

## Hypocentral Parameter Inversion using Hybrid Grey Wolf Optimizer

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### ABSTRACT

Hypocentral inversion is a crucial branch of seismology that estimates earthquake spatial coordinates from seismographic data. This paper presents a comparative study on hypocentral parameter inversion using various metaheuristic approaches. The optimization techniques investigated include *Gauss-Newton with Multi-Start (GM)*, *Genetic Algorithm (GA)*, *Particle Swarm Optimization (PSO)*, *Grey Wolf Optimizer (GWO)*, and *Hybrid Grey Wolf Optimizer (HGWO)*. The study evaluates the effectiveness of these methods in determining hypocentral parameters and compares their performance, specifically assessing whether *HGWO* outperforms the others. Results indicate that *HGWO* achieves a lower total cost of the objective function for average solutions, while *GWO* shows better performance in terms of mean objective function costs across stabilized iterations and computational time. *PSO* demonstrates superior convergence speed. These findings highlight the strengths and weaknesses of each method, aiding in the selection of inversion control parameters and coefficients. This work contributes to advancing optimal strategies for hypocentral inversion and offers valuable insights for researchers and practitioners in seismology.

**KEYWORDS.** Earthquake Location, Metaheuristic, Bio-inspired Optimization.

### 1. Introduction

The precise determination of earthquake hypocentral locations is crucial in geophysics, and computational methods are fundamental for this purpose. Various techniques, including metaheuristics and neural networks, have been extensively explored to address the challenging problem of hypocentral location [Sambridge and Gallagher, 1993; Muramatsu and Nakanishi, 1997; Deep et al., 2011; Perol et al., 2018; Smith et al., 2022].

This wealth of studies plays a pivotal role in geophysics, influencing both tectonic understanding and essential industrial applications, such as hydrocarbon field stimulation, effective management of carbon and radioactive waste storage, and the characterization of faults and fractures at depth [Eisner et al., 2010; Pytharouli et al., 2011]. Additionally, obtaining geological models of crustal velocities through inversion using multiple hypocenters is another crucial application [Pavlis and Booker, 1980; Spencer and Gubbins, 1980].

As outlined by Sen and Stoffa [2013], there are two main categories of approaches to solve the inverse problem, including the hypocentral location problem: local gradient-based methods and global search methods. Local methods converge quickly to the nearest local minimum, whereas global search methods extensively explore the model space for a satisfactory minimum.

This study focuses on comparing five methods: *Gauss-Newton with Multi-Start (GM)*, *Genetic Algorithm (GA)*, *Particle Swarm Optimization (PSO)*, *Grey Wolf Optimizer (GWO)*, and the *Hybrid Grey Wolf Optimizer (HGWO)* with genetic operators.

The *GM* proves effective as a local approach, initiating from an initial estimate and iterating toward the local minimum using a linear approximation based on the Taylor series. Particularly effective in well-conditioned problems, it ensures efficient convergence to an optimal solution. Multi-Start, which relies on a set of uniformly distributed solutions, helps obtain an acceptable minimum and saves computational resources [Meju, 1994].

The *GA*, a metaheuristic simulating the natural evolutionary process, explores the global search space using genetic operators such as mutation and recombination. It is especially useful in scenarios with multiple local minima and incorporates genetic diversity [Holland, 1992].

The *PSO* draws inspiration from the social behavior of birds and insects. Particles move in the search space, adjusting their positions based on individual performance and collective knowledge. This method efficiently explores multidimensional search spaces [Kennedy and Eberhart, 1995].

The *GWO* is a bio-inspired optimization algorithm based on the social behavior of grey wolves. It classifies agents according to the cost of the objective function and uses predatory strategies such as hunting, encircling, and capturing to simulate the selection of the fittest agent [Mirjalili et al., 2014]. Its effectiveness comes from its ability to explore the search space thoroughly and ensure convergence to optimal solutions.

To enhance *GWO*'s performance, a hybridization with genetic operators is proposed, resulting in the *HGWO*. Techniques such as mutation, crossover, elitism, and tournament selection introduce genetic diversification and further exploration of the search space. Mutation introduces variations, crossover combines promising features, elitism preserves the best solutions, and tournament selection fosters competition among solutions. This combination of bio-inspired and genetic strategies provides a powerful and versatile approach to solving a wide range of optimization problems.

