LVI SBPO SIMPÓSIO BRASILEIRO DE PESQUISA OPERACIONAL
DE 04 ATÉ 07 NOVEMBRO 2024
FORTALEZA | CEARÁ

Measuring Paradigm Shifts in dynamic efficiency frameworks of public administrations: A Directional Distances Malmquist Index approach

Thyago C. C. Nepomuceno
Department of Statistics, Federal University of Pernambuco
Av. Jorn. Aníbal Fernandes, 497-629 - Cidade Universitária, Recife - PE, 50740-540
thyago.nepomuceno@ufpe.br

Cinzia Daraio
Dep. of Computer, Control and Management Engineering, Sapienza University of Rome
Via Ariosto, 25, 00185 Roma RM, Itália
daraio@diag.uniroma1.it

ABSTRACT

The values and priorities of public institutions frequently shift in response to election outcomes in many democracies. Consequently, there is a recurring need to incorporate subjective policies and varying preference structures in efficiency and productivity assessments. We propose a novel Malmquist directional framework for measuring paradigm shifts in response to these challenges. We extend current directional approaches by accounting for changes in subjective perspectives, input contraction or output expansion uncertainties, and the dynamic nature of technical and technological changes over time. This proposal can measure directed recovery, preference shifts, and paradigm shifts and offer a comprehensive tool for evaluating subjective efficiency changes over time, enhancing the robustness and adaptability of efficiency measurements particularly relevant for evaluating the impact of changes in political contexts and policy preferences on the performance of public decision-making units.

KEYWORDS. Data Envelopment Analysis. Multicriteria Decision Aid. Malmquist Directional Index.

DEA – Análise Envoltória de Dados

1. Introduction

In the dynamic landscape of public administration, decision-makers often face the challenge of balancing multiple, sometimes conflicting, priorities. Nevertheless, from the seminal work of (Charnes et al., 1978), the measurement of efficiency in decision-making units has traditionally relied on unidimensional approaches. These approaches align with production technologies that involve a single criterion or policy dimension, which does not directly influence decision-makers' preferences in the production process.

Although some interesting approaches introducing weight restrictions to capture preferences and value judgments have addressed the issue of subjective performance evaluations (see, e.g. Dyson & Thanassoulis, 1988; Zhu, 1996). Those nonparametric projections fail to capture the multidimensional nature inherent in many industries, including public administrations. Production perspectives related to profitability, client satisfaction, managerial preferences, sustainability, and people's well-being are distinct yet interrelated dimensions that are affected differently by the different combinations of input and output. Consequently, accurately assessing the efficiency of decision units in such environments requires considering the relative importance and preferences associated with each dimension.

Directional Distance Functions (DDF) offer flexible and consistent analytical formulations to construct composite indicators of efficiency, effectively addressing the subjective nature of evaluations. Composite indicators of public services can provide efficiency and effectiveness classifications of public banks, public hospitals, schools, universities, courts and police stations, non-governmental businesses, telecommunications, energy companies, among other entities, that can be relevant to researchers, governments, industry, and stakeholders by considering the multidimensional output perspective of each particular production system. However, the specification of appropriate direction vectors when preferences change over time due to changes in the political context and their relative importance remains a critical challenge yet to be explored in the productivity and efficiency analysis literature. In the next section, we discuss a new approach based on a Malquist index, including directions to cope with this issue.

2. Measuring dynamic subjective efficiencies over time

The Malmquist productivity index was initially introduced by the Swedish economist and statistician Sten Malmquist (1953) and extended to the multi-input and output production set by Fare, Grosskopf and Lovell (1994) to measures the total factor productivity of decision-making units (DMUs) comparing input-output data across different time periods. The index assesses how efficiently a DMU transforms inputs into outputs over time, offering insights into productivity changes. The DEA-based Malmquist productivity index can be decomposed into two main components: Technical Efficiency Change (TEC): measuring the catch-up or recovery effect of decision units over time which is related to how well a DMU improves its use of inputs to produce outputs without considering changes in the technology itself, and Technological Change (TC) representing how the entire production possibility set evolves (frontier shift), impacting all DMUs' potential performance.

