

A PROPOSAL OF AN ADAPTED ANALYTIC HIERARCHY PROCESS METHOD TO DETERMINE THE SHORTAGE COSTS OF REPAIRABLE SUPPLIES.

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ABSTRACT

This paper aims to propose the application of an adapted Analytic Hierarchy Process method to the problem of determining the shortage cost of a repairable supply. Repairable supplies are spare parts, capable of being repaired, belonging to equipment. The mathematical models seek to minimize the logistic cost of the supply chain, which is provided by the sum of ordering cost, storage cost, acquisition cost, and shortage cost. Of these types, the only one with a total degree of certainty is the acquisition cost, while the others are uncertain. Two scenarios were designed: Firstly, one with the ordering costs, storage costs, and acquisition costs are known and another where only the acquisition cost is known. Finally, it was concluded that the AHP method is easily implementable and can be applied to this problem if a different scale from the original is adopted.

KEYWORDS. Repairable Supplies, Shortage Cost, Analytic Hierarchy Process Method.

Paper topics: Logistics and Transportation, OR in Production Management and Administration, Other Areas.

1. Introduction

Nowadays, both private organizations and public sector entities have shown special attention to their supply chains. Even if a given company does not primarily focus on the supply of spare parts, its activities will be directly impacted by the lack of such items. For example, imagine a hospital where the CT scanner is unavailable due to the lack of a spare part. Certainly, managing the supply chain for that spare part is not the hospital's main activity, which is to attend to patients, but it will be affected by the lack of the spare part.

Spare parts are classified into repairable and non-repairable items, the latter also being called consumable supplies. Repairable supplies are of greater importance compared to non-repairable items because the logistical costs of a repairable item are significantly higher than those of consumable items, as observed in [Igawa 2006] and [Machado 2017].

Repairable items appear quite frequently in the maintenance of aeronautical items such as helicopters and airplanes. For example, a helicopter turbine is considered a repairable item because, when it malfunctions, it is sent to a repair center for intervention and then restored to working condition.

However, such items appear in a wide range of activities. As previously mentioned, in hospitals, equipment such as CT scanners, ultrasound machines, and X-ray machines are also subject to interventions, and their components can be sent for repair. Items in the telecommunications and naval industries also fall into the category of repairable items. A more detailed analysis of the subject will also identify such items in bus company fleets and fleets of vehicles, aircraft, and vessels of the Armed Forces and Auxiliary Forces, such as the Military Police, Firefighters, and Municipal Guards.

The most common approaches to the managing supply chain of repairable items found in the literature involve determining a closed-form mathematical expression for the total cost of the supply chain and minimizing it through non-linear optimization techniques. To do so, these expressions assume that the logistical costs are known and seek to determine trivial aspects of an inventory policy, such as determining the Economic Order Quantity (EOQ) and the Reorder Point.

According to [Taha 2008], the main logistical costs involved in a supply chain are the ordering cost, the acquisition cost, the holding cost, and the shortage cost. The first comprises the fixed charges incurred when a purchase order is issued, regardless of its size. These costs include the salary of the employee who places the order, and expenses for electricity, internet, and telephone, for example. The second is quite intuitive and refers to the price per unit of a given item.

The third cost, holding cost, encompasses expenses related to maintaining the item in stock. Typically, these costs include expenses related to renting physical space (warehousing), handling costs (forklifts, cranes, robots, etc.), as well as maintenance costs. The last type of cost involves expenses incurred due to the absence of a spare part in stock.

Models found in the literature generally assume that such costs are known. The ordering cost is relatively simple to determine since it deals with tangible things. The holding cost is also relatively straightforward to determine, as companies typically outsource item storage by renting a warehouse or special facility for storing repairable items. The acquisition cost is the easiest to determine, as it is presented to the customer at the time of quotation, being the only known logistical cost and free of uncertainties.

The biggest issue, however, lies in determining the shortage costs. The shortage costs involve various intangible impacts. One of them would be the company or organization's image. If the company is unable to serve its clientele due to the unavailability of its equipment, surely customer confidence in the business will decline, and consequently, the company's profit as well.

