit planning. The next section presents the problem setting and the modeling assumptions.

3. Problem Description

Urban transit agencies face the complex task of allocating a limited and heterogeneous bus fleet to meet time-varying passenger demand across multiple routes. Since ridership fluctuates substantially between peak and off-peak periods, failure to align supply with demand can lead to long waiting times, overcrowding, or underutilized vehicles. At the same time, operators must manage operational costs (such as fuel, labor, and maintenance) while also minimizing environmental impacts, particularly CO₂ emissions, and operating within fixed fleet-size constraints.

In this context, planners must determine, for each route and time interval, the appropriate number and type of buses to deploy. These decisions must simultaneously ensure that: (i) passenger waiting times remain within acceptable thresholds; (ii) onboard crowding does not exceed vehicle capacity multiplied by an allowed load factor; (iii) total service frequency aligns with the temporal demand profile, especially during peak hours; and (iv) buses allocated to long or multi-period routes remain committed throughout their service, thereby preventing scheduling conflicts.

Moreover, each allocation incurs a composite cost, including operational expenses, emissions, and user-related penalties for waiting, overcrowding, or required transfers. Therefore, this study proposes a multiperiod optimization model that jointly minimizes operational costs, environmental impacts, and social discomfort, while satisfying service quality targets and operational constraints, thereby supporting more adaptive and equitable transit planning.

4. Mathematical Modeling

This section introduces the proposed optimization model for bus route allocation. The model is formulated as a mixed-integer linear program (MILP) that aims to minimize a composite total cost. This cost incorporates operational expenses, environmental impact, social discomfort (including passenger waiting and overcrowding), transfer penalties, fare-based costs, and penalties for unmet passenger demand. The remainder of the section presents the sets, parameters, decision variables, objective function, and constraints that compose the model.

4.1. Sets and Parameters

This subsection presents the fundamental sets and parameters employed in the mathematical formulation of the mixed-integer linear programming (MILP) model. Table 1 details the definitions of the indices, cost coefficients, operational parameters, demand variables, and system constraints that structure the decision-making framework. These elements provide the formal basis for representing vehicle operations, passenger flows, and resource limitations within the optimization model.

4.2. Decision Variables

To make a linear formulation of the bus allocation problem, we discretize the possible number of buses assigned to each route $r \in R$ at each time period $t \in T$. This is done by defining, for each route r , a finite set of candidate fleet sizes $K_r = \{1, 2, \dots, k_{\max}^r\}$, where $k_{\max}^r$ is given by:

$$k_{\max}^r = \min \left(\alpha \cdot \left\lceil \frac{\max_{t \in T} Q_{r,t}}{\min_{b \in B} (LF_b \cdot SC_b)} \right\rceil, \sum_{b \in B} AU_b, \left\lfloor \frac{P}{CT_r} \right\rfloor \cdot \sum_{b \in B} AU_b \right) \quad (1)$$

The model adopts this discretization approach to ensure a scalable and computationally tractable formulation for the bus allocation and scheduling problem, while effectively capturing the complexity of selecting optimal frequencies for each route. This strategy has been employed in previous studies, such as Martínez et al. [2014]. Moreover, the model is linearized through the introduction of the variable $w_{r,t}$, since the binary variable $y_{r,t}$ would otherwise need to multiply $\sum_{b \in B} q_{b,r,t}$ in the transfer cost component of the objective function. This linearization is implemented

Table 1: Sets and parameters of the mathematical model.

Notation	Description
R	The set of bus routes on the transit network.
T	The set of time periods.
B	The set of bus types.
OC_b	Operational cost (BRL per kilometer) of vehicle type $b \in B$.
WC	Waiting cost (BRL per minute).
AC	Agglomeration cost (BRL per passenger).
TC	Transfer cost (BRL per passenger).
FC	Fare cost (BRL per passenger).
CC	Carbon emission tax cost (BRL per kilogram of CO ₂).
P	Duration of a time period (minutes).
LE_r	Length (kilometers) of route $r \in R$.
CT_r	Cycle time (minutes) of route $r \in R$.
$Q_{r,t}$	Passenger demand (passengers per period) on route $r \in R$ at time $t \in T$.
$\Delta_{t,r}$	Inactivity periods for route $r \in R$, defined as $\lceil CT_r/P \rceil - 1$.
SC_b	Seat capacity (passengers) of vehicle type $b \in B$.
LF_b	Maximum load factor (dimensionless) of vehicle type $b \in B$.
CB_b	CO ₂ emission (kilograms of CO ₂ per kilometer) of vehicle type $b \in B$.
AU_b	Available units (dimensionless) of vehicle type $b \in B$.
AW	Maximum acceptable waiting time (minutes).
M	Large positive constant (dimensionless).
K_{max}^r	Max frequency of route $r \in R$
α	Discretization factor (dimensionless)

