

Analysis of uncertainty levels using dynamic programming for an integrated production and inventory management problem of a perishable product

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ABSTRACT

In this paper we present a sensitivity analysis in relation to demand uncertainty considering a dynamic programming approach for an integrated production and inventory management problem. The model considers restrictions related to production, storage and flow of materials between production and availability of products to retailer's customers. We propose a rolling horizon strategy for generating plans for a perishable product. At each decision-making stage, there is adaptation of expected demand levels and re-planning in the face of the fulfillment of initially uncertain demands. We carried out an investigation of the relationship between different levels of uncertainty and the indicators: destroyed items, service level and expected profits and compared the results with the results obtained considering a static approach. The results highlight a large performance gain in the dynamic approach, compared to the static approach, and make it possible to estimate the gains obtained when investing in more accurate demand forecasts.

KEYWORDS. Rolling Horizon; Perishable Product; Production and Inventory Management

AD&GP – PO na Administração e Gestão da Produção; POI – PO na Indústria;

1. Introduction

Management and integration on Supply Chain (SC) are fundamental in contemporary production processes. An SC is made up of a group of organizations and facilities that, through the exchange of materials and information, provide products or services. SC encompasses all activities related to the flow and transformation of goods, as well as the exchange of information that flows in both directions of SC [Ballou, 2007]. According to Geunes and Pardalos [2006] Supply Chain Management (SCM) is the study of strategies and methodologies that allow organizations responsible for SC activities to meet their objectives effectively. SCM plays a fundamental role in business viability and obtaining competitive results.

Inspired by a practical problem, faced by a large dairy Brazilian industry, we address a problem of integrated management of production and storage of perishable products. This problem considers four levels of the supply chain and involves production activities in multiple factories, storage integrated into factories, supply to several distribution centers and meeting demands from a retail network. Two critical features of this problem are the perishability of products and the uncertainty of demand over the planning horizon.

A detailed description of the complete problem, as well as the mathematical model considering a static approach to carrying out the Production and Inventory Management Plan (PIMP) can be seen in Cruz et al. [2024]. Uncertainty in demand leads managers to adopt measures such as the use of safety stock, inventory control and re-planning of stock replenishment. In turn, the perishability of products leads to destroy items that reach their maximum marketing time (m). The challenge for decision makers lies in determining when to produce products and what quantities to keep in stock, finding a balance between serving customers and minimizing costs associated with lost sales, storage and destruction of items due to expiration of the maximum time for products to be marketed.

To deal with uncertainties and possible fluctuations around expected demand, some approaches such as dynamic, stochastic and robust programming are explored in the literature. The choice of the approach to be used in this work involved the reaction time to the fulfillment of initially uncertain demands and the processing time of the mathematical model. Therefore, we propose a dynamic approach using the Rolling Horizon (RH) strategy to carry out a Dynamics Production and Inventory Management Plan (DPIMP).

RH iterative solves sub-problems delineated by study horizons linked along a planning horizon. Managers make decisions based on information and demand forecasts available for the study horizon of the sub-problem considered. Planning is implemented only for the specified periods until the next planning is carried out. As planning is carried out, inventory levels and forecast adherence to actual demands are checked. If necessary, the plans made for subsequent periods are corrected by "rolling over" the study horizon [Chand et al., 2002; Sethi and Sorger, 1991; Wang et al., 2019; Funda Sahin and Robinson, 2013]. This horizon is adjusted with each new sub-problem requiring additional forecasts according to the most recent demand information available. Thus, a dynamic production plan is designed to deal with uncertainties, enabling a flexible and agile system response to unexpected fluctuations in demand [Wang et al., 2019; Funda Sahin and Robinson, 2013].

The adaptation of the mathematical model to the dynamic approach, in addition to being extensive, is currently under analysis by a renowned scientific journal. Due to the extension of the model and the ongoing review process, it will not be possible to present it in full in this work. However, we will share an overview of the dynamic approach and present some preliminary results obtained so far. The aim is to provide a comprehensive understanding of the dynamic approach and present an analysis of uncertainties in relation to expected demands.

2. Literature review

Cruz et al. [2024] present the complete problem addressed in this work and propose a static approach for carrying out integrated planning. In the work, the authors use a combination of mathematical modeling with Monte Carlo simulation to carry out analyzes of performance indicators of the proposed approach. They also illustrate the PIMP developed, presents a case study and a sensitivity analysis around possible uncertainties at the time of decision making.

Even though in real problems, managers and decision makers have to dynamically deal with the results of their decisions, works considering RH are relatively scarce in the literature [Chand et al., 2002; Funda Sahin and Robinson, 2013]. This gap can be explained by the relative low number of academic works involving practical situations approaches to integrated production planning and inventory management, especially for perishable products [Perez and Torres, 2020]. Chand et al. [2002], Funda Sahin and Robinson [2013], Jans and Degraeve [2007] and Gunasekaran and Ngai [2005] present reviews of works involving SCM using RH. All authors point out the low number of works based on practical situations and the large number of opportunities that addressing them using dynamic planning can provide.