Let's suppose an aircraft is not available for a specific rescue operation. A catastrophe occurs, and this aircraft is not available for rescue or patient transfer to the hospital. The life of that patient should (or at least should) be taken into consideration in the cost of shortages for the item.

Because the shortage cost involves many uncertainties, important intangibles, and is heavily influenced by the subjectivity of the decision-maker, it is suggested in this study to adopt a Multicriteria methodology based on the Analytic Hierarchy Process - AHP, which is widely

explored in various Operations Research books such as [Taha 2008] and [Shimizu 2006], for example, although there are several criticisms in the literature regarding this method.

Based on the above, the problem described can be summarized in the following research questions:

- 1) How can the unit shortage cost of a repairable supply be determined, considering the other known logistical costs? And if these other logistical costs are not known?
- 2) How can the AHP method be applied to this problem? What modifications are imperative for the method to be effective?

To address these questions, two scenarios were proposed. The first considers that the ordering cost, holding cost, and acquisition cost are fully known and free from uncertainties. In the second scenario, it is assumed that only the acquisition cost of the item is fully known and free from uncertainties.

2. Literature Review

The concern to determine the optimal purchase quantity and its corresponding reorder point based on minimizing logistical costs is relatively new, dating back to the early 20th century with the emergence of the EOQ formula. The calculation of EOQ depends primarily on ordering costs and holding costs. This model does not consider stochastic item demand and therefore assumes that no shortages will occur, making the total shortage cost equal to 0.

Several models related to supply management emerged in the subsequent years. A classic literature on the subject is the work of [Hadley and Whithin 1963], which compiles various conventional supply management models.

The first model related to the management of repairable supplies only appears in the work of [Schrady 1967]. In this article, a formula quite similar to EOQ is developed, which depends solely on ordering and holding costs. The following year, [Sheerbrooke 1968] developed a probabilistic model that considers the demand for repairable items to follow a Poisson distribution but does not consider the logistical costs of the supply chain.

In the late 1970s, [Nahmias and Rivera 1979] refined the model of [Schrady 1967] and proposed a model based on all the costs of previous models. For these authors, the holding cost is also divided into two types: the holding cost of items in service and the holding cost of items in need of repair.

From the 1990s to the present day, models related to the management of repairable supplies have been growing exponentially. Some noteworthy works include: [Mabini et al. 1992], [Richter 1996], [Koh et al. 2002], [Wong et al. 2005], [Konstantaras et al. 2010], [Basten et al. 2012], [Tracht et al. 2013], and [Arts 2017].

Another work elaborated in the last three decades that deserves attention is [Teunter 2004], which refines the models developed by [Schrady 1967] and [Nahmias and Rivera 1979]. The proposed model depends on ordering and holding costs, without mentioning the shortage cost or acquisition cost.

However, the practical application of the mathematical models presented in the literature requires that the shortage cost be fully known, which is not a reality in many organizations. Some works, in literature, addressed the measurement of the impact of shortage cost in the supply chain. [Rossi et al. 2012], [Xu, 2016] and [Xu and Chan, 2019] are some examples.

Concerning the AHP method, it was originally presented in [Saaty 1977]. This model is applied to various problems, such as supplier selection, for example. In the field of supply chain management, particularly in the evaluation of logistical risks, the AHP method is widely used. However, the applicability of the AHP method in the determining logistical costs of the supply chain has a rather scarce literature, which justifies the development of this work.

[Salehi et al. 2018] apply the Fuzzy AHP method to the logistical costs of a supply chain and state that the traditional AHP method may not accurately reflect human preferences. [Vergara et al. 2020] use the traditional AHP method to determine which criteria most impact the delivery of value to the customer by the supply chain.

3. Method

To determine the shortage cost of a given repairable supply, this study proposes the adoption of the AHP method, with some adaptations to the peculiarities of the presented problem. As mentioned earlier, the AHP method was proposed by Saaty in the 1970s in the United States and has been employed in situations such as priority definitions, cost-benefit evaluations and resource allocation, among numerous other applications.