While the recovery effect measures how efficiency changes compared to the best practice frontier in the same period, frontier shifts reflect changes in the shape or location of the production frontier itself, indicating technological improvements or declines. Nevertheless, from time to time, changes in the industry perspectives based on several decision criteria can be observed in addition to technical or technological changes. Public administrations, political influence, and other exogenous demands provoke a change of paradigms in service units that are used to conduct processes and activities in a certain way.

A paradigm shift can be defined as a fundamental change in the underlying assumptions, methodologies, or perspectives within a scientific dimension or other framework. To the best of our knowledge, the current options for estimating efficiency changes over time based on two single-period and two mixed-period measures fail to consider this perspective. In this work, we introduce the concept of paradigm shift as a measure of how changes in values, decisions, polices

or preferences can impact the production of decision-making units, assuming innovation (technology frontier) does not change in the short term. The following equations formalize this proposition, including the Directed Recovery, Preference Shift and Paradigm Shift indicators to the traditional Malmquist productivity index, illustrating a frontier application.

$$\text{Recovery: } \frac{Eff_{ot_1|T_1}}{Eff_{ot_2|T_2}} \qquad \textbf{(1)}$$

Where (for VRS input-oriented model):

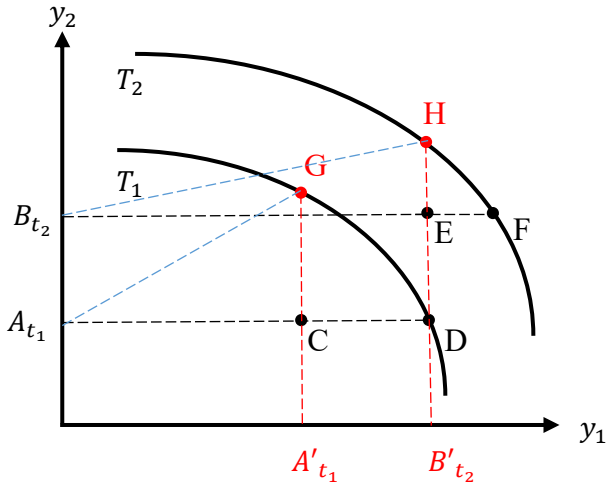
$$Eff_{ot|T} = \sup\{\theta(x_i, y_r) \in T \mid \sum_{j=1}^{n} \lambda_j y_{jrt} \geq y_{ort}, \sum_{j=1}^{n} \lambda_j x_{jit} \leq \theta x_{oit}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\} \qquad \textbf{(2)}$$

and (for directional models):

$$Eff_{og_t t|T} = \sup\{\beta(x_i, y_r) \in T \mid \sum_{j=1}^{n} \lambda_j y_{jrt} \geq y_{ort} + \beta g_{y_{ort}}, \sum_{j=1}^{n} \lambda_j x_{jit} \leq x_{oit} - \beta g_{x_{oit}}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\} \qquad \textbf{(3)}$$

$$\text{Directed Recovery: } \frac{Eff_{og_1 t_1|T_1}}{Eff_{og_2 t_2|T_2}} \quad \textbf{(4)}, \quad \text{Preference shift } (t)\text{: } \frac{Eff_{og_1 t|T}}{Eff_{og_2 t|T}} \quad \textbf{(5)}$$

