

PIECEWISE LINEAR FUNCTIONS APPLIED TO RESERVOIR OPERATION RULES FOR POWER SYSTEM PLANNING

Luiz Raphael B. Lopes

luizraphael@psr-inc.com

PSR

Praia de Botafogo, 370, 1st floor

Thiago Correa