To estimate the unit shortage cost (C_i) of a repairable item, one must first compare it with the other logistical costs of the supply chain, including the ordering cost (C_p), the holding cost (C_s), and the acquisition cost of the item (C_a). Among these three types of costs, the only cost that does not involve uncertainties is the acquisition cost, as it can be obtained from the supplier through a price quote, giving the exact value for the acquisition of a new item. The other two types of costs, the ordering cost and the holding cost, although easier to determine than the shortage cost, involve some relatively uncommon calculations.

At this point, was developed a methodology to obtain the shortage cost considering the exact values of the ordering cost and holding cost, free from uncertainties. Subsequently, the methodology will be expanded by considering only the acquisition cost as a known value, with the other two types of costs subject to uncertainties and inaccuracies.

3.1 – Application of AHP considering other logistical costs known.

To begin the application of the AHP method, one must first construct the pairwise comparison matrix, which in this case will be a 4 x 4 square matrix, as there are four types of costs involved. This matrix should be developed by eliciting from the decision-maker the importance they attribute to the shortage cost relative to the ordering cost, the holding cost, and the acquisition cost.

1) Reciprocal Property:

$$a_{ij} = \frac{1}{a_{ji}} \quad \forall i, j. \quad (1)$$

This means that if cost i is twice (for example) as important as cost j , then cost j is half as important as cost i .

2) Diagonal Elements:

$$a_{ij} = 1, \forall i = j. \quad (2)$$

This reflects the fact that any cost compared to itself has equal importance, hence the value of 1.

As a variant of the AHP method, considering the ordering cost, holding cost, and acquisition cost which is the only calculated accurately and free from uncertainties, we have:

$$a_{21} = \frac{C_s}{C_p}, \quad (3)$$

$$a_{31} = \frac{C_a}{C_p}, \quad (4)$$

$$a_{32} = \frac{C_a}{C_s}. \quad (5)$$

In this way, it is now necessary to elicit from the decision-maker only the values to be assigned to a_{41} , a_{42} and a_{43} positions, since all elements of the pairwise matrix adhere to the properties of equalities (1) and (2).

To elicit the above values from the decision-maker, it is necessary to establish an importance index of the shortage cost relative to the other costs. At this point, a modification of

the traditional scale proposed by Saaty is necessary. The traditional scale consists of integer values from 1 to 9. In this case, as the acquisition cost of the item is generally thousands of times greater than the ordering cost and holding cost, then, assigning a value from 1 to 9 loses its meaning, as will be discussed later.

In this way, a completely variable scale is used, where the values that can be assumed may be fractional and do not have an upper limit. Adopting a scale of this nature will require more cautious elicitation from the decision-maker, making it clear to them the values of the known costs (ordering, holding, and acquisition) and requesting that they indicate how many times the shortage cost will be greater (or smaller) than each of these costs.

To illustrate, let's consider that the acquisition cost of the item is \$50,000.00, the ordering cost is very low, around \$20.00, and the holding cost is \$50.00 per unit per month. In this case, the acquisition cost is 2,500 times greater than the ordering cost and 1,000 times greater than the holding cost. Therefore, if the decision-maker considers the ordering cost to be more important than the acquisition cost, the shortage cost relative to the ordering cost should be more than 2,500 times greater than the ordering cost and more than 1,000 times greater than the holding cost. Thus, using the traditional scale would not be feasible in this type of problem.

After eliciting the preferences mentioned above and completing the 16 positions of the pairwise comparison matrix, it is necessary to verify the consistency of the model.

At this point, [Saaty 1980] defines that the consistency index (IC), which is given by equation (6), should be divided by the random consistency index (IR), thus providing the Consistency Ratio (RC) as per equation (7). The IC is calculated based on the maximum eigenvalue of this matrix and the order n of the matrix, in this case, $n = 4$. According to [Gomes et al. 2011], the IR is a random index calculated for square matrices of order n by the Oak Ridge National Laboratory in the United States. For the case at hand, when $n = 4$, the matrix has an IR of 0.90, as observed in [Triantaphyllou and Mann 1995].