In this study, the strategic choice of these methods reflects the need to evaluate their effectiveness in specific hypocentral inversion scenarios. The goal is to determine whether the proposed Hybrid Grey Wolf Optimizer offers advantages over the other methods. Given that neither the standard *GWO* nor the hybridized version (*HGWO*) has been applied to this problem, applying both represents a novel approach. The careful selection of these strategies is crucial for obtaining accurate results in the dynamic and challenging field of seismology.

## 2. Hypocentral Inversion

Hypocentral inversion is essential for comprehending seismic activity and evaluating earthquake risks. The solution to the inverse problem relies on precise information about seismic wave arrival times at seismographic stations, providing data for determining hypocentral coordinates [Geiger, 1910].

Analyzing arrival times at various seismographic stations establishes relationships between wave travel time, station distance, and hypocenter. Despite simplifications, assumptions of straight wave propagation paths through the Earth guide this process, considering phenomena like refraction and reflection.

Straight-ray modeling methods determine hypocentral location by constructing theoretical seismic ray paths based on velocity models describing wave velocity variation with depth. Coupled with arrival times, these models allow adjusting parameters (latitude, longitude, and depth) to closely match observed and calculated arrival times.

## 2.1. Direct Problem

The direct problem in hypocentral location deals with predicting earthquake travel times at seismic stations, involving estimating travel times using equations describing the observed arrival time  $t_k$  at the  $k$ -th seismic station.

The travel time equation  $t_k$  calculates travel times by summing layer travel times, considering hypocentral coordinates  $(x_0, y_0, z_0)$ , layer interface coordinates  $(x_j^k, y_j^k, z_j^k)$ , and seismic wave velocities  $v_j$  for  $m$  layers and  $n$  stations:

$$t_k = \sum_{j=1}^n \sum_{j=1}^m \frac{\sqrt{(x_j^k - x_0)^2 + (y_j^k - y_0)^2 + (z_j^k - z_0)^2}}{v_j}. \quad (1)$$

## 2.2. Inverse Problem

The inverse problem entails identifying earthquake coordinates by minimizing the disparity between the observed travel times vector  $\vec{t}_{obs}$  and the calculated travel times vector  $\Phi(\vec{s})$ . This is achieved through the formulation of an objective function, denoted as  $\mathcal{L}(\vec{s})$ , which is based on squared normalized errors:

$$\mathcal{L}(\vec{s}) = \frac{\|\vec{t}_{obs} - \Phi(\vec{s})\|_2^2}{n}. \quad (2)$$

Here,  $\Phi(\vec{s})$  computes travel times based on the model parameters  $\vec{s}$ , as determined by the Equation 1. Minimizing the objective function  $\mathcal{L}(\vec{s})$  by adjusting the hypocenter coordinates  $\vec{s}$  ensures a close approximation between the calculated and observed travel times.

## 3. Optimization

Optimization, as defined by Beveridge and Schechter [1970], involves identifying the best solution to a given problem by minimizing or maximizing an objective function within specified constraints. This fundamental concept permeates Mathematics, Computer Science, and Engineering. Mathematically, the optimization problem can be expressed as follows:

$$\begin{aligned}
 &\text{Minimize } f(x), \\
 &\text{Subject to } g_i(x) \leq 0, \quad i = 1, 2, \dots, m \\
 &\quad \quad \quad h_j(x) = 0, \quad j = 1, 2, \dots, n \\
 &\quad \quad \quad t_k(x) < 0, \quad k = 1, 2, \dots, p
 \end{aligned}$$

Here,  $f(x)$  denotes the objective function to minimize, and  $g_i(x)$ ,  $h_j(x)$ , and  $t_k(x)$  represent inequality and equality constraints. The goal is to determine the values of  $x$  that minimize  $f(x)$  while adhering to the specified constraints. In our study, we aim to minimize the objective function of Equation 2 to identify hypocentral parameters explaining the observed data. The objective is to ensure that the obtained solution, when evaluating the objective function, has a smaller image, i.e., a lower cost, across the entire search space.