Kian et al. [2021] considers the problem of production planning in the presence of economies of scale in relation to production costs using a dynamic RH approach for non-perishable products. Vieira and Favaretto [2006] proposes a practical heuristic for creating Master Production Scheduling for non-perishable products. Jing and Lan [2017] studies a multi-item lot sizing problem for perishable products with inventory spoilage rates in a dynamic approach. Jing and Mu [2019] investigates a dynamic lot sizing problem under replacement of perishable products considering stock deterioration rates, maintenance costs in each period depending on the age of the stock and the possibility of product replacement. In our research, we did not find any work in the literature that addresses the set of practical specifications present in our work.

3. The integrated problem of production and inventory management for a perishable product

This work is an extension of a study presented by Cruz et al. [2024] that originated from a practical challenge faced by a large company in the Brazilian dairy industry. The problem aims to determine production days and stored quantities considering factories, warehouses (PW) and distribution centers (SDC) as shown in Figure 1. The solution aims to meet uncertain demand, seeking to minimize costs due to the storage and destruction of items throughout the planning horizon. A Safety Stock Relative to Expected Demand (SSLRED or γ^{st}) is used to absorb possible fluctuations around expected demand (d^{st}). With the use of safety stock, the quantity available (S_2^{st}) in each distribution center must be given by:

$$S_2^{st} \geq d^{st} \times (\gamma^{st} + 1) \quad \forall s \in S, \forall t \in T \quad (1)$$

Where: S is the set of all distribution centers s , F is the set of all integrated factories and warehouses f and T the set of all periods t of the planning horizon.

The conditions of the operating environment require the consideration of a series of restrictions such as: transportation time between PWs and SDCs, minimum interval between subsequent productions, specific days for receiving lots at the SDCs, disposal of items that reach the maximum commercialization age and the consideration of a First Expired, First Out (FEFO) policy in inventory management. Using a FEFO policy in Pws and SDCs ensures that the product with the earliest expiration date is shipped first, guaranteeing that older items are shipped before newer ones, keeping inventory rejuvenated and helping to reduce waste. In relation to the problem presented

by Cruz et al. [2024] we did not consider the restriction on sending lots with minimum quantities and multiple items because, for the purpose of sensitivity analysis in relation to uncertainties in demand, this condition is not very relevant, and consideration of this restriction would impact processing times and require more extensive and in-depth analyses.

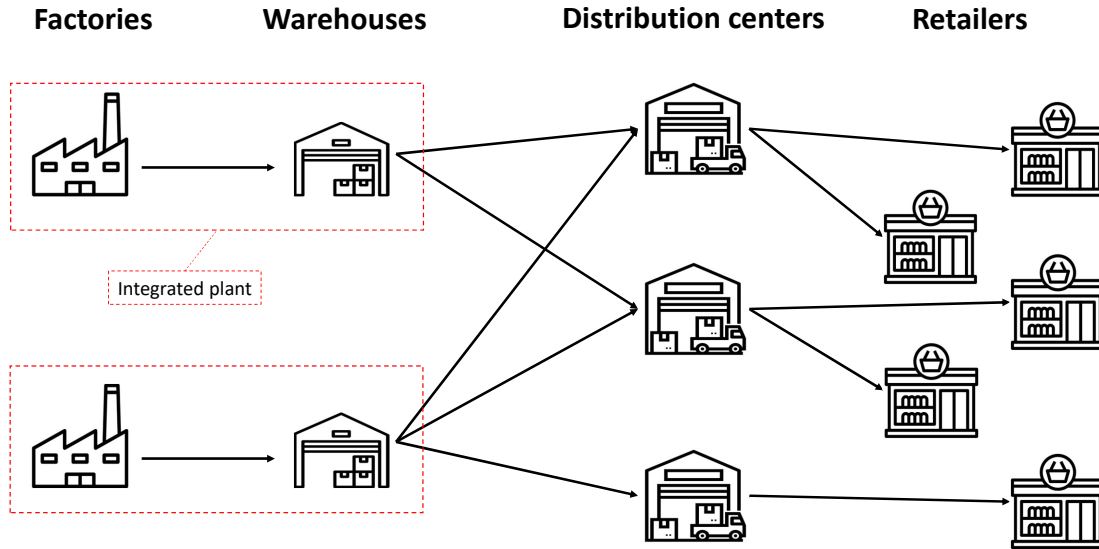


Figure 1: Simplified supply chain of the addressed problem.

4. Proposed method

The method proposed to address the problem presented in this work structures a deterministic MILP model in a dynamic approach using RH. The flowchart in Figure 2 presents the sequence of steps for solving the sub-problems generated in each decision making and the subsequent full accounting of the entire planning horizon. The first sub-problem is determined for the study horizon (rng) from $t_{start} = t_0$ to t_{end} . After the periods (stp) until the next decision have elapsed, stock levels and compliance with the demand forecast are checked and a new planning is carried out at the time $t_{start} = t_0 + stp$. The process is repeated until the end of the considered planning horizon (ph) is reached.