$$IC = \frac{(\lambda_{\max} - n)}{n - 1}. \quad (6)$$

$$RC = IC/IR. \quad (7)$$

According to [Saaty 1980], if the value of RC is less than 0.09 for matrices of order 4, the model is considered consistent. Otherwise, there is inconsistency in the model, and elicitation with the decision-maker should be conducted again. A major reason for inconsistencies in the model is the adoption of highly disparate values. For example, in the example, the decision-maker assigns a value of 3,000 for the relationship between the shortage cost and the ordering cost. For the relationship between the shortage cost and the holding cost, this value should be lower by about 2.5 times, as the holding cost is 2.5 times higher. Therefore, if the decision-maker assigns a value much lower than 2.5 times less than 3,000 (i.e., a value around 1,200), inconsistency in the model may occur.

It is worth noting that by substituting equation (6) into (7) and adopting RI equal to 0.9, we have:

$$\frac{\lambda_{\max} - 4}{(4 - 1) \cdot 0.9} < 0.09. \quad (8)$$

Developing this inequality, we have:

$$\lambda_{\max} < 4.243. \quad (9)$$

[Saaty 1977] enunciates a theorem that states that the matrix is consistent if and only if $\lambda_{\max} \geq n$. Therefore, for the present case, the lower bound of the maximum eigenvalue will be 4. Thus, for the consistency of the model, it must have:

$$4 \leq \lambda_{max} \leq 4,243. \quad (10)$$

After validating the model by measuring its inconsistency, the pairwise comparison matrix must be standardized by dividing each element by the sum of the values in the column to which it belongs. Then, the relative priority of each cost is calculated by taking the arithmetic mean of the row values.

Since the values of the ordering, holding, and acquisition costs are known and considered accurate, the shortage cost is obtained by taking the arithmetic mean of these three types of costs and using a simple proportionality rule. To exemplify and clarify understanding, consider the following example:

Supposing that $C_p=20$, $C_s=50$ e $C_a=50.000$. During elicitation, the decision-maker mentioned that the shortage cost, in their opinion, would be about 3,000 times greater than the ordering cost, 1,500 times greater than the holding cost, and 1.4 times greater than the acquisition cost. Thus, the pairwise comparison matrix (M) would have the following format:

$$M = \begin{pmatrix} 1 & 0.4 & 0.0004 & 0.00033 \\ 2.5 & 1 & 0.001 & 0.00067 \\ 2500 & 1000 & 1 & 0.7142857 \\ 3000 & 1500 & 1.4 & 1 \end{pmatrix}. \quad (11)$$

At this point, the consistency index of the model is determined by finding the largest eigenvalue of matrix M , which is easily obtained by subtracting the eigenvalue (λ) from the main diagonal and setting the determinant of this matrix equal to 0:

$$\begin{vmatrix} 1 - \lambda & 0.4 & 0.0004 & 0.00033 \\ 2.5 & 1 - \lambda & 0.001 & 0.00067 \\ 2500 & 1000 & 1 - \lambda & 0.7142857 \\ 3000 & 1500 & 1.4 & 1 - \lambda \end{vmatrix} = 0. \quad (12)$$

Resolving the determinant, we have the characteristic polynomial of the matrix, which when set to zero gives us:

$$\lambda^4 - 4\lambda^3 + 0.00500002\lambda^2 - 0.0730714\lambda = 0. \quad (13)$$

It is easily observed that one root is $\lambda = 0$, which is disregarded, and we have:

$$\lambda^3 - 4\lambda^2 + 0.00500002\lambda - 0.0730714 = 0. \quad (14)$$

Solving this equation, we have $\lambda \approx 4.00331$, which is situated within the interval defined by expression (10), leading us to believe that the model can be considered consistent. If a value outside the range $[4 - 4.243[$ were observed, the model would be considered inconsistent, and the elicitation of values for the pairwise matrix should be conducted again. The next step would be to standardize the matrix, obtaining the matrix M^* :

$$M^* = \begin{pmatrix} 0.000181703 & 0.00015991 & 0.000167 & 0.000194 \\ 0.000454256 & 0.000399776 & 0.000416 & 0.000389 \\ 0.454256382 & 0.399776125 & 0.416424 & 0.416424 \\ 0.545107659 & 0.599664188 & 0.582993 & 0.582993 \end{pmatrix}. \quad (15)$$