Source: Created by the authors.

through an implicit Big-M formulation, where the constant $Q_{r,t}$ represents the maximum feasible value in each constraint. This approach avoids the numerical instability typically associated with arbitrarily large Big-M values, as discussed in Asghari et al. [2022], and is considered good modeling practice in mixed-integer linear programming formulations.

Table 2: Decision variables of the mathematical model.

Notation	Description
$v_{b,r,t} \in \mathbb{Z}^+$	Number of buses of type $b \in B$ assigned to route $r \in R$ at time $t \in T$.
$f_{b,r,t} \in \mathbb{Z}^+$	Operation frequency (buses per hour) of bus type $b \in B$ on route $r \in R$ at time $t \in T$.
$q_{b,r,t} \in \mathbb{Z}^+$	Passenger load (passengers) on bus type $b \in B$ on route $r \in R$ at time $t \in T$.
$h_{r,t} \in \mathbb{R}^+$	Headway (minutes) for route $r \in R$ at time $t \in T$.
$e_{r,t} \in \mathbb{Z}^+$	number of passengers who traveled on foot on route $r \in R$ at time $t \in T$
$y_{r,t} \in \{0, 1\}$	Binary variable indicating if waiting time on route $r \in R$ at time $t \in T$ exceeds AW .
$w_{r,t} \in \mathbb{Z}^+$	Auxiliary variable representing number of passengers that waited more than AW minutes in route $r \in R$ at time $t \in T$
$u_{b,t} \in \mathbb{Z}^+$	Total buses of type $b \in B$ in use or inactive at time $t \in T$.
$\delta_{k,r,t} \in \{0, 1\}$	Binary variable of discretization frequency per route $r \in R$ at time $t \in T$.

Source: Created by the authors.

4.3. Objective Function

The objective function (2) minimizes the total system cost across all buses, routes, and periods. It includes the operational cost of running buses (fuel, driver, maintenance), carbon emission penalties, passenger waiting times (which increase with headways), penalties for overcrowded buses (agglomeration), transfer penalties between routes, fare-related costs proportional to transported passengers, and a large penalty to discourage unserved passenger demand. Each term targets a different component of the real-world performance of the bus system.

$$\begin{aligned}
 \min \quad & \underbrace{\sum_{b \in B} \sum_{r \in R} \sum_{t \in T} OC_b \cdot LE_r \cdot f_{b,r,t}}_{\text{Operational cost}} + \underbrace{\sum_{b \in B} \sum_{r \in R} \sum_{t \in T} CC \cdot CT_r \cdot f_{b,r,t} \cdot CB}_{\text{Carbon emission cost}} \\
 & + \underbrace{\sum_{r \in R} \sum_{t \in T} \frac{H_{r,t}}{2} \cdot WC}_{\text{Passenger waiting cost}} + \underbrace{\sum_{b \in B} \sum_{r \in R} \sum_{t \in T} e_{b,r,t} \cdot AC}_{\text{Agglomeration penalty}} \\
 & + \underbrace{\sum_{r \in R} \sum_{t \in T} CT \cdot w_{r,t}}_{\text{Transfer penalty}} + \underbrace{\sum_{b \in B} \sum_{r \in R} \sum_{t \in T} FC \cdot q_{b,r,t}}_{\text{Fare-based cost}} + \underbrace{M \cdot \sum_{r \in R} \sum_{t \in T} \left(Q_{r,t} - \sum_{b \in B} q_{b,r,t} \right)}_{\text{Penalty for unmet demand}}
 \end{aligned} \tag{2}$$

In summary, the objective function seeks to minimize: (i) operational costs, (ii) emissions, (iii) passenger waiting time, (iv) crowding discomfort, (v) transfer penalties, (vi) fare-related costs, and (vii) penalties for unmet demand.