For each new sub-problem, information on decisions already made and stock levels sensitized by the realization of demand (from previous periods) are considered. The condition of materials flow conservation between successive planning and respect for restrictions that would be affected in a future condition of each sub-problem are also considered. To do this, we define the parameters $t_{extend0}$, $t_{extend1}$ and $t_{extend2}$ which are used to define the intervals in which there must be considerations before t_{start} , after t_{end} or between the periods t_{start} and t_{end} , respectively. The parameter $t_{extend0}$ considers a period prior to t_{start} in order to guarantee the minimum production interval condition. The parameter $t_{extend1}$ considers a period after t_{end} in order to guarantee the condition of no shipping if it is not possible to receive the items. The parameter $t_{extend2}$ considers a period after t_{start} in order to record decisions already made and ensure that lots previously shipped, from PWs to SCDs, are correctly received at the beginning of each sub-problem. Figure 3 presents an illustration of the dynamic approach and defined intervals for structuring the rolling horizon developed for this work.

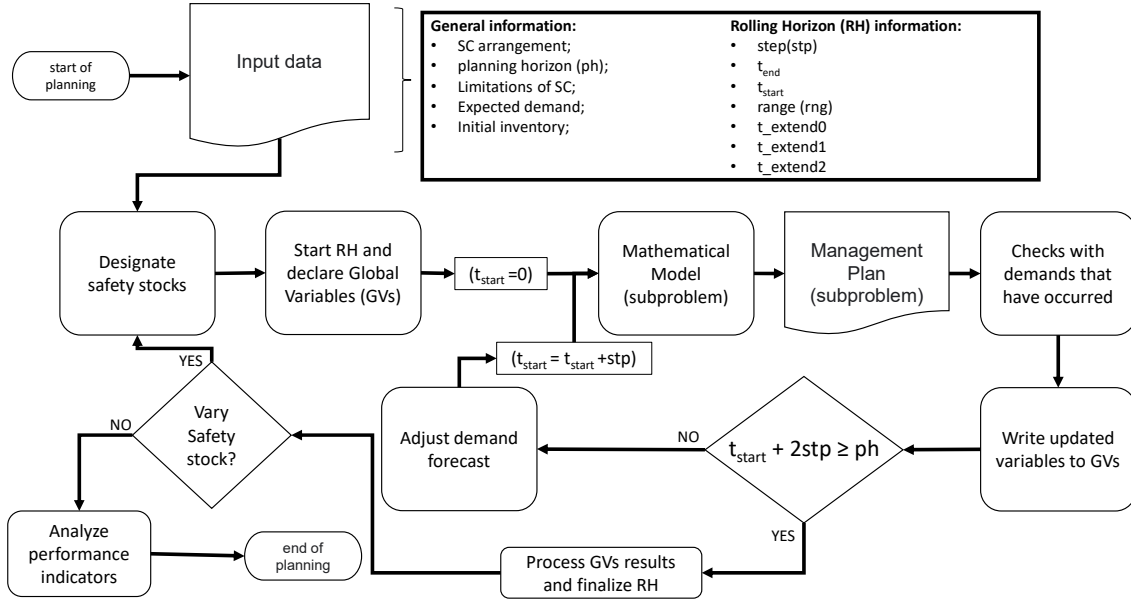


Figure 2: Flowchart of the proposed method to address the problem considered in this work.

As mentioned previously, the complete mathematical model, in addition to being extensive, is under evaluation by a renowned scientific journal and cannot be presented in this manuscript. The mathematical model of the static approach can be seen in Cruz et al. [2024]. The objective function presented in Equation 2 seeks to maximize profits considering the following elements: unit sales prices (sp^s) at the distribution center s , production preparation cost (sc^f) at factory f , unit production cost (uc^f) at factory f , handling and transportation cost (tc^{fs}) between factory f warehouses and the production center distribution s , storage and destruction costs of items in the factory-integrated warehouse f (h_1^f and w_1^f) and storage and destruction costs of items in the distribution center s (h_2^s and w_2^s). The decision variables define the quantities produced in the factory f in the period t (Q^{ft}), the quantities commercialized in the SDC s in the period t (C^{st}), the inventory level in the SDC s with age v in the period t (I_2^{svt}), the quantities sent from the PW f to the SDC s with age v in period t (E^{fsvt}) and the inventory level in PW f with age v in period t (I_1^{fvt}). The variable Y^{ft} defines a binary variable to indicate the occurrence of production and the variable A^s defines a slack variable to indicate the missing quantities to meet the condition of an inventory level in the last period equal to the initial inventory level.

