

A REVIEW ON PROJECT PORTFOLIO SELECTION CONSIDERING INTERDEPENDENCIES

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ABSTRACT

In an increasingly competitive market, companies must strive to outperform their competitors. Hence, organizations must carry out effective project portfolio management to achieve strategic benefits for their organizations and add value to customers. Project portfolio selection is crucial in this process, and the interdependencies between projects add complexity to its activities. Nevertheless, it is relevant to incorporate these relationships into the model, given the possibility of seizing opportunities and improving resource management. This paper describes a systematic literature review on project portfolio selection with interdependencies. We covered the following topics regarding portfolio selection considering interdependencies between projects: terminology applied in project relationships, main approaches, and application scenarios, among others. Based on our results, we present promising research trends for future research regarding this topic and to improve project portfolio selection amongst practitioners in organizations.

KEYWORDS. Project Portfolio Selection. Interdependency. Systematic Literature Review.

Paper topics: AD&GP – PO na Administração e Gestão da Produção and OA – Outras aplicações em PO.

1. Introduction

With globalization and increased competition, a company must seek ways to ensure its competitiveness in the market. One way to add value to customers is through project implementation. Projects are undertaken to meet specific market needs, seize strategic opportunities, comply with environmental and legislative requirements, and promote technological advancement, among other reasons. Organizations typically implement multiple projects simultaneously to obtain benefits from their collective management, thus performing Project Portfolio Management to leverage organizational results. [PMI 2017].

Project Portfolio Management (PPM) focuses on a more holistic view for efficiently achieving the organizational objectives through the projects. It relies upon a Project Portfolio Selection (PPS) process that ensures that the projects selected for the portfolio comply with stakeholder requirements, resource constraints, and business strategies. PPS is often quite complex, given the uncertainties associated with projects regarding information, multiple objectives, risks, and interdependencies between projects, among others. [Benaija and Kjiri 2015].

Interdependencies (sometimes referred to in the literature as “interactions” or “synergies”) refers to the relationships between projects. We say that two projects are interdependent when an aspect of one influence an aspect of another, either positively or negatively [Zhai et al. 2021]. These relationships may involve resource sharing, increase/decrease in expected benefits, risks, or other project peculiarities. It is of utmost importance to know and consider interdependencies during portfolio selection to make the problem more similar to reality, seize generated opportunities, reduce harm in project set management, and assist in program implementation.

Most academic papers do not incorporate interdependencies in PPS [Martinsuo 2013]. Authors argue that it is difficult to obtain accurate information and that interdependencies increase considerably the complexity of the model. Moreover, our previous research identified a gap in the literature: there was no previous paper with a thorough review of interdependencies between projects in PPS.

Hence, we noticed that a literature review could help organizations with ideas about possible improvements in their portfolio management processes, propose new research avenues within the topic, and clarify pre-existing knowledge about models that can leverage PPS activities within the organizations.

Moreover, we sought to understand how authors have used terms such as interdependence, interaction, and synergy throughout the literature. We acknowledged that this knowledge could help us propose a novel terminology with a clear distinction among these terms.

Therefore, we conducted a systematic literature review on PPS selection with interdependencies to fill the previously mentioned gap. This study discussed some theoretical concepts on the topic, the most common approaches in the literature to deal with PPS by considering interdependencies, a proposal for defining relationships between projects in PPS, and some future research trends.

This article comprises five sections. This introduction presents our motivation for conducting this study. Section 2 provides contextualization on the area of project portfolio management and interdependencies. Section 3 summarizes how we conducted the literature review process. Next, Section 4 presents the main results of our review. Finally, Section 5 presents the final considerations and discusses our contributions.

2. Portfolio Management

2.1 Project Portfolio Selection

According to [PMI 2021], a project is a temporary endeavor to generate a unique outcome. Additionally, projects are investments made to assist in achieving organizational strategic objectives [Vetschera and Almeida 2021]. When projects compete for resources, organizations usually manage them as a project portfolio [PMI 2017].