When it comes to local optimization, a specific focus within this realm involves seeking the minimum or maximum value of a function within a defined region, proving particularly beneficial when a reasonable initial solution is available. This approach finds applicability in scenarios featuring complex, multidimensional objective functions and constraints, leveraging heuristics or biases in the search process [Beveridge and Schechter, 1970]. For example, in machine learning, local optimization strives to discover parameter values minimizing a specific loss function in the vicinity of the current solution [Ruder, 2016].

An example of a widely used local algorithm in geophysical inverse problems is the Gauss-Newton method. This method, applied in nonlinear optimization tasks like curve fitting and hypocentral inversion, aims to minimize the mean squared error between observed data and a modeled function. It linearizes the current parameter estimate, transforming the nonlinear problem into a linear one for enhanced computational efficiency. The algorithm then utilizes least squares techniques to iteratively update parameters Meju [1994].

In the present study, we combine the Gauss-Newton method with the Multi-Start method, a prevalent optimization technique in nonlinear problems, especially for global minimum search in complex spaces. This approach is valuable when the objective function's topology is unclear or the search space is large and irregular. The Multi-Start method involves initiating optimization from multiple starting points to explore diverse regions for local and global minima [Martí et al., 2013].

In contrast, global optimization, a field within computer science, seeks the best solution for problems in extensive search spaces. While local optimization targets local minima, global optimization endeavors to find the global minimum, considering all feasible solutions [Horst and Pardalos, 2013]. Employing intelligent strategies such as heuristic search techniques, genetic algorithms, and particle swarm optimization, global optimization methods efficiently explore the search space. This approach finds applications in complex problems across various domains, including hypocentral inversion [Sambridge and Gallagher, 1993; Han and Wang, 2009].

A significant advantage of global optimization lies in its ability to efficiently explore the search space, identifying solutions that other methods might overlook. These methods often guarantee convergence properties, meaning they can find the optimal solution or a solution close to the optimum with sufficient time [Sen and Stoffa, 2013; Hein, 1994]. However, global optimization can be computationally intensive, especially for large-scale problems, necessitating the development of efficient algorithms and advanced techniques [Horst et al., 2000].

In this study, we utilize the *GA*, introduced by Holland [1992], a stochastic global optimization technique inspired by biological evolution. It mimics natural selection by favoring the survival and reproduction of models with higher fitness values. In this algorithm, physical parameters are combined into chromosomes, representing potential solutions or model states. These chromosomes, analogous to individuals in a population, are randomly generated and signify hypocentral coordinates in the specific application discussed.

Additionally, we employ the *PSO*, proposed by Kennedy and Eberhart [1995], a meta-heuristic inspired by the social behavior of swarms, such as flocks of birds or schools of fish. *PSO* has been widely applied to solve global optimization problems in various fields. The algorithm begins by creating a swarm of particles, where each particle represents a candidate solution in the search space, with a random initial position and velocity.

### 3.1. Grey Wolf Optimizer

The Grey Wolf Optimizer (*GWO*) is inspired by the hunting behavior of grey wolves. Introduced by Mirjalili et al. [2014], this algorithm models decision and control parameters based on the wolf pack's food chain principles. *GWO* involves social categorization, prey hunting, surrounding, and attacking.

*GWO*'s pack-based approach enables broad exploration of the search space, increasing the likelihood of finding the global minimum. It's easy to implement and adaptable, making it suitable for various optimization problems.

Wolves, like other social animals, have a defined hierarchy. The alpha wolf ( $\alpha$ ) leads, beta wolves ( $\beta$ ) are potential leaders, delta wolves ( $\delta$ ) monitor dangers, and omega wolves ( $\omega$ ) are the most subordinate.

During hunting, wolves use strategies mathematically modeled by the Equation 3:

$$\vec{s}(t+1) = \vec{s}_p(t) - \vec{A} \cdot |\vec{C} \cdot \vec{s}_p(t) - \vec{s}(t)|, \quad (3)$$

where  $\vec{s}$  is the wolf's position vector,  $\vec{s}_p$  is the prey's position vector,  $t$  is the current iteration, and  $\vec{A}$  and  $\vec{C}$  are coefficient vectors. These coefficients are calculated using Equations 4 and 5:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a}, \quad (4)$$

$$\vec{C} = 2 \cdot \vec{r}_2, \quad (5)$$

where  $\vec{r}_1$  and  $\vec{r}_2$  are uniformly distributed vectors in  $[0, 1]$ , and  $\vec{a}$  is a vector linearly described over iterations.