It is important to note that the sum of the elements in each column of the matrix is equal to 1, indicating that the matrix is standardized. The relative priority of each cost is provided by the arithmetic mean of the values in each row, where row 1 represents the priority of the ordering cost,

row 2 the priority of the holding cost, row 3 the priority of the acquisition cost, and in the last row, the priority of the shortage cost. Therefore, we have the following relative priorities:

	C_p	C_s	C_a	C_f	Relative Priority
C_p	0.000181703	0.00015991	0.000167	0.000194	0.000176
C_s	0.000454256	0.000399776	0.000416	0.000389	0.000415
C_a	0.454256382	0.399776125	0.416424	0.416424	0.42172
C_f	0.545107659	0.599664188	0.582993	0.582993	0.57769
Total	1	1	1	1	1

Table 1 – Standardized matrix with the relatives' priorities of each cost type. Source: Authors

From reading Table 1, it can be observed that the ordering cost (C_p), in the decision-maker's view, represents 0.000176 or 0.0176% of the total cost. The holding cost (C_s) represents 0.000415 or 0.0415% of the total cost, the acquisition cost represents 42.172%, and the shortage cost represents 57.769% of the total cost. All costs are known, except for the shortage cost, therefore, the sum of all costs can be determined through a simple proportionality rule, where in this case, \$20.00 represents 0.0176%, \$50.00 represents 0.0415%, and \$50,000.00 represents 42.172%. Then, we would obtain the following values for the sum of costs (100%):

	Relative Priority	Cost Value	Sum of the Cost Value (100%)
C_p	0.000176	20.00	113,876.80
C_s	0.000415	50.00	120,545.95
C_a	0.42172	50,000.00	118,562.08
C_f	0.57769	-	-
Mean	-	-	117,661.61

Table 2 – Sum of the cost determination. Source: Authors

From Table 2, it can be observed that we have three different values for the sum of costs, which is perfectly normal since the decision-maker will hardly be able to elicit the values of the pairwise matrix linearly. However, due to the high consistency of the model, it is noted that the values found for the sum of costs are quite close, and in this case, the average will provide a good approximation for it.

The final step consists of determining the shortage cost, knowing the average of sum of the costs (\$ 117,661.61), which corresponds to 100%, through a simple proportionality rule. In the example in question, we obtain the shortage cost value of \$ 67,971.89 (sixty-seven thousand, nine hundred and seventy-one dollars and eighty-nine cents). It is noted that this value is higher than the acquisition cost, which is fully compatible with the decision-maker's intention since it was elicited that the shortage cost would be about 3,000 times greater than the ordering cost, while the acquisition cost is 2,500 times greater than the ordering cost.

3.2 – Application of AHP considering only the acquisition cost known.

The methodology to be presented considers that only the acquisition cost is known, while the other costs (ordering cost and holding cost, as well as shortage cost) are uncertain. Such methodology can be adapted in case any of the other two costs are known with exactitude.

In this scenario, as the ordering cost and holding cost are unknown, there is a need to elicit from the decision-maker the relationship between these costs taken pairwise. To summarize, consider Table 3 below, where all possible combinations are shown, and the X marks indicate the elicitation that must be done with the decision-maker:

	C_p	C_s	C_a	C_f
C_p	-	-	-	-
C_s	X	-	-	-
C_a	X	X	-	-
C_f	X	X	X	-

Table 3 - Elicitations to be conducted with the decision-maker. Source: Authors.

It is important to note that it is not necessary to elicit all possible combinations, since properties (1) and (2) hold in the pairwise comparison matrix. Generally, the number of elicitations to be conducted in the AHP method is given by the expression:

$$\frac{n(n-1)}{2}, \text{ where } n \text{ is the order of the matrix.} \quad (16)$$

In this case, we have $(4 \times 3)/2 = 6$ elicitations.

Since only the acquisition cost of the material is known and reliable, efforts should be made to elicit the values from Table 3, highlighting to the decision-maker the value of the item's acquisition cost.