4.4. Model Constraints

The model ensures operational feasibility through a set of constraints governing bus allocation, passenger management, and fleet usage:

$$\sum_{b \in B} f_{b,r,t} = \sum_{k \in K_r} k \cdot \delta_{k,r,t}, \tag{3} \quad v_{b,r,t} = \begin{cases} f_{b,r,t} & \text{if } CT_r > P, \\ \lfloor \frac{P}{CT_r} \rfloor \cdot f_{b,r,t} & \text{otherwise} \end{cases} \tag{9}$$

$$\sum_{k \in K_r} \frac{P}{k} \cdot \delta_{k,r,t} = h_{r,t}, \tag{4} \quad \begin{cases} e_{b,r,t} \geq q_{b,r,t} - SC_b \cdot f_{b,r,t}, \\ e_{b,r,t} \geq 0, \end{cases} \tag{10}$$

$$\frac{h_{r,t}}{2} \leq AW + (P - AW) \cdot y_{r,t}, \tag{5} \quad \sum_{b \in B} q_{b,r,t} \leq Q_{r,t}^{\max}, \tag{11}$$

$$\sum_{k \in K_r} \delta_{k,r,t} = 1, \tag{6} \quad \sum_{r \in R} v_{b,r,t} \leq AU_b, \quad \forall b \in B, t \in T, \tag{12}$$

$$\begin{cases} w_{r,t} \leq \sum_{b \in B} q_{b,r,t}, \\ w_{r,t} \leq Q_{r,t} \cdot y_{r,t}, \\ w_{r,t} \geq \sum_{b \in B} q_{b,r,t} - Q_{r,t} \cdot (1 - y_{r,t}) \end{cases} \tag{7} \quad \begin{cases} \tau_{init} = \max(0, t - \Delta_t[r]) \\ u_{b,t} = \sum_{r \in R} v_{b,r,t} + \sum_{r \in R} \sum_{\tau=init}^{t-1} v_{b,r,\tau}, \\ u_{b,t} \leq AU_b, \quad \forall b \in B, t \in T, \\ \text{if } CT_r > P, \quad u_{b,t} + \tau \geq v_{b,r,t}, \quad \forall \tau \leq \Delta_{t,r} \end{cases} \tag{13}$$

$$q_{b,r,t} \leq LF_b \cdot SC_b \cdot f_{b,r,t}, \tag{8}$$

Equation (3) ensures consistency between the number of buses assigned to a route and the selected fleet configuration. Equation (4) defines the bus headway (interval between departures) according to fleet size. Equation (5) imposes an upper bound on expected passenger waiting time, relaxed if necessary via binary variable $y_{r,t}$. Equation (6) enforces that only one fleet size configuration is chosen per route and period.

Equations (7) model standing passengers $w_{r,t}$ using a big-M logic, activating only when crowding is present. Constraint (8) ensures that passenger assignments do not exceed each bus's standing capacity. Equation (9) determines how many buses are actively operating, differentiating between long-cycle and short-cycle routes. Equation (10) quantifies overloads when assigned passengers exceed seated capacity. Constraint (11) ensures total assigned passengers respect the demand limit. Equation (12) guarantees that the number of buses in operation does not exceed the available units for each time step. Finally, equations (13) introduce time-coupled constraints to enforce rolling fleet availability: it accumulates the use of buses over recent intervals for long routes (through $u_{b,t}$), and ensures that vehicles on such routes remain unavailable for reuse until their full travel cycle is completed. These constraints ensure temporal consistency and resource continuity.

5. Experimental Setup

The mathematical models were implemented and tested on a machine equipped with an Intel Core i7-12700F processor (20 threads), 32GB of RAM (4800MHz), and running Ubuntu 22.04. All experiments were performed using Gurobi Optimizer version 12.0.1, interfaced through the Python 3.11.9 API. No custom heuristics or decomposition techniques were applied. The models were solved using Gurobi's default settings, with a time limit of 60 minutes per instance.