[Mohagheghi et al. 2019] argue that Project Portfolio Selection (PPS) is a periodic and continuous effort to support organizational goals. It aims at the optimal allocation of resources among a subset of actions [Liu and Zhang 2019]. In summary, PPS selects a portfolio that comprises those projects that best align with the organization's objectives within the resource constraints of the company.

However, PPS is a complex Portfolio Management activity. It often involves selecting a set of projects among multiple possibilities that comprise information uncertainty and risks. It also frequently requires multi-objective analysis and the assessment of the effects of interdependencies between projects, among other issues [Benaija and Kjiri, 2015].

2.2 Project interdependency

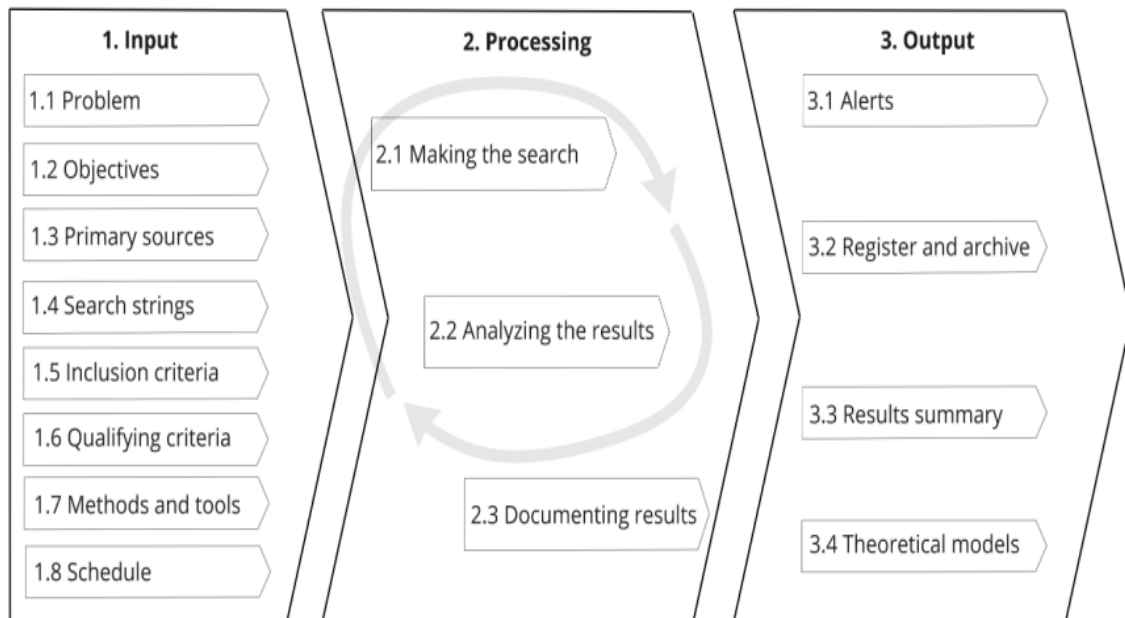
Interdependencies (interactions or synergies) are relationships between projects that are ubiquitous in portfolio selection [Alvarez-García and Fernández-Castro 2018]. They can influence aspects such as costs, success, value, etc. Consequently, the total benefit of a portfolio might not be identical to the sum of individual contributions of projects [Schmidt 1993] when these relations are present.

Although sometimes they can benefit the portfolio results, interdependencies typically increase the complexity of PPS decision-making [Vieira et al. 2024]. Thus, traditional portfolio selection approaches may not be sufficient [Killen 2013], and managers face challenges in developing models that adequately incorporate interdependencies between projects. In this scenario, many PPS studies discard these relationships or ignore conceptual aspects that could be useful to PPS processes. Hence, the following section presents the procedures undertaken to find the articles on portfolio selection with interdependencies among projects.

3. Literature Review Protocol

We applied an adaptation of the model proposed by [Conforto et al. 2011], called RBS Roadmap to execute the literature review. The protocol comprises three phases, totaling 15 stages, as shown in Figure 1 and detailed below.

Figure 1 - Systematic Literature Review Model



Source: [Vieira et al. 2024]

3.1 The Phase 1 – Input

Phase 1 consists of eight stages, focusing on defining key research parameters, such as objectives, inclusion criteria, and keywords.