In the hunting process, the goal is to surround the prey with the assistance of agents, led by the alpha wolf ( $\alpha$ ). Mathematically, obtaining three optimal solutions ( $\alpha$ ,  $\beta$ , and  $\delta$ ) and updating agent positions involve Equations 6 - 12:

$$\vec{D}_\alpha = |\vec{C}_1 \cdot \vec{s}_\alpha - \vec{s}| \quad (6)$$

$$\vec{D}_\beta = |\vec{C}_2 \cdot \vec{s}_\beta - \vec{s}| \quad (7)$$

$$\vec{D}_\delta = |\vec{C}_3 \cdot \vec{s}_\delta - \vec{s}| \quad (8)$$

$$\vec{s}_1 = \vec{s}_\alpha - \vec{A}_1 \cdot \vec{D}_\alpha \quad (9)$$

$$\vec{s}_2 = \vec{s}_\beta - \vec{A}_2 \cdot \vec{D}_\beta \quad (10)$$

$$\vec{s}_3 = \vec{s}_\delta - \vec{A}_3 \cdot \vec{D}_\delta \quad (11)$$

$$\vec{s}(t+1) = \frac{\vec{s}_1 + \vec{s}_2 + \vec{s}_3}{3} \quad (12)$$

The predatory process concludes when the prey is immobilized. Mathematically, vector  $\vec{a}$  is restricted and gradually decreased relative to floating point  $\vec{A}$ , as in Equation 13:

$$\vec{a} = 2 - \frac{2 \cdot t}{\tau}, \quad (13)$$

where  $\tau$  is the maximum number of iterations.

### 3.2. Hybridization with Genetic Operators

This study integrates genetic operators into the *GWO* for optimization, incorporating mutation, crossover, tournament selection, and elitism, resulting in the *HGWO*. This hybrid approach combines the grey wolf heuristic search with genetic strategies, enhancing the exploration and exploitation of the solution space.

The literature already records the inclusion of genetic operators in *GWO*, as demonstrated by Gupta and Deep [2019]; Bouzary and Frank Chen [2019]; Miao et al. [2020]. However, this work proposes applying the adapted genetic operators to the hypocentral inversion problem, an approach that is novel in the literature, as is the application of the *GWO* in its standard form.

Pseudocode 1 illustrates the algorithm, beginning with the generation and evaluation of initial grey wolf positions using the objective function. Dynamic updates occur throughout iterations, guided by the alpha, beta, and delta concepts, with their influence weighted by the decreasing parameter ' $a$ '.

To explore new solutions, a crossover process randomly selects parents, applies crossover to produce offspring, and introduces additional variation through mutation. Tournament selection maintains population diversity, ensuring promising solutions advance to the next generation.

Elitism preserves the best solutions across generations by ranking and retaining top-performing individuals. Checks are in place to keep wolf positions within defined search space limits, ensuring adherence to solution boundaries. After each iteration, the best individual, represented by the alpha wolf, is recorded along with its score for analysis and monitoring of the optimization process.

#### 4. Methodology

In this study, we used the *Octave* programming language for hypocentral modeling and inversion algorithms [Octave, 2020]. We employed the straight-ray method to estimate seismic hypocenter coordinates. The main goal was to determine hypocenter locations using the *HGWO*, and to compare its performance with *GM*, *GA*, *PSO*, and *GWO*. This comparative analysis aimed to assess whether *HGWO* outperforms these alternative methods.

To validate this approach, we conducted experiments using synthetic data distributed in a hypothetical region. The origin of this region is defined at (0,0,0) in Cartesian coordinates, serving as a reference point for the study. The region is represented by a regular grid of 20 km by 17 km on the surface, extending to a depth of 10 km with a two-layered subsurface crustal model. The first layer had a P-wave velocity of 5.9 km/s and a thickness of 4 km, over a half-space with a velocity of 6.1 km/s, following Santana et al. [2012].