$$\begin{aligned}
 &Max \sum_{s=1}^{|S|} \left\{ \sum_{t=1}^{|T|} \left[sp^s \times C^{st} - \left(h_2^s \times \sum_{v=1}^{m-1} I_2^{svt} + w_2^s \times I_2^{smt} \right) \right] - uc^f \times A^s \right\} + \\
 &- \sum_{t=1}^{|T|} \sum_{f=1}^{|F|} \left[\sum_{s=1}^{|S|} \sum_{v=1}^{m-\tau^{fs}} tc^{fs} \times E^{fsvt} + sc^f \times Y^{ft} + uc^f \times Q^{ft} + \sum_{v=1}^{m-1} h_1^f \times I_1^{fvt} + w_1^f \times I_1^{fmt} \right]
 \end{aligned} \quad (2)$$

At the beginning of each sub-problem, the information generated by the fulfillment of simulated demands in previous periods is verified. The simulated demands were constructed point by point considering a normal probability distribution. For the probability distribution, the value of

real demands, provided by the company that presented us with the problem situation, was adopted as an average value and different degrees of uncertainty according to the standard deviations presented in Section 5.

At the end of the planning horizon considered, the following performance indicators are obtained: total processing time, total production, lost sales, destroyed items, service level and profit that would be obtained with the safety stock levels considered. From this data, graphs are constructed that, with careful visual analysis, make it possible to identify trends, patterns and designate safety stock levels that bring the best results in performance indicators. These management insights can guide strategic decisions, further refining production planning and inventory management to optimize overall system performance.

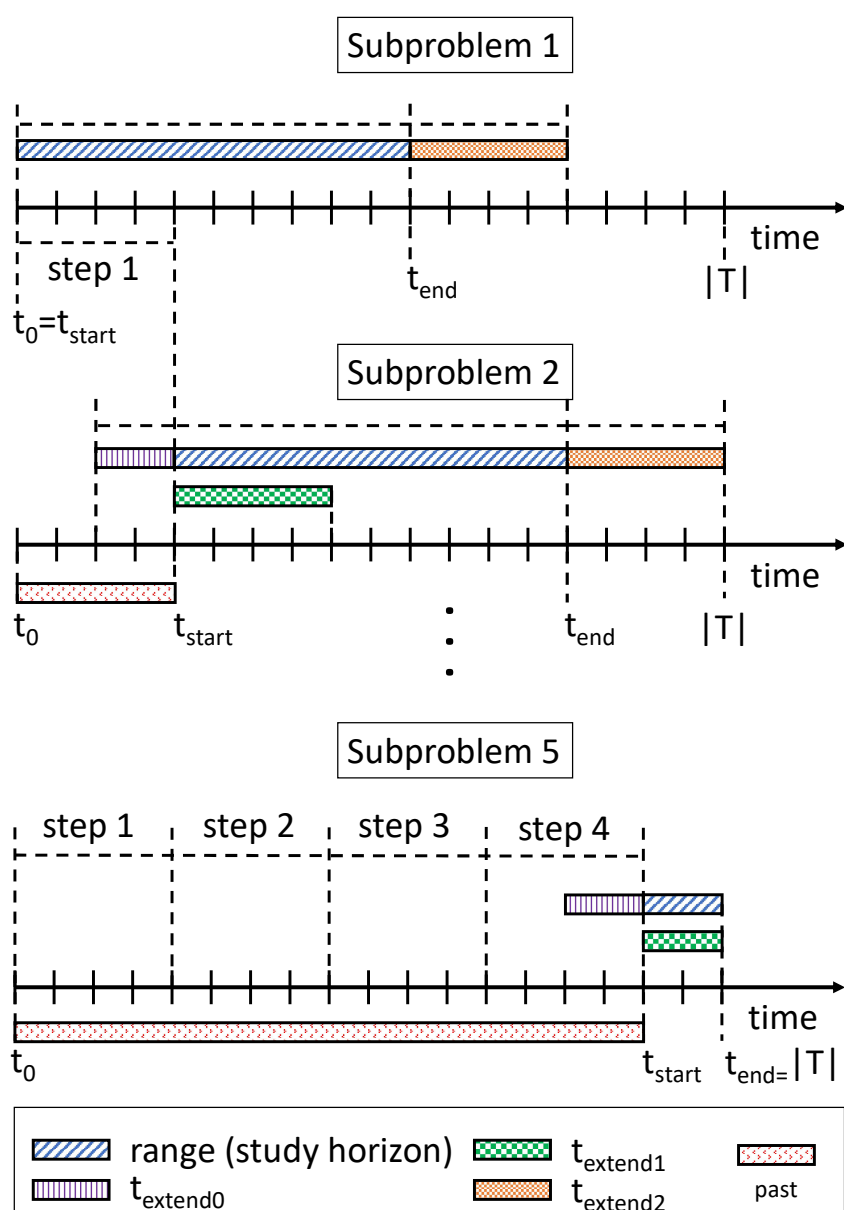


Figure 3: An HR example showing the relevance parameters structured.