Stage 1.1 Problem - The study's scope was delimited by the question: "Are there PPS approaches for organizations that consider interdependencies between projects in the decision model?"

Stage 1.2 Objectives - The main research objective was defined as: "Identify works with applications that address project portfolio selection methods and explicitly consider interdependency between projects in the proposed model."

Stage 1.3 Primary Sources - The initial studies in the area analyzed were [Almeida and Duarte 2011] and [Duarte 2007]. Relevant keywords for the research were identified through these studies, and various combinations were tested in databases.

Stage 1.4 Search Strings – Following the previous step, the following string was considered: ("portfolio problem" OR "portfolio decision analysis" OR "portfolio decision problem" OR "project portfolio selection" OR "project selection" OR "portfolio analysis" OR "project Portfolio") AND ("project*" AND ("interaction*" OR "interdependen*" OR "synerg*")).

Stage 1.5 Inclusion Criteria - The following criterion was defined for article inclusion: "papers related to project portfolio selection that identify case studies related to interdependency (or synergy or interaction) between projects." Articles dealing solely with mutual exclusion and complementary relationships were excluded, given their extensive exploration in knapsack problem studies. These relationships should be between projects, not between problem criteria, and should be clearly incorporated into the model in the study. Additionally, articles on investment portfolio project portfolio management (without portfolio selection) were not considered.

Stage 1.6 Qualifying Criteria – The five qualitative criteria were: (i) publication in selected databases (Web of Science or Scopus), irrespective of publication year; (ii) publication in an English-language scientific journal; (iii) addressing interdependency relationships beyond mutual exclusivity and complementarity; (iv) exclusion of papers presenting portfolio management approaches without portfolio selection methodology; and (v) inclusion of papers specifically addressing project portfolios, excluding other portfolio types.

Stage 1.7 Methods and Tools – The filters used were similar to those proposed by [Conforto et al. 2011], but as mentioned in the previous stage, two databases were used. Therefore, a preliminary filter was adopted to remove possible duplicate article copies. Thus, the following filters were considered:

- Filter 0: identify the duplicated papers and retain a single copy;
- Filter 1: read the Title, Abstract and Keywords;
- Filter 2: read the Introduction and Conclusion;
- Filter 3: full reading of the paper.

Stage 1.3 Schedule – The database search was conducted on 11/17/2022 at 11:25 (GMT-3). Article analysis was conducted during the months of November 2022, December 2022, and January 2023.

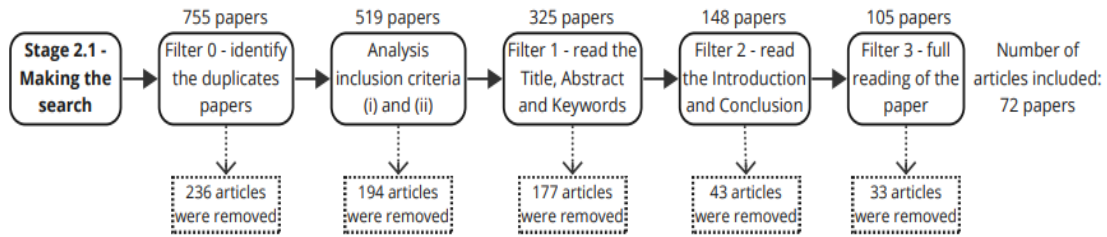
3.2 The Phase 2 – Processing

This phase involves executing the search in the databases and analyzing the articles.

Stage 2.1 Making the search – we found 331 articles in Web of Science and 424 in Scopus, resulting in a total of 755 articles. When we applied Filter 0, 236 duplicate articles were removed as they appeared in both databases. Regarding the analysis of qualitative criterion (ii), we realized that 15 articles were not in English and were thus excluded from the set of articles. In Filters 1 and 2, we identified 177 and 43 articles, respectively, were beyond the scope of the investigation. In Filter 3, we applied the qualitative criteria (iii), (iv), and (v) and we identified 31 articles that did not meet at least one of the criteria, and 2 articles were disregarded due to inaccessible content. After we applied all filters and inclusion criteria, a total of 72 articles were included in the literature review.