5.1. Data Description

The real data used in this study were obtained from previous works, such as those presented in Caetano et al. [2022, 2025]. Since the available data cover only a single period, temporal demand variations were modeled using standard profiles provided by National Academies of Sciences, Engineering, and Medicine [2022], which are widely adopted in transportation research to simulate realistic intraday fluctuations. This strategy allows for controlled variation of key parameters while addressing the lack of granular real-world datasets. The objective of the experiments is to assess the computational viability of the proposed model and to analyze how different configurations of demand, operational costs, and fleet size affect solution quality.

5.2. Scenarios Generation

The base scenario uses information from Table 3, 4 and 5 for fleet, costs and demand distribution, respectively. The methodology for defining costs follows the article of Caetano et al. [2022]: waiting cost is based on minimum wage (1518.00 BRL) divided by 44 hours (minimum weekly working hours) [Valor Econômico, 2025]. Agglomeration cost was decided to be twice the waiting cost; the fare cost is the current one in the real data city (Niterói), and the transfer cost is twice the fare cost [Prefeitura de Niterói, 2024]. The value of AW was set to 5 minutes, as this is the value considered acceptable in the literature Caetano et al. [2025]. The value of M was decided after running the base scenario, ranging from 10^2 to 10^8 . From 10^6 onwards, there were problems of numerical instability detected. Therefore, the value $M = 10^4$ was chosen, as it was the one that took the least time to run. The value of discretization factor ($\alpha = 5$) was decided empirically after tests comparing results of non-linear and linearized models.

Table 3: Bus Fleet Characteristics

B	SC	OC	CC	LF	AU	$AU \cdot 2.4$
1	42	4.20	0.38	1.90	78	187
2	37	4.83	0.43	1.84	14	33
3	35	4.72	0.42	2.00	18	43
4	42	4.16	0.37	1.93	37	88
5	43	3.94	0.36	1.98	18	43
6	39	3.99	0.35	2.15	7	16
7	32	3.68	0.33	2.81	18	43

Source: Adapted from Caetano et al. [2025]

Table 4: Costs

Costs	Value
WC - Waiting cost	0.5750
AC - Agglomeration cost	1.1500
FC - Fare cost	5.1500
TC - Transfer cost	10.3000
CC - Carbon emission tax cost	1.2800

Source: Created by the authors.

Note: B – Bus type; SC – Seating capacity (units); OC – Operational cost (BRL/km); CC – Environmental costs (BRL/kg of CO₂); LF – Maximum load factor (dimensionless); AU – Available fleet (units).

Table 5: Demand Distribution of Passengers Throughout the Day

Hour	Demand Distribution	Hour	Demand Distribution	Hour	Demand Distribution
6 a.m	0.017	12 p.m	0.071	6 p.m	0.063
7 a.m	0.024	1 p.m	0.071	7 p.m	0.052
8 a.m	0.035	2 p.m	0.072	8 p.m	0.044
9 a.m	0.046	3 p.m	0.073	9 p.m	0.038
10 a.m	0.056	4 p.m	0.073	10 p.m	0.033
11 a.m	0.054	5 p.m	0.073	11 p.m	0.026

Source: Adapted from National Academies of Sciences, Engineering, and Medicine [2022], Exhibit 17-5.

Moreover, the algorithm 1 distributes each route's total passenger demand across 18 time periods (from 6 AM to 11 PM) based on a predefined temporal demand distribution vector. It normalizes the base demand of each route by the peak value of the distribution and allocates demand proportionally to each period. The result is a matrix that represents the estimated maximum demand per route and period, which serves as input to the optimization model.

Algorithm 1: Period Demand Generator using Normalized Route Capacity

Input : Scenario scn ; demand distribution vector $dist$
Output: Demand matrix $Q_{\max}[(r, t)]$ for 18 periods

```

1 InitializeParameters( )
2 foreach  $r \in Routes$  do
3    $base\_demand \leftarrow scn.routes[r].passengersPerHour$ 
4    $scaled\_demand \leftarrow base\_demand / \max(dist)$ 
5   foreach  $t \in Time$  do
6      $Q_{r,t} \leftarrow scaled\_demand \cdot dist[t]$ 
7   end
8 end

```

Although the model takes into account situations in which the fleet is insufficient for all passengers, it is necessary to define a base scenario with sufficient capacity to meet total demand. After a series of preliminary simulations, a multiplication factor of 2.4 was found to be sufficient to cover the multi-period base demand, which was used to create the base scenario. Others scenarios use the base scenario as a reference, with modifications described in Table 6.