Experiments used five seismographic stations in the region, focusing on four earthquakes. Stations were positioned a quarter of the minimum distance between the grid's width and length apart, forming a pentagon on the surface plane. Hypocenters were generated with coordinates  $x_h$  and  $y_h$  from Gaussian distributions. For  $x_h$ , the mean was half the grid's width, with a standard deviation of one-fourth. For  $y_h$ , the mean was half the grid's length, with a standard deviation of one-fourth. Depth had a mean of -8 and a standard deviation of 2.

Observed data (travel times) had pseudo-random Gaussian noise (5%) for inversion method testing. For the Gauss-Newton algorithm, we chose a step size ( $\epsilon$ ) of 1 and a perturbation on hypocentral parameters ( $\gamma$ ) of 1%. We set 10 iterations as the stopping criterion for all algorithms. Additionally, for the *Multi-Start* approach, we initiated the optimization process from 10 randomly chosen points within the search space.

The lower and upper bounds used by optimization algorithms for searching solutions in the search space were, respectively,  $\delta_{lb} = [1, 1, -10]$  and  $\delta_{up} = [20, 17, -5]$ .

For the *GA*, key coefficients include the maximum number of iterations ( $\tau$ ), population size ( $Q_{size}$ ), lower ( $\delta_{lb}$ ), and upper ( $\delta_{up}$ ) bounds, crossover rate ( $P_c$ ), mutation rate ( $P_m$ ), mutation magnitude ( $\sigma_m$ ), and tournament size ( $\rho_t$ ). The specific values assigned to these parameters, determined through empirical experimentation, are, respectively, 10, 100, 0.75, 0.01, 0.1.

This study employs the Arithmetic Crossover or Blending Crossover for genetic algorithm crossover. This method combines genes from two parents to generate offspring, promoting genetic diversity in the population [Michalewicz, 1992].

The *PSO* algorithm's optimization process relies on parameters including the maximum number of iterations ( $\tau$ ), population size ( $Q_{size}$ ), minimum inertia factor ( $\omega_{min}$ ), maximum inertia factor ( $\omega_{max}$ ), individual learning coefficient ( $c_1$ ), and collective learning coefficient ( $c_2$ ). The values associated with these parameters are respectively 10, 100, 0.4, 0.9, 2, and 2.

The linear decay factor ( $\psi$ ) maintains an equivalent value to the maximum number of iterations ( $\tau$ ), introducing a decay mechanism aligned with the overall optimization process. This set of parameters collectively shapes the dynamics of the *PSO*, contributing to its effectiveness in finding optimal solutions.

The classic *GWO* employs coefficients such as the number of wolves and the number of iterations. For both *GWO* and *HGWO*, the same values as those used for the other algorithms were

**Control Parameters:** Population Size ( $Q_{size}$ ), Lower Limit ( $\delta_{lb}$ ), Upper Limit ( $\delta_{ub}$ ), Crossover Rate ( $P_c$ ), Mutation Rate ( $P_m$ ), Mutation Magnitude ( $\sigma_m$ ), Tournament Size ( $\rho_t$ );

**Output:** Best wolf  $\vec{s}_\alpha$ ;

Generate wolves of the pack  $Q$  randomly within limits ( $\delta_{lb}$ ) and ( $\delta_{ub}$ );

Create coefficient vectors  $a$ ,  $A$ , and  $C$ ;

Evaluate fitness  $\Phi(\vec{s})$  of each search agent  $\vec{s}$  in the pack  $Q$ ;

Number of crossovers  $\Gamma \leftarrow \text{round}\left(\frac{Q_{size} \cdot P_c}{2}\right) \cdot 2$ ;

$\vec{s}_\alpha$  = Best agent in the pack;

$\vec{s}_\beta$  = Second-best agent in the pack;

$\vec{s}_\delta$  = Third-best agent in the pack;

**while** *Termination criterion not satisfied* **do**

**foreach** wolf  $s_i$  in the Pack **do**

Update position of each search agent;

**end**

Update coefficients  $a$ ,  $A$ , and  $C$ ;

Evaluate fitness of each search agent in the pack

Update agents  $\vec{s}_\alpha$ ,  $\vec{s}_\beta$ , and  $\vec{s}_\delta$ ;

**for**  $j = 1$  **to**  $\Gamma$  **do**

Randomly select two wolves  $\vec{s}_a$  and  $\vec{s}_b$  from the pack  $Q$ ;