Figure 2 presents a summary of the results of each step of Stage 2.1.

Figure 2 - Summary of Stage 2.1 analyzes



Source: [Vieira et al. 2024]

Stage 2.2 Analyzing the results – we conducted multiple analyses of the articles, including: (i) years of the publications; (ii) application scenarios; (iii) countries of application; (iv) definitions for project relationships in PPS; (v) theoretical concepts related to interdependencies; (vi) important concerns to be addressed in PPS; (vii) presentation modes; (viii) degrees of interdependencies; (ix) different perspectives of the influence of project interdependencies in PPS; and (x) approaches for PPS that considered interdependencies between projects.

Stage 2.3 Documenting results - we documented all the results of Stage 2.2. A summary of this documentation presented in Section 4.

3.3 The Phase 3 – Output

This phase consists of “Stage 3.1 - Alerts”, “Stage 3.2 - Register and Archive”, “Stage 3.3- Results Summary” and “Stage 3.4 - Theoretical Models”. Section 5 describes the recommendations extracted from the literature review along with supporting evidence and important alerts for researchers regarding future trends in the field.

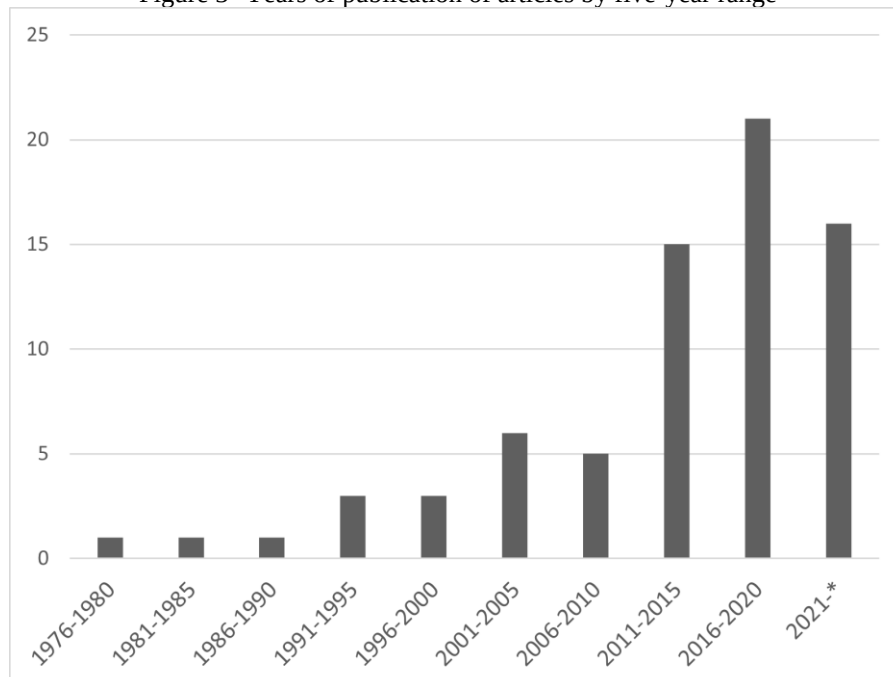
4. Results

This section summarizes the results of the literature review. For a complete description of these results, the reader can refer to [Vieira et al, 2024].

4.1 Years of the publications

The number of articles published per five-year range is shown in Figure 3.

Figure 3 -Years of publication of articles by five-year range



Source: Authors (2024)

We highlight that only three papers were published in the first three periods (from 1976 to 1990). The first was in 1978, followed by a six-year interval until the next one in 1984, and two more years for a new study in 1986. Subsequently, the numbers increased slightly in the following year ranges: (i) three papers were published between 1991 and 1995 and between 1996 and 2000; (ii) six papers were released between 2001 and 2005; and (iii) there was a slight decrease in the period 2006-2010, with only five publications. From 2011 to present, there was a significant growth in the number of publications: 15 papers were published between 2011 and 2015, 21 papers between 2016 and 2020 and, finally, 15 papers were published in the last two years.