Table 6: Description of Evaluated Scenarios

Scenario	Description
1	Base scenario
2	Passenger demand increased to 120%
3	Passenger demand increased to 150%
4	Passenger demand increased to 200%
5	Fleet availability reduced to 50% of units
6	Fleet availability reduced to 70% of units
7	Agglomeration cost (AC) and waiting cost (WC) increased fivefold
8	Operational cost (OC) of each bus type equals 0.1 BRL

Source: Created by the authors.

5.3. Performance Metrics

To evaluate the computational performance and solution quality of the model, we measure several key metrics. The execution time is the total time (in seconds) taken by Gurobi to solve each instance, reflecting the computational efficiency of the models. The gap quantifies the quality of the solution, defined as $\frac{|z_P - z_D|}{|z_P|}$, where z_P is the best known primal (feasible) objective value, and z_D is the best dual bound obtained during the solution process. This metric indicates how close the solution is to optimality, with a smaller gap corresponding to a better solution.

Table 7: Route Characteristics Used in Tests

Route	Length	Time	Base Demand	Route	Length	Time	Base Demand	Route	Length	Time	Base Demand
1	18 km	65 min	200	9	38 km	115 min	700	17	11 km	37 min	550
2	22 km	75 min	900	10	21 km	66 min	200	18	32 km	100 min	70
3	18 km	61 min	300	11	21 km	68 min	400	19	40 km	131 min	300
4	8 km	38 min	80	12	14 km	48 min	600	20	11 km	32 min	50
5	32 km	100 min	600	13	49 km	155 min	1050	21	40 km	129 min	350
6	25 km	79 min	500	14	36 km	116 min	1000	22	47 km	139 min	300
7	22 km	72 min	200	15	39 km	118 min	200	23	43 km	148 min	350
8	57 km	186 min	500	16	24 km	81 min	100				

Source: Adapted from [Caetano et al., 2025].

6. Results and Discussion

The result was the optimization of the aforementioned scenarios, with an overview in Table 8. The percentages of the costs were normalized in relation to the baseline scenario. The 50% fleet scenario shows an 18.55% reduction in operational and carbon emission costs, but this economy is overshadowed by substantial increases in waiting costs (59.33%) and agglomeration costs (23.92%). The extraordinary rise in transfer costs (1188.39%) indicates severe deterioration in network connectivity. Critically, the penalty for unserved demand (2.02% of passengers) contributes significantly to the extreme increase in the objective function (2493.93% of baseline). The average headway increase from 7.52 to 11.98 minutes confirms the degradation in service quality. In contrast, the 70% fleet scenario presents a total cost only 0.89% higher than baseline, with an average headway of 7.75 minutes, suggesting a near-optimal fleet dimensioning point for decision-makers seeking operational efficiency without compromising service quality.

Table 8: Comparison of Values Between Scenarios

	OC (%)	CC (%)	WC (%)	AC (%)	TC (%)	FC (%)	Total Objective (%)	Headway Mean (min)	Passengers Not Attended (%)	Runtime (s)	Gap
Baseline Scenario	100.00	100.00	100.00	100.00	100.00	100.00	100.00	7.52	0.00	97	0.000000
50% of Fleet	81.45	81.45	159.33	123.92	1288.39	97.98	2493.93	11.98	2.02	3600	0.003376
70% of Fleet	101.48	101.52	103.01	100.29	119.72	100.00	100.89	7.75	0.00	3600	0.000012
120% Demand	108.10	108.13	91.73	137.22	85.60	119.86	115.98	6.90	0.00	3600	0.000003
150% Demand	123.17	123.23	68.05	193.51	26.33	149.91	140.31	5.12	0.00	3600	0.000164
200% Demand	144.70	144.76	71.69	281.44	101.13	196.36	4470.38	5.39	1.82	3600	0.002702
5x WC and AC	130.49	130.56	431.01	90.34	87.53	100.00	110.03	6.48	0.00	322	0.000000
OC = 0.1 BRL	3.38	138.80	73.13	8.23	74.74	100.00	68.38	5.50	0.00	3600	0.000004

Note: OC - Operational Cost; CC - Carbon Emission Cost; WC - Waiting Cost; AC - Agglomeration Cost;

TC - Transfer Cost; FC - Fare Cost

Source: Created by the authors.