Figure 1: Illustration of recovery effect and preference shift



$$\text{Recovery: } \frac{Eff_{ot_2|T_2}}{Eff_{ot_1|T_1}} = \frac{\text{BF}}{\text{BE}} / \frac{\text{AD}}{\text{AC}}, \quad \text{Directed Recovery: } \frac{Eff_{og_2 t_2|T_2}}{Eff_{og_1 t_1|T_1}} = \frac{\text{B'H}}{\text{B'E}} / \frac{\text{AD}}{\text{AC}}$$

$$\text{Preference shift } (t_1)\text{: } \frac{Eff_{og_1 t_1|T_1}}{Eff_{og_2 t_1|T_1}} = \frac{\text{AD}}{\text{AG}}, \quad \text{Preference shift } (t_2)\text{: } \frac{Eff_{og_1 t_2|T_2}}{Eff_{og_2 t_2|T_2}} = \frac{\text{BF}}{\text{BH}}$$

Where $\text{AG} = \sqrt{\text{AC}^2 + \text{CG}^2}$ and $\text{BH} = \sqrt{\text{BE}^2 + \text{EH}^2}$

Assuming no technological change, the recovery effect greater than 1 (one) indicates an advancement in the relative efficiency from periods 1 to 2, the same as the Malmquist Index (Cooper et al., 2007, Fare et al. 1994). Recovery effects equal to 1 (one) mean lack of technical efficiency change and recovery effects below 1 (one) indicate the unit regressed in technical efficiency over time. Directed recovery effects based on preferences, values, or policies have similar interpretations: units scoring above one indicate an improvement in the subjective efficiency from period 1 to 2, units scoring equal to or less than one have no technical efficiency change or have deteriorated. Preference shifts assess whether units become more or less capable after including the new directions. Preference shifts greater than 1 indicate the new directions are beneficial for the units under evaluation. In contrast, preference shifts equal or less than, respectively, means that the new directions have no effect or are disadvantageous for the units under evaluation (i.e. making it more difficult attain efficient prospects or making units less capable of producing optimally).

The paradigm shift (PS) is computed as the product of directed recovery and the geometric mean of preference shifts for each j = 1, 2, 3, …, n units operating in times t_1 and t_2:

$$\text{Paradigm shift} = \frac{Eff_{og_2t_2|T_2}}{Eff_{og_1t_1|T_1}} * \sqrt{\frac{Eff_{og_1t_1|T_1}}{Eff_{og_2t_1|T_1}} * \frac{Eff_{og_1t_2|T_2}}{Eff_{og_2t_2|T_2}}} \quad (6)$$

3. Conclusion

The paradigm shift indicator represented can be interpreted as a measure of how a change in the overall operational perspective, for instance, by including or removing objectives, assumptions, value judgments, decision dimensions, polices or preferences on the way production process should be conducted, affects the production capacity of decision-making units. Assuming innovation in the processes, technologies or methods are kept constant (i.e. do not change considerably from t_1 and t_2), paradigm shifts greater than 1 (one) point to a positive evolution in the operational and strategic management of the organization. On the other hand, decision units scoring paradigm shifts less than 1 (one) have deteriorated both operational and strategic management. Paradigm shifts equal to one indicate similar levels of subjective efficiency over time.

The framework's ability to measure directed recovery, preference shifts, and paradigm shifts provides a comprehensive tool for evaluating the dynamic efficiency of public institutions. This approach enhances the accuracy and relevance of efficiency assessments and offers valuable insights for researchers, governments, and stakeholders in understanding and improving the operational and strategic management of public services.

References

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, *2*(6), 429-444.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software* (Vol. 2, p. 489). New York: springer.

Dyson, R. G., & Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *Journal of the operational research society*, *39*, 563-576.

Fare, R.S., Grosskopf, S., Lovell, C.A.K.. (1994). *Production Frontiers*. Cambridge University Press.

Malmquist, S. (1953). Index numbers and indifference surfaces. *Trabajos de estadística*, *4*(2), 209-242.

Zhu, J. (1996, January 1). Data Envelopment Analysis with Preference Structure. Palgrave Macmillan, 47(1), 136-150. https://doi.org/10.1057/jors.1996.12