4.2 Application scenarios

Most papers (39%) devised hypothetical data to illustrate their approach, while simulations were the second most common scenario (25%), often presenting comparisons between newly proposed methods and other ones from previous publications. Some papers (6%) retrieved data from earlier work to illustrate their approach. Other papers (30%) presented practical applications implemented in real organizations, such as IT projects (four papers), Energy and Infrastructure (three papers), and two papers each for Improvement Projects, R&D, and Technology. Other scenarios, such as "Air traffic," "Consulting Projects," "Health," "Mining," "Oil and Gas," and "Startup Projects," had one paper each.

4.3 Definitions and implications from project interdependencies in PPS

We identified three primary terms to describe relationships: "project synergy," "project interaction," and project interdependency. These terms are used interchangeably across the literature on PPS. We investigated the terms applied and the specific types of interdependence addressed in each study. Additionally, we mapped the correspondences between the terms employed and allocated them into categories, such as Value/Benefit, Resource, Time-Related, Technology, etc.

After analyzing the definitions retrieved from the selected articles, we proposed the following definitions as a common terminology for these relationships:

- "Project interdependency" refers to any relationship between projects in a PPS process, where two or more projects can generate a combined effect different from the sum of their individual effects.
- "Project interaction" represents the mechanism by which an interdependency influences the PPS process. Each interdependency may involve one or more interactions pertaining to pre-defined categories: value, resource, time-related, technology, accomplishment, technical, knowledge, strategic, fiscal, mutual exclusivity, and complementary interactions.
- Project synergy (and cannibalization) describes the effect of a project interaction. Synergy enhances projects' performance measures or decreases resource consumption within a portfolio, while cannibalization reduces projects' performance measures or increases consumption due to interaction.

Figure 4 presents an overview of the terminology proposed.

While not exhaustive, these definitions provide a starting point for standard terminology to describe project relationships. This proposed notation can aid researchers and practitioners in identifying and analyzing interdependencies by mapping their interactions and effects.

4.4 Degree of the interdependencies between projects

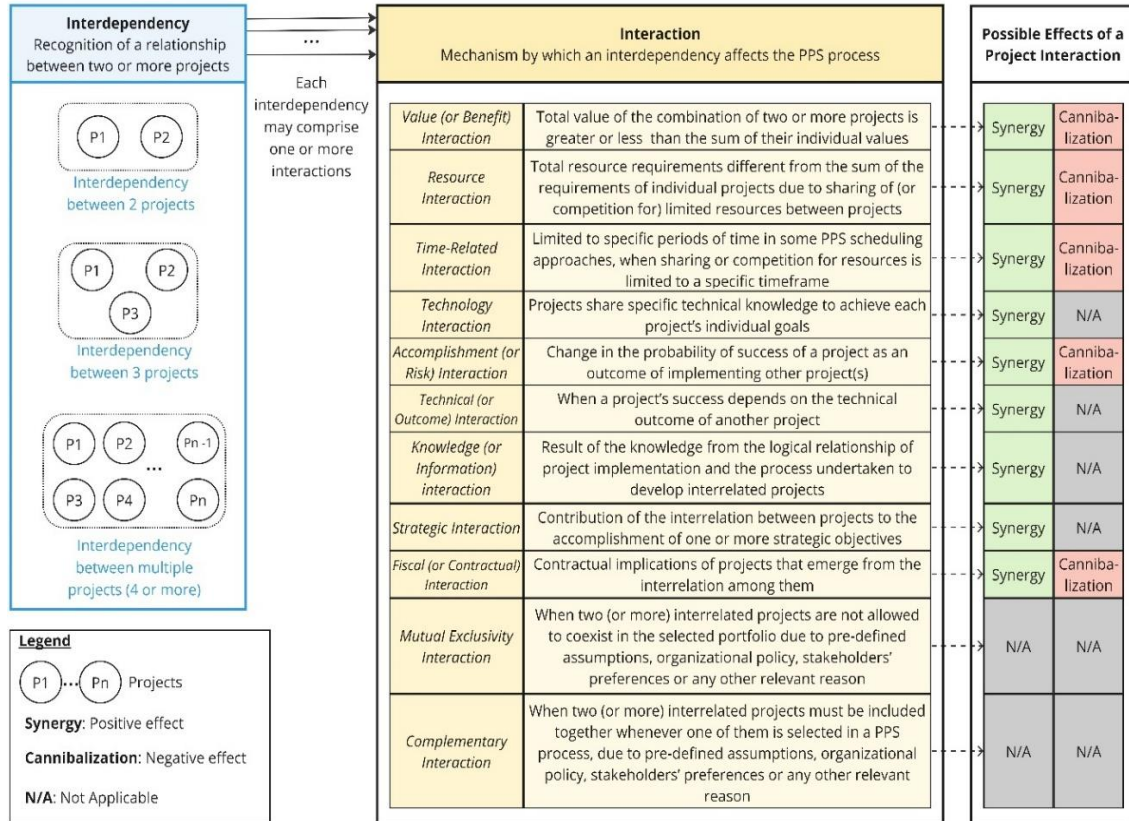
Three degrees of interdependencies were identified: pairs (involving two projects), pairs/trios (up to three projects), or subgroups (involving two or more projects, considering all possible subsets). Most relationships were between pairs (57% or 41 papers). Other relationships included pairs/trios (11% or 8 papers) and subgroups (32% or 23 papers).