As demand increases to 120% and 150%, headways decrease (6.90 and 5.12 minutes respectively), indicating higher service frequency. This results in reduced waiting costs: -8.27% for 120% demand and -31.95% for 150% demand. However, at 200% demand, the system shows

clear saturation signs. Despite maintaining a relatively low headway (5.39 minutes), 1.82% of passengers remain unserved, contributing significantly to the extreme increase in total objective cost (4470.38%). This demonstrates that both insufficient fleet capacity and excessive demand can push the system beyond its functional limits, a critical consideration for long-term transit planning.

Furthermore, the scenario with quintupled waiting and agglomeration costs demonstrates how objective function weights direct the model toward different operational trade-offs. The average headway decreases to 6.48 minutes, despite a 30.49% increase in operational costs. Simultaneously, agglomeration costs decrease by 9.66%, indicating system reconfiguration to minimize crowding in response to higher penalties. Finally, in the reduced operational cost scenario (0.1 BRL), the average headway drops to 5.50 minutes, demonstrating that optimization of operational costs can yield system configurations with higher service frequency and improved user experience.

This behavior exemplifies how modifying objective function weights can serve as a policy instrument to prioritize different service quality aspects according to public needs. For instance, decision-makers concerned primarily with passenger comfort might accept higher operational costs to reduce crowding and waiting times. These results provide decision-makers with valuable insights into the non-linear relationships between costs, fleet size, demand levels, and service quality. They demonstrate that proper calibration of cost components in the objective function can effectively achieve different balances between operational efficiency and service quality, allowing transit managers to align system performance with established priorities and available resources.

Across all extreme cases, the solver reaches the 3600 s limit yet maintains low optimality gaps (≤ 0.0034), confirming that observed cost surges arise from genuine capacity–demand imbalances rather than numerical artifacts. These dynamics validate that the model effectively captures critical real-world nuances, capacity thresholds, adaptive frequency control, and cost trade-offs, offering robust decision support.

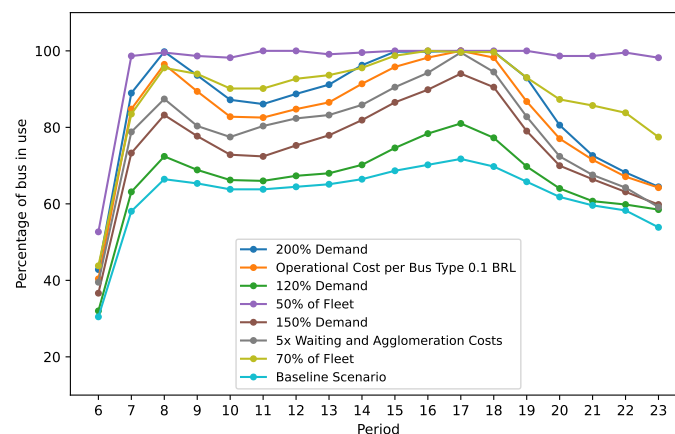


Figure 1: Bus Utilization by Period in Different Scenarios

Examining the temporal dynamics depicted in Figure 1, it is evident that the model deliberately avoids allocating too many buses outside peak hours to ensure vehicle availability during peak periods, when the risk of overcrowding is higher. Fortunately, this precaution does not negatively affect waiting times (as seen in Figure 3) or overcrowding (Figure 2), especially during off-peak hours, indicating that the model is also able to meet both peak and off-peak demand effectively. The scenarios produced the expected outcomes, where an increase in demand can lead to the concentration of buses on certain routes to mitigate overcrowding, at the expense of routes

with lower passenger volumes (e.g. routes 17 and 19). Furthermore, lower operational costs or higher penalties related to waiting time and overcrowding increase overall fleet utilization (Figure 1), which consequently reduces the proportion of standing passengers relative to the total number of passengers (Figure 2).