For example, one way to elicit the value a_{31} would be by presenting the decision-maker with the following question: "Considering the value of \$50,000.00 for the item's acquisition cost, how many times is the ordering cost less than the acquisition cost?"

The elicitation should start with the values involving the acquisition cost, which is the known cost (a_{31} , a_{32} and a_{43}). These values will be more reliable than the other three values of the pairwise comparison matrix, being less susceptible to errors since the decision-maker estimated them based on the known cost.

Having conducted the above-described elicitations, the process proceeds similarly to the previous case, constructing the pairwise comparison matrix and determining the matrix's eigenvalue, which should be situated within the values defined by the expression (10). If it does not, the elicitation process should be repeated, paying attention to possible changes in the values that do not involve the acquisition cost (in this case, a_{21} , a_{24} and a_{42}).

After verifying the model's consistency, the pairwise comparison matrix is standardized, and the relative priority of each cost is determined. To determine the total cost value and the values of the ordering, storage, and shortage costs, only a single proportional rule is used, based on the acquisition cost, as it is the only known cost.

For example, let's assume that the relative priorities of the costs found are the same as in Table 1, where the acquisition cost priority is 42.172% of the total cost and its value is \$50,000. Therefore, the total cost would be 100%, equivalent to $\$50,000/0.42172 = \$118,562.08$. Note that at this point, we cannot take the average since the acquisition cost is the only known one. Therefore, the total cost is considered to be \$118,562.08. Using the rule of three, the other costs are determined (Ordering cost = \$20.87, Storage cost = \$49.20, and Shortage cost = \$68,492.13).

4. Results and Discussions

From the analysis of the presented methodology, it can be observed that the proposal to adopt a different scale from that defined by [Saaty 1977], where values range from 1 to 9, may be subject to some criticism. According to [Triantaphyllou and Mann 1995], Saaty developed this scale based on studies in the theory of Psychology developed by Miller in 1956, which states that individuals cannot simultaneously compare more than seven objects with a tolerance of two objects more or less (that is, from 5 to 9 objects).

However, it is understood that, in the case at hand, it is not a matter of comparing more than 9 objects, since the decision maker compares only 4 types of costs. Furthermore, both cases mentioned in the previous section assume that at least the cost of acquiring the item is known, and

the decision maker will estimate the other costs based on comparing them with the cost of acquisition. In fact, it is not a preference between costs but rather a valuation.

The question of adopting a different scale from the original proposal of the AHP method has been addressed by other authors, among whom we can mention [Triantaphyllou et al. 1994], who propose 78 different scales. [Lootsma 1990] proposes a scale ranging from -8 to +8 in a variation of the AHP method, called Multiplicative AHP. Saaty himself in [Saaty 2008] proposes the adoption of a scale from 1 to 9, including intermediate values between 1 and 2, varying by 0.1 each. Thus, it is evident that the use of a scale of values different from the traditional one should not be seen as a deficiency of the model.

In practical terms, considering the example presented, the decision maker could understand that the cost of the order and the cost of storage have the same importance, thus having identical values. The cost of shortage would be slightly more important than the cost of acquiring the item, so the cost of acquisition compared to the cost of the order would receive a score of 8, as well as the cost of acquisition compared to storage. Meanwhile, the cost of shortage would receive a score of 9 when compared to both the cost of the order and the cost of storage. In this case, we would have the following pairwise comparison matrix:

$$N = \begin{pmatrix} 1 & 1 & 1/8 & 1/9 \\ 1 & 1 & 1/8 & 1/9 \\ 8 & 8 & 1 & 1 \\ 9 & 9 & 1 & 1 \end{pmatrix}. \quad (17)$$

Applying the described methodology, we obtain an eigenvalue $\lambda_{\max} = 4.00173$, which is situated within the range predicted in expression (10), indicating that the model is considered consistent. Standardizing matrix N and determining the relative priorities of each cost, we have $C_p = C_s = 5.2705\%$, as the decision maker assigned equal importance to both costs, $C_a = 43.4137\%$, and $C_f = 46.0453\%$.