4.5 Approaches for PPS

This subsection outlines the findings regarding approaches to PPS considering interdependencies between projects. They were categorized into five groups: "Multicriteria" (27 articles), "Linear Programming" (16 articles), "Non-linear Programming" (15 articles), "Heuristics and Metaheuristics" (14 articles) and "Other approaches" (12 articles). It is worth noting that the

sum of approaches is greater than the number of articles, but this is because many papers employ a combination of methods.

Figure 4 - Overview of the Terminology proposed for Project Relationships



Source: [Vieira et al. 2024]

Table 1 compares the main approach categories, detailing their advantages, disadvantages, and applications. It serves as a valuable tool for identifying suitable approaches for future research. Researchers can use the Table to select the approach category that best addresses their research problem.

5. Important findings and gaps from literature review

Based on the information gleaned from the literature review articles, we identified some significant conclusions about the subject and interesting topics for future research.

Regarding research focused on applications, only 30% of the articles conducted case studies (as mentioned in the previous section). Therefore, there is an urge to encourage researchers to present more papers with real case studies. Future studies in areas with few applications up to now, such as sustainable projects, health support projects, and military applications could be especially relevant.

We also found that the number of publications has increased over the years, particularly in the last 13 years. One possible reason for this is the growth of the computational resources available for organizations due to continuous technology improvements in hardware and software. Some PPS instances deal with many projects that commonly yield a higher number of interactions, and higher computational capacity can benefit the resolution of such complex problems.

We highlight that when proposed projects have a significant strategic impact and can contribute decisively to the overall goals, organizations often realize the benefits of adopting reliable and high-performance computational resources to undertake "PPS" since the expected return is far greater than the computational infrastructure cost.

Table 1 - Approaches for PPS

APPROACH	BENEFITS	DRAWBACKS	APPLICABILITY
Multicriteria Approaches	- Allows for integration of multiple decision objectives; - Can be combined with other methods; - Facilitates inclusion of decision makers' opinions and preferences.	- May encounter issues like incomparability, non-transitivity, and cyclic preferences; - Decision makers may struggle to grasp the scientific rationale behind multi-objective-based approaches.	- Suitable for decision problems involving multiple objectives. - Incorporates uncertainties through fuzzy variables (as discussed in various literature approaches). - Frequently cited in multicriteria approaches.
Linear Programming Approaches	- Facilitates complex relationship mapping between decision variables as model constraints; - Most modern commercial solvers offer efficient, exact, or approximate solutions for linear programming models.	- Real-world problems often exhibit nonlinear behavior, making accurate modeling challenging with linear functions; - Nonlinearities typically arise when considering project relationships.	- Applicable to a wide range of domains where problems can be modeled using linear functions alone; - Linear programming algorithms are generally simpler than those of other approaches, with multiple tools available for their solution.
Non-Linear Programming Approaches	- Allows for flexible modeling. - Numerous mathematical solvers and application programming interfaces are available for nonlinear programming models; - Real-world problems often exhibit nonlinear behavior, requiring nonlinear models for accurate representation.	- Nonlinear models are frequently more complex, making finding suitable solutions challenging; - Some authors resort to linearizing the model or using heuristics due to its complexity.	- Applicable to problems across various domains where nonlinear function models accurately reflect the problem's characteristics; - Many older papers focused on nonlinear programming models, with several addressing Research and Development.
Heuristics and Metaheuristics Approaches	- Suitable for highly challenging problems; Particularly beneficial for cases with strict scheduling constraints or high information elicitation costs; - Less affected by the mathematical properties of objective functions and problem constraints.	- May not find a globally optimal solution to the PPS problem within a reasonable timeframe; - Risk of heuristic becoming trapped in a local maximum or minimum within a large solution region.	- Genetic Algorithm (GA) frequently applied in PPS problems for optimized solution search; - Suitable for handling large-scale instances.