5. Uncertainty analysis

The analysis presented below aims to carry out an investigation into the impact of uncertainty level of expected demand on the performance indicators indicated in Section 4. For all tests, we adopted a naive forecast, considering the demands provided to us, as the expected demand for running DPIMP. Using the demands that occurred as an average value and considering the probability patterns presented in Figure 4, we constructed, with the demands simulated in each period t and SDC s , the scenarios: cold, regular and warm. For each scenario, uncertainties were also considered according to the standard deviations: $2\sigma = \mu$, $4\sigma = \mu$ and $6\sigma = \mu$.

To verify the effectiveness of the dynamic method, PIMP static planning tests for the cold, regular and cold scenarios are also considered. Thus, 12 tests are analyzed. We evaluate the benefits derived from investments aimed at reducing forecast uncertainty across various performance indicators, including expected profit, service level, lost sales, destroyed items and total production. Due to the large amount of data, the information that makes up the analyzed instance will not be presented in the body of this work, but can be consulted at Cruz et al. [2024] or made available by contacting the authors of this work.

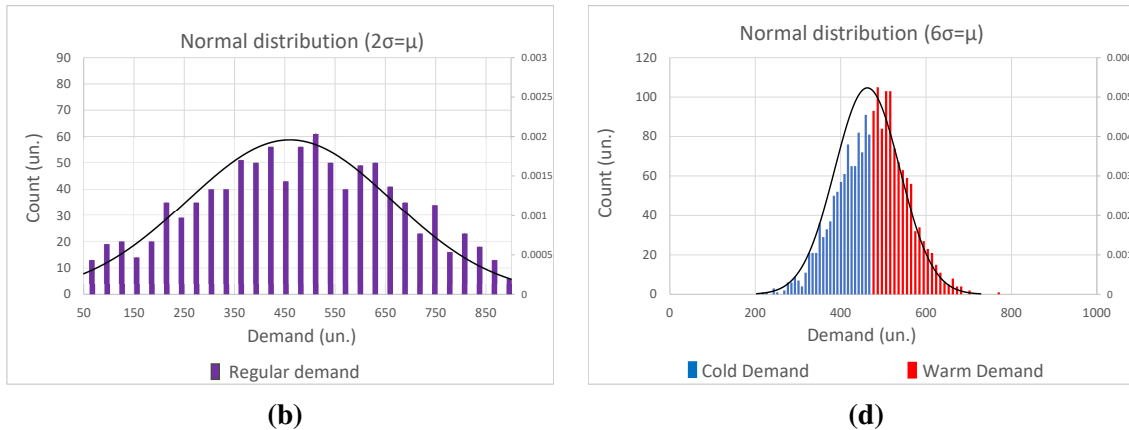


Figure 4: Examples of data generated for the scenarios: regular cold and warm considering a normal probability distribution with mean value $\mu = 467kg$ and standard deviation: (a) $\pm 2\sigma = \mu$; (b) $\pm 6\sigma = \mu$.

Figures 5 – 9 present the results of PIMP and DPIMP planning considering simulated demands constructed with $\pm 2\sigma = \mu$. Figures 5 and 6 present the results of expected profit and service level of these tests. Analyzing the results that would be obtained without the adoption of a safety stock ($\gamma = 0$) we observed a significant improvement in expected profits and service levels from the DPIMP approach in the cold and warm scenarios, respectively. Analyzing the regular scenario, we observed that the results are very close for the static and dynamic approaches. This occurs because in the regular scenario there are no major differences between the average simulated demands and the expected demands, providing similar planning between PIMP and DPIMP.

Analyzing the cold scenario, we observe that for the static PIMP approach, even without the adoption of safety stocks, the economic indicator presents a negative result (loss $\approx R\$48,000$) and the increase in safety stock only leads to greater losses. In the DPIMP approach, we observe an expected positive result (profit $\approx R\$50,000$) with the adoption of $\gamma = 0$ and can only generate losses with $\gamma \approx 3.5$. This occurs due to higher total production than total simulated demand and the lack of plans adaptation, resulting in high costs and large quantities of destroyed items.

Analyzing the warm scenario, we observed that for the static PIMP approach for $\gamma = 0$ the service level indicator is $SL \approx 0.75$, reaching $SL = 1$ only with $\gamma \approx 9$. As for the dynamic

approach, with $\gamma = 0$ the DPIMP provides $SL \approx 0.91$ reaching $SL = 1$ with $\gamma \approx 2$. As in this scenario, the average of simulated demands is higher than expected demands, all production ends up being used to meet simulated demands. Consequently, the anticipated profits from both static and dynamic planning approaches appear similar, primarily because we are not considering losses incurred due to lost sales opportunities.

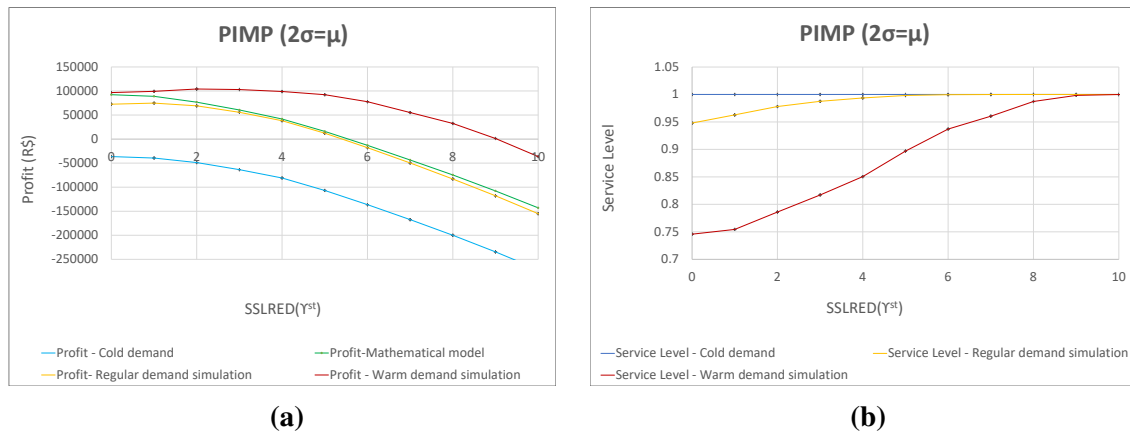


Figure 5: (a) Profit and (b) service level results for PIMP tests considering $\pm 2\sigma = \mu$.

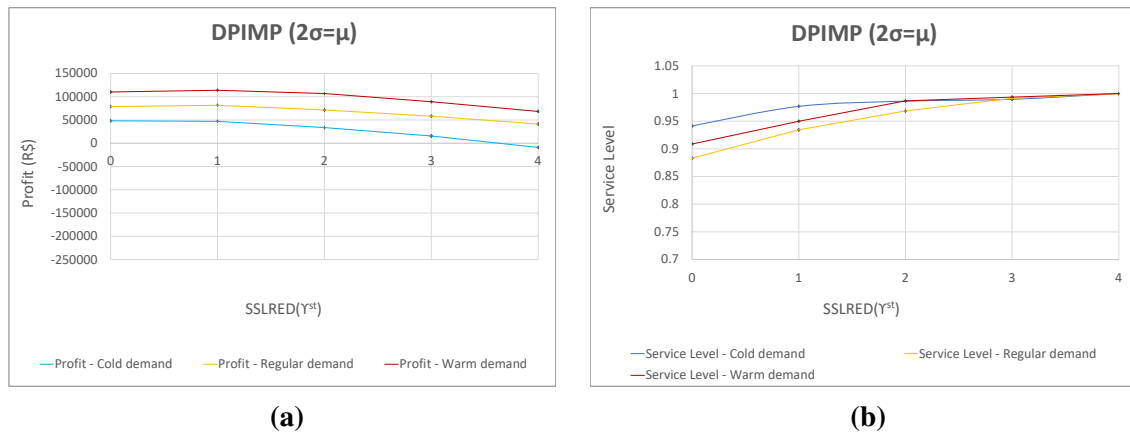
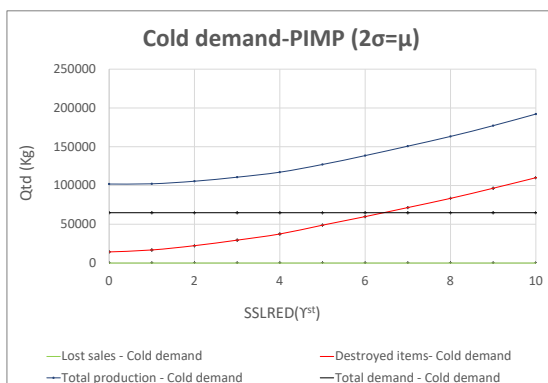
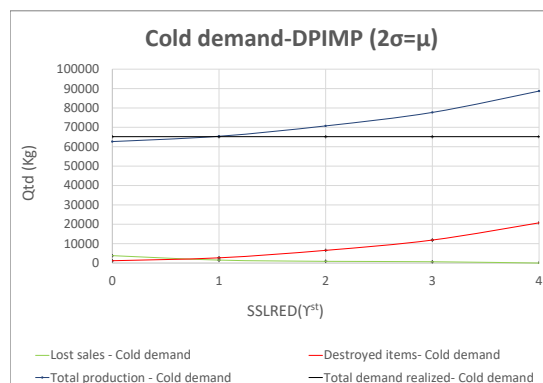


Figure 6: (a) Profit and (b) service level results for DPIMP tests considering $\pm 2\sigma = \mu$.

Comparing performance indicators: lost sales, destroyed items and production between static and dynamic planning, we observed the great gains provided by dynamic approach in cold and warm scenarios. In these scenarios, average demand is much lower (cold scenario) or higher (warm scenario) the expected average demand. Under these conditions the dynamic planning adapted stock levels to the demands that occurred during the successive planning, reducing or increasing production to the levels necessary to supply simulated demands. These gains are easily observed in Figures 7 and 9. In both scenarios the level of production approaches the simulated total demand without consideration of safety stocks, enabling demands to be met, without the occurrence of large quantities of items destroyed.

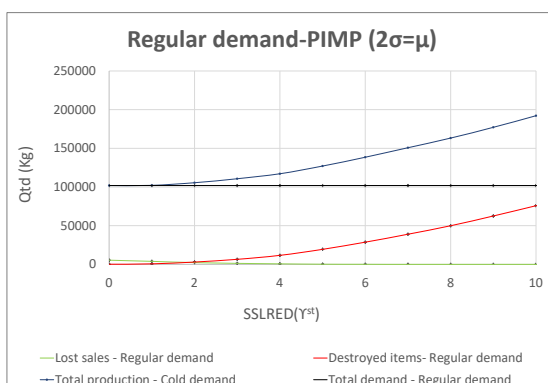


(a)

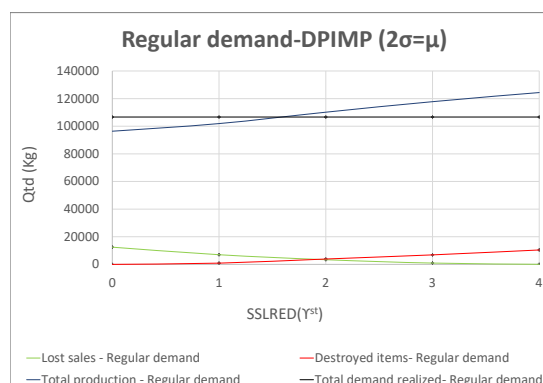


(b)

Figure 7: Total production, destroyed items and lost sales for the "cold" scenario considering $\pm 2\sigma = \mu$ in approaches (a) PIMP and (b) DPIMP.

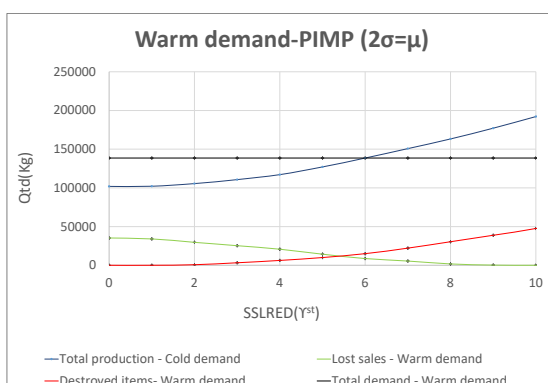


(a)

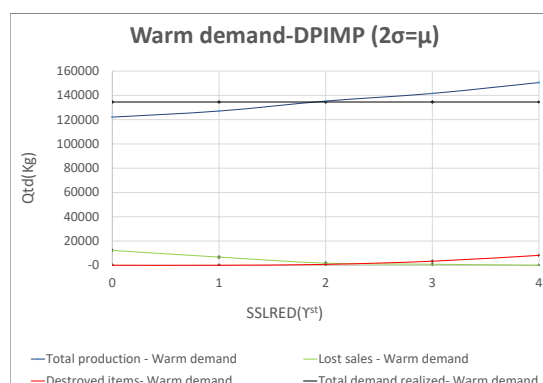


(b)

Figure 8: Total production, destroyed items and lost sales for the "regular" scenario considering $\pm 2\sigma = \mu$ in approaches (a) PIMP and (b) DPIMP.



(a)



(b)

Figure 9: Total production, destroyed items and lost sales for the "warm" scenario considering $\pm 2\sigma = \mu$ in approaches (a) PIMP and (b) DPIMP.

Figures 10 and 11 shows results obtained with dynamic planning considering the decrease of uncertainty degrees in the analyzed scenarios. We observed that the lower the standard deviation value, the closer the curves for the cold, regular and warm scenarios become. Considering the warm scenario and the uncertainty levels: $\pm 2\sigma = \mu$, $\pm 4\sigma = \mu$ and $\pm 6\sigma = \mu$, the service levels obtained with $\gamma = 0$ are $SL \approx 0.91$, $SL \approx 0.93$ and $SL \approx 0.96$ respectively. These results demonstrate the potential of the dynamic approach developed because, even if no safety stock is considered, the execution of plans would result in high service levels, even with high uncertainty around the average ($\pm 2\sigma = \mu$) for the 3 scenarios.

The decrease in uncertainty promotes improved results, requiring even lower levels of safety stock to achieve higher service levels. However, due to the high costs associated with improving demand forecasts, managers responsible for the operation may consider it more convenient to use more uncertain forecasts and still guarantee more advantageous results. This is a subjective assessment and depends on the objectives of each operation. It may be important to achieve targets for other performance indicators, such as levels of destroyed items, making the accuracy of forecasts a decisive factor in achieving the desired objectives.

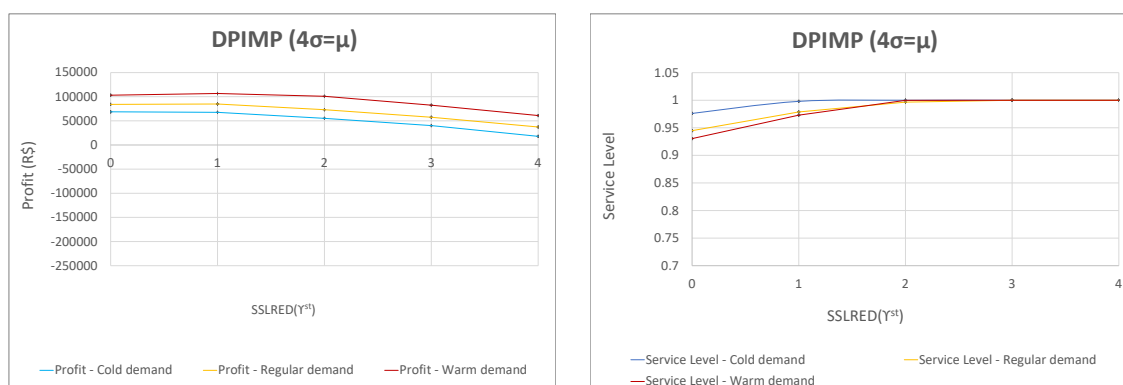


Figure 10: (a) Profit and (b) service level results for DPIMP tests considering $\pm 4\sigma = \mu$.

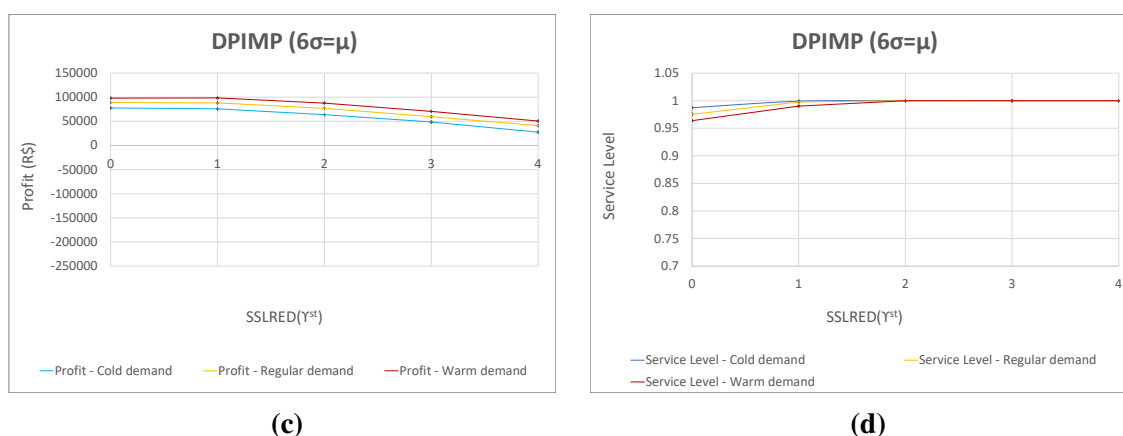


Figure 11: (a) Profit and (b) service level results for DPIMP tests considering $\pm 6\sigma = \mu$.

6. Conclusion and further work

In this work we perform a sensitivity analysis in relation to demand uncertainty considering a dynamic programming approach for an integrated production and inventory management problem in a dairy company. We also present an overview of the RH method developed in an integrated way with the deterministic MILP model.

In the sensitivity analysis we check the performance indicators: profit, service level, destroyed items and total production in three scenarios (cold, regular, warm) and 3 degrees of uncertainty ($\pm 2\sigma = \mu$, $\pm 4\sigma = \mu$ and $\pm 6\sigma = \mu$). The method proposes to iterative solve linked sub-problems over a planning horizon. For each sub-problem, a plan is generated for a study horizon and, period by period, the execution of the plan is verified in the face of the fulfillment of initially uncertain demands. The periods until the next planning have elapsed. the plans made for subsequent periods are corrected, “rolling over” the study horizon. A dynamic plan that is flexible to possible changes in demand is created, enabling agile responses even in scenarios with demand patterns deviate significantly from the expected ones.

The results obtained demonstrated that, in comparison with tests with static planning, dynamic planning showed a significant improvement in all performance indicators analyzed, enabling results close to optimal results, especially in scenarios with large fluctuations around expected demand. Regarding the analysis of uncertainty in demands, we found that the smaller the dispersion of the data considered, the closer the profit and service level curves became for the scenarios with cold, regular and warm demand. Although more accurate models have generated better results, the combination of the dynamic approach with the deterministic model has been shown to provide excellent results in adverse forecasts. This flexibility of the tool, conditioned by the high costs of obtaining more accurate forecasts, may result in managers making less precise estimates and still obtaining a better overall result for the economic performance indicator.

In the sequence of the work, we will implement a dynamic approach to the problem considering multiple products and, as the processing cost becomes an impediment, we will develop approaches that make it possible to obtain results in feasible times for running DPIMP.

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