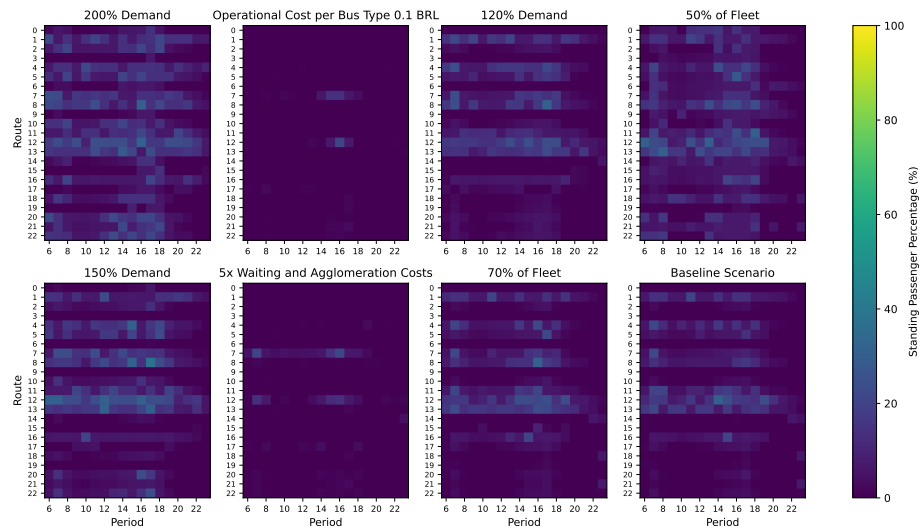


Figure 2: Percentage of Standing Passengers in Different Scenarios

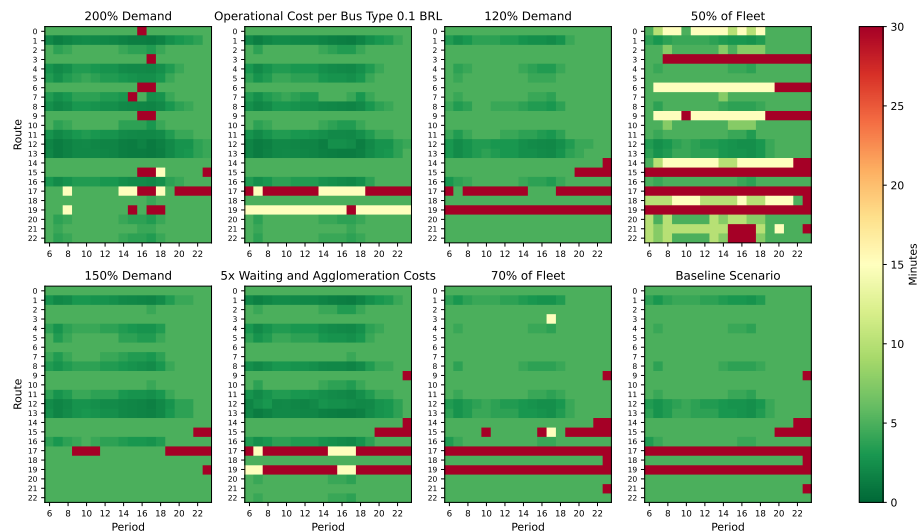


Figure 3: Percentage of Standing Passengers in Different Scenarios

These patterns offer valuable insights for urban mobility planning. For instance, the model's behavior suggests that dynamic reallocation of buses based on time-of-day demand is an effective strategy to maximize service efficiency without compromising user experience. This could support the implementation of time-dependent service schedules, where vehicle allocation is adapted in real time or semi-real time. Moreover, the ability to preserve performance under varying demand levels strengthens the case for demand-responsive transit systems, especially in cities with

fluctuating passenger flows throughout the day.

7. Conclusions

This work proposes a multiperiod optimization model for bus frequency and fleet allocation in urban transport systems. The model addresses key operational challenges by jointly minimizing operational costs, passenger discomfort, and emissions through time-resolved decision variables and realistic service constraints. Results across simulated scenarios show the model's ability to dynamically adapt to demand variations, particularly during peak periods, ensuring efficient resource use and improved passenger service. For transport operators, this translates into actionable benefits: reduced overcapacity, mitigation of overloads, and improved fleet utilization without compromising service standards.

A key contribution is the integration of headway constraints, crowding penalties, and time-coupled vehicle availability, allowing realistic modeling of fleet commitments across periods. Despite relying on fixed demand profiles and fleet assumptions, the model provides a flexible foundation for tactical planning. As future work, incorporating demand uncertainty through stochastic modeling and integrating real-time data sources could improve adaptability to disruptions and enhance operational robustness in real-world deployments.

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