Source: Adapted from [Vieira et al. 2024]

Only one paper presented an approach considering project selection for a pre-implemented portfolio, addressing interdependencies within new projects and those already in the portfolio. Hence, this presents an interesting gap for future research, as many organizations deal with similar issues in their PPS processes.

Another finding was that more than half of the articles address the degree of interdependence in pairs, possibly due to the difficulty of comparing multiple projects. Thus, a promising research avenue would be the investigation of interdependencies among more than two projects. Authors could propose alternative mathematical models to deal with higher degrees of interdependencies and novel approaches, such as visual tools for identifying and measuring these relationships.

We also identified other promising research branch: the assessment of information regarding the costs of interdependence between projects. Only two articles in the review addressed this topic, which is particularly relevant for situations with many projects and interdependencies. It is also noteworthy the potential to develop procedures to identify the concerns of stakeholders or organizations and to map the impact of the interdependencies in the achievement of these concerns (or strategic objectives).

Researchers can also delve into relationships for specific project domains not explored in the literature, which primarily focuses on IT projects. Furthermore, we noticed some additional gaps in the literature, for instance, the identification of interactions stemming from knowledge sharing between projects within a portfolio, the assessment of the influence of strategic guidelines and stakeholder perspectives on identifying interdependencies, and the investigation to verify if contractual issues can lead to unforeseen interdependencies.

In our study, we introduced a new terminology with precise definitions for "interdependency," "interaction," and "synergy". However, our proposed terminology is not exhaustive. Hence, a promising direction for future research would be to explore additional types of interactions and their impacts.

Additional work is also necessary to depict the relation between PPS considering interdependencies and the implementation of programs within organizations, thereby promoting knowledge that can assist managers in achieving benefits and mitigating drawbacks of each program.

Figure 5 presents the previously mentioned gaps into five categories. The dashed lines in the figure indicate related topics, and their combination might represent complementary research avenues.