It is observed that the values of the cost of order and the cost of storage show a very high percentage. Let's assume that $C_a = \$50,000.00$ as in the example. The values obtained for the costs would be $C_p = C_s = \$6,070.45$ and $C_f = \$53,030.81$. The cost of order appears to be significantly overestimated, as nowadays, such costs are quite insignificant compared to the value of the material. If the item costs $\$1,000,000.00$, which is entirely possible for items like a helicopter turbine or a main transmission box, the ordering cost would jump to an astonishing $\$121,400.90$, which would be completely unrealistic, although the model is consistent.

One could counter-argue that an alternative to the original AHP scale would be the modified scale proposed by [Saaty 2008], containing intermediate values between 1-2. In this case, a possible pairwise comparison matrix is presented below:

$$P = \begin{pmatrix} 1 & 5/6 & 1/8 & 1/9 \\ 1.2 & 1 & 1/8 & 1/9 \\ 8 & 8 & 1 & 1 \\ 9 & 9 & 1 & 1 \end{pmatrix}. \quad (18)$$

Applying the described methodology, the model appears to be consistent with a $\lambda_{\max} = 4.00589$. However, the order cost remains extremely high, accounting for 5.05% of the total cost, totaling $\$5,817.48$.

Thus, it is observed that the use of the traditional AHP scale or even the one modified by [Saaty 2008] shows to be inadequate to model the problem, generating consistent but unrealistic values for the cost of order mainly.

Another aspect that makes the original scale unsuitable for modeling the problem would be the fact that the decision-maker can assign greater importance and consequently obtain a higher cost of shortage. Let's suppose that the decision-maker considers the cost of shortage to be much

more important than the cost of acquisition. In this case, we could have the following pairwise comparison matrix:

$$Q = \begin{pmatrix} 1 & 5/6 & 1/6 & 1/9 \\ 1.2 & 1 & 1/5 & 1/8 \\ 6 & 5 & 1 & 0.67 \\ 9 & 8 & 1.5 & 1 \end{pmatrix}. \quad (19)$$

The model above would yield a $\lambda_{\max} = 4.00052$, being considered consistent, but the order cost will represent 5.51% of the total cost, which would amount to \$8,333.33, remaining at high levels.

Given the above, the use of a scale of values different from the traditional scale ranging from 1-9 with integer values is justified. At this point, the assertion of [Salehi et al. 2018] is corroborated, where the traditional AHP method may not be able to fully express human thought.

5. Conclusion

Given what has been presented, it is observed that the AHP methodology can be used in the problem of determining costs, especially the cost of lack of a repairable supply, provided adaptations are made regarding the scale of values presented to the decision-maker. These adaptations should not be seen as a limitation of the model, but rather as a way to make the method viable for use in the present problem.

Special attention should be given to the elicitation phase of the decision-maker's preferences, where they will relate one cost to another in a pairwise manner. The known costs should be presented to the decision-maker to avoid inconsistencies in the model. If inconsistencies are found, elicitation should be conducted again, which can lead to the decision-maker's disbelief in the method, as well as consuming precious time. It is worth noting that a new elicitation should be made for each item analyzed.

Regarding the application of the method, it is observed that it is quite simple and can be implemented in a spreadsheet. To verify the consistency of the model, it is necessary to calculate the largest eigenvalue related to the pairwise comparison matrix. For this purpose, determinant calculators for matrices available online can be used, such as Matrixcalc, which can be found at the link: <https://matrixcalc.org>, and solvers for algebraic equations like Symbolab, available at the link <https://pt.symbolab.com>.

The method proposed here aims to fill gaps in the literature regarding the determination of logistic costs. Mathematical models in the literature that depend on the determination of such costs to define the best inventory policy become viable, as the impact of each specific cost on the total logistic cost can now be measured and understood.

Obviously, the topic presented here is far from exhausted, since besides the 04 (four) logistic costs discussed here, other types of costs may arise over time, or as a result of particularities in the supply chain, leading to new adaptations of the AHP method.

As a future improvement opportunity, an Excel spreadsheet or even programming in a language like Python and R, for example, can be created so that the decision-maker enters the data related to the known costs and answers the elicitation questions, receiving as output the values of the logistic costs, thus implementing the present methodology. Some simulations using this tool also can be a powerful improvement to this work.

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