Generate  $\vec{s}_c$  and  $\vec{s}_d$  through crossover of  $\vec{s}_a$  with  $\vec{s}_b$ ;

Save  $\vec{s}_c$  and  $\vec{s}_d$  in the pack  $Q^*$ ;

**end**

**for**  $j = 1$  **to**  $\Gamma$  **do**

Select an agent  $\vec{s}_j$  from the pack  $Q^*$ ;

Mutate each bit of  $\vec{s}_j$  under the rate  $P_m$  and magnitude ( $\sigma_m$ );

Generate a new wolf  $\vec{s}_j$ ;

Update  $\mathbf{m}_j$  with  $\vec{s}_j$  from the pack  $Q^*$ ;

**end**

**for**  $j = 1$  **to**  $\Gamma$  **do**

Randomly generate the  $\rho_t$  indices of wolves  $\vec{s}$  from the pack  $Q^*$  that will participate in the tournament;

Compare the  $\rho_t$  wolves based on their fitness  $\Phi(\vec{s})$  and select the fittest;

Save the tournament winner in the pack  $Q^{**}$ ;

**end**

Select the best  $Q_{size} - \Gamma$  wolves from the old generation to form the new generation  $Q$ ;

Update pack  $Q$  with the best  $Q_{size} - \Gamma$  wolves from the old generation with the offspring  $Q^{**}$ ;

**end**

Return  $\vec{s}_\alpha$ ;

**Algorithm 1:** Hybrid Grey Wolf Optimizer

applied: 10 wolves and 100 iterations, respectively. Genetic operators share parameters with the *GA* in this study, using identical values to test if *HGWO* outperforms *GA* with these same values, incorporating *GA* mechanisms into the spatial coordinates estimation process.

## 5. Results and Discussions

To evaluate solution quality and algorithm equivalence, Kruskal-Wallis hypothesis tests were performed at a 5% significance level ( $\alpha$ ) [Kruskal and Wallis, 1952].

Table 1 displays the geographical locations of seismographic stations in the region, labeled as  $S^*$  where  $*$  represents the station number. The coordinates, given in Cartesian format, assume a depth of 0 since the stations are situated on the Earth's surface. This information facilitates the precise identification of each station's position and its spatial relationships in the study, which is crucial for the subsequent analysis and interpretation of seismic data.

In Table 2, the real coordinates of the hypocenters located in the delimited region are recorded, totaling four selected hypocenters. These real coordinates serve as a reference for comparing the estimated hypocentral parameters obtained through different optimization techniques. Table 3 showcases the estimated hypocentral parameters for each algorithm, providing insights into the performance of these techniques in determining seismic event characteristics. In the initial

| Station       | S1    | S2      | S3      | S4     | S5     |
|---------------|-------|---------|---------|--------|--------|
| Latitude (X)  | 12.75 | 9.8133  | 5.0617  | 5.0617 | 9.8133 |
| Longitude (Y) | 10    | 14.0420 | 12.4981 | 7.5019 | 5.9580 |
| Depth (Z)     | 0     | 0       | 0       | 0      | 0      |

Table 1: Locations of Seismographic Stations

|   | Real                     |
|---|--------------------------|
| A | 11.1167, 8.1166, -9.3961 |
| B | 5.6459, 4.7750, -6.0685  |
| C | 9.9927, 5.6386, -8.7963  |
| D | 8.0145, 11.8474, -9.2372 |

Table 2: Locations of Real Hypocentral Parameters

|   | GM                    | GA                    | PSO                   | GWO                   | HGWO                  |
|---|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| A | 10.635, 7.894, -9.283 | 10.801, 8.139, -9.232 | 10.523, 8.280, -9.405 | 10.699, 7.998, -9.295 | 10.643, 7.933, -9.270 |
| B | 5.725, 4.993, -6.061  | 5.782, 4.927, -5.955  | 5.805, 5.489, -6.175  | 5.685, 4.962, -6.015  | 5.703, 5.977, -6.010  |
| C | 9.990, 5.317, -8.655  | 10.048, 5.385, -8.689 | 10.462, 5.463, -8.685 | 10.076, 5.232, -8.613 | 10.161, 5.164, -8.575 |
| D | 8.353, 11.465, -9.399 | 8.168, 11.572, -9.355 | 8.257, 11.465, -9.616 | 8.271, 11.681, -9.359 | 8.431, 11.627, -9.378 |

Table 3: Locations of Hypocentral Estimates for *GM*, *GA*, *PSO*, *GWO* and *HGWO*.

criterion, we evaluated the equivalence of mean solutions across different algorithms. The null hypothesis ( $H_0$ ) states that there is no difference between the mean solutions obtained for each algorithm. With a  $p$ -value of 1, we fail to reject  $H_0$ , indicating no significant differences among the algorithms. This result is further corroborated by Fig. 1 and Table 3, which highlight the comparable effectiveness of all algorithms in producing similar mean solutions. In the second criterion, we scrutinized potential discrepancies in the calculated data of mean solutions from the five algorithms. The null hypothesis ( $H_0$ ) asserts that there is no difference between the calculated data of mean solutions obtained for each algorithm. With a  $p$ -value of 0.9995,  $H_0$  is upheld, as supported by Fig. 2, demonstrating accurate data adjustments with no apparent differences among the algorithms.

For the third criterion, we evaluated the equivalence of the objective function costs for obtaining average solutions. The null hypothesis ( $H_0$ ) posits that there is no difference between the costs of average solutions obtained for each algorithm. The test yielded a  $p$ -value of 0.4101, indicating that  $H_0$  cannot be rejected.

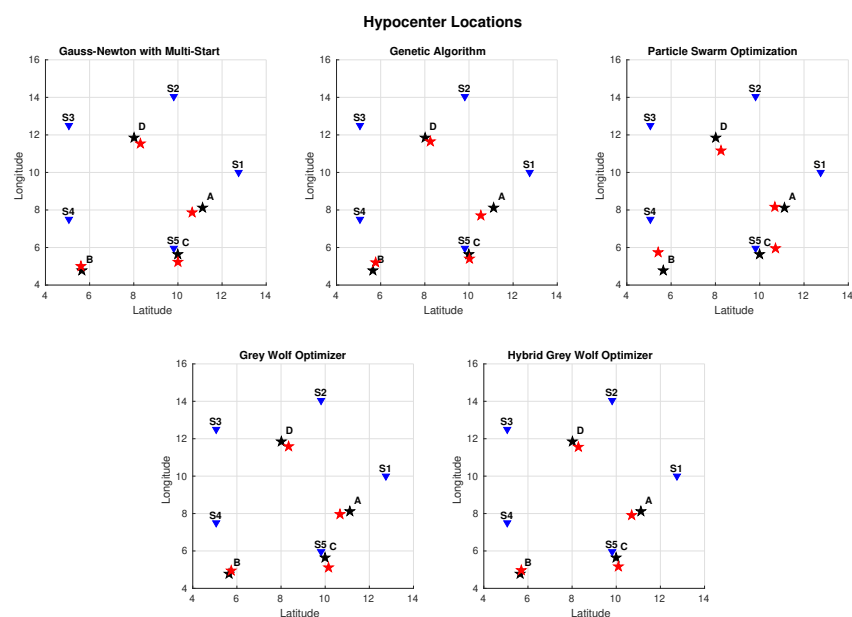


Figure 1: Hypocentral estimation for each algorithm and for each hypocenter, named from A to D. The black stars represent the true synthetic hypocenters, and the red stars represent the solutions estimated by the algorithms.

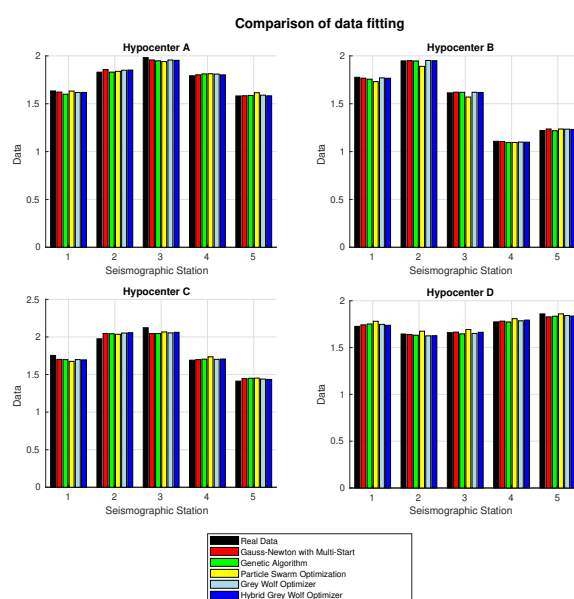


Figure 2: Calculated and real data for each seismic event for each algorithm.

In the fourth criterion, we evaluated the comparability of the total cost in achieving average solutions among the algorithms. The null hypothesis ( $H_0$ ) posits that there is no difference between the sums of costs of solutions obtained for each algorithm. The test produced a  $p$ -value of 0.0049, leading to the rejection of  $H_0$ . This rejection suggests a disparity in the sums of solution costs, with *HGWO* demonstrating a quicker descent towards the function minimum compared to the other algorithms. This distinction is evident in Fig. 3. In the fifth criterion, we examined the

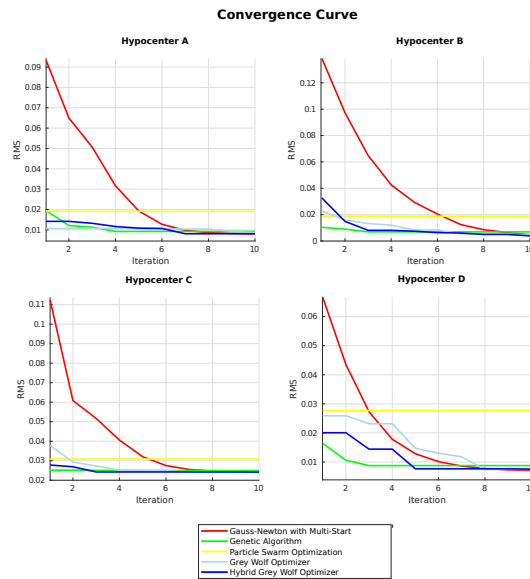


Figure 3: Comparison of the objective function cost across iterations for estimating each hypocenter using the different addressed algorithms.

equivalence of the mean objective function costs across stabilized iterations for different algorithms. The null hypothesis ( $H_0$ ) states that there is no difference between the mean costs in the iterations where each algorithm has stabilized. Given a  $p$ -value of 0.0022, we reject  $H_0$ , indicating a significant difference in the mean costs during the stabilized iterations. Among the algorithms, *GWO* achieved lower objective function costs, effectively minimizing the values and producing reasonable solutions within the search space.

In the sixth criterion, we examined the equivalence of algorithm convergence. The null hypothesis ( $H_0$ ) posits that there is no difference between the iterations in which each algorithm converged. With a  $p$ -value of 0.0009,  $H_0$  is rejected, indicating divergent convergence iterations. Specifically, *PSO* achieves stability more rapidly towards the function minimum, as evident in Fig. 3.

In the seventh and final criterion, we examined the equivalence of computational costs required to obtain average solutions among the algorithms. The null hypothesis ( $H_0$ ) posits that there is no difference in computational time needed to achieve average solutions with each algorithm. With a  $p$ -value of 0.0015,  $H_0$  is rejected, indicating significant variations in computational times for solution acquisition, with *GWO* being the fastest of all.

## 6. Conclusion

This study implemented hypocentral modeling and inversion algorithms in the *Octave* programming language to estimate seismic hypocenter locations. The algorithms addressed include

*GM*, *GA*, *PSO*, *GWO*, and *HGWO*. Experiments with synthetic seismic data, based on a two-layer crustal velocity model within a 20 km by 17 km grid at a depth of 10 km, yielded promising results.

The algorithms effectively estimated the hypocentral parameters, as evidenced by the close proximity between the hypothetical and actual locations. A comprehensive analysis, including fit metrics, adjusted data graphics, hypocenter location graphics, convergence curves, and computational times, confirmed the stability of the solutions and their effective convergence.

The *HGWO* demonstrated superiority in minimizing the total cost of the objective function to achieve average solutions, while the *GWO* excelled in reducing the mean objective function costs across stabilized iterations and computational time. However, the *PSO* algorithm was the fastest in terms of convergence speed.

In summary, this study significantly enhances the understanding and prediction of seismic events. The methodology effectively locates seismic hypocenters, supported by detailed solution analyses that validate the quality and reliability of the results. Future work will focus on processing and analyzing real seismic data.

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