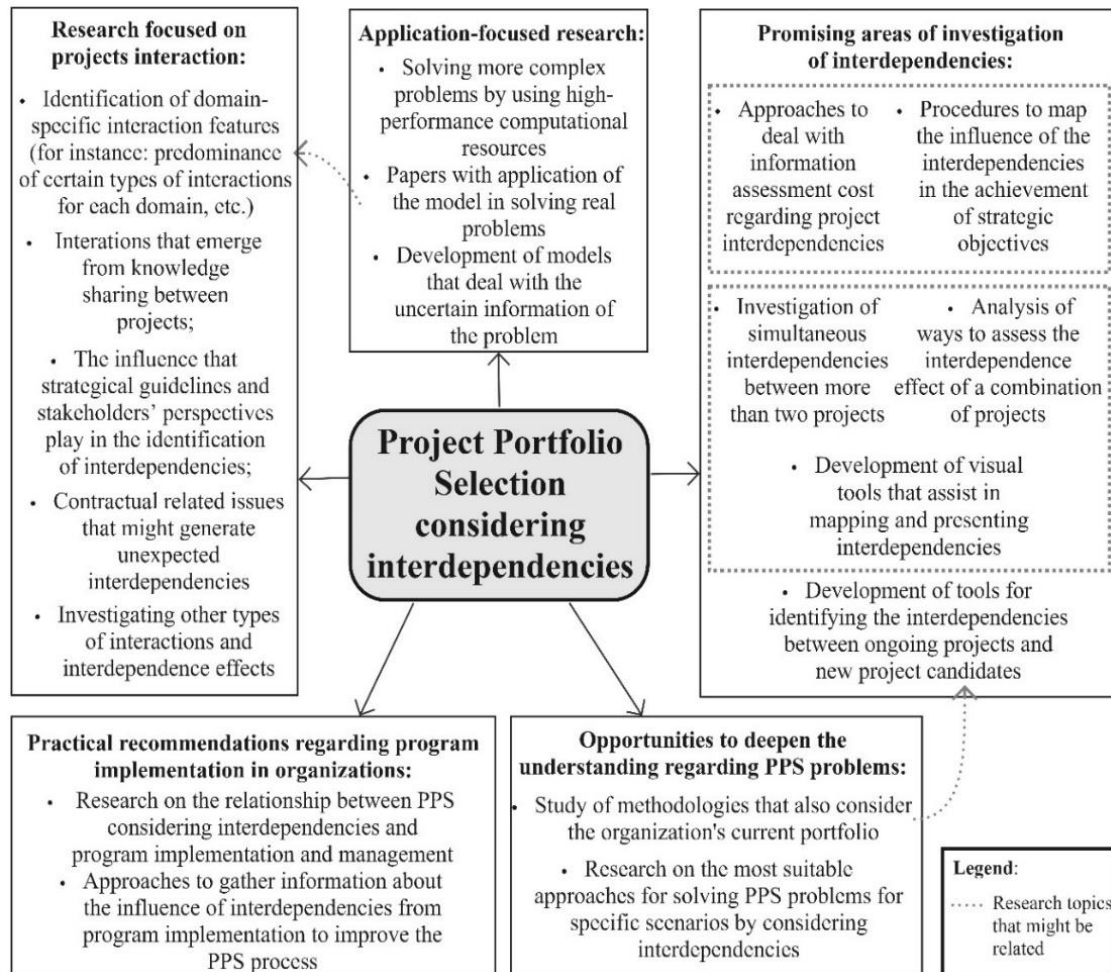
6. Conclusion

Operations and investments within organizations are often subject to a finite amount of available resources. Therefore, project selection and implementation decisions should rely on a formal process that addresses all relevant concerns and issues. Thus, Project Portfolio Selection is required to achieve strategic objectives considering resource constraints and is particularly complex when there are interdependencies between projects. Recognizing these interdependencies is crucial as they can provide insights to identify opportunities for leveraging overall project results within the organizations.

Hence, we presented a literature review throughout this work that depicted the main contributions and recommendations for future work in PPS with interactions. We sought and analyzed articles that address PPS methods and consider interdependencies within their proposed model.

The first main contribution of this paper was the summary of the approaches identified in the literature to deal with this type of problem, which can help researchers and professionals select the most suitable approach for their PPS problem. Additionally, we proposed a terminology definition for interdependence, interaction, and synergy to reduce misunderstandings in conceptual and practical discussions about the relationships between projects. We also provided valuable insights to guide research paths for researchers that might drive new research developments in this topic and assist organizations in portfolio and program management.

Figure 5 - Conceptual framework with trends and models for future research



Source: [Vieira et al. 2024]

Among the limitations of this study, we highlight that we considered only papers from two databases (Scopus and Web of Science), and we included only those written in English. Therefore, we might have missed some insights from papers from other databases and written in different languages.

Additionally, we depicted some recommendations for future work, such as new practical applications in areas still not explored in previous studies, the investigation of interdependencies between more than two projects, the development of tools to assist in obtaining interdependencies, the analysis of the costs of obtaining information about the impact of interdependencies in PPS processes, and research to generate a broader understanding of interaction mechanisms, among others.

Lastly, we believe we have filled a gap in the literature regarding terminology and knowledge in portfolio selection with interdependence between projects, thus assisting managers in improving their processes to leverage their expected returns from portfolios, programs, and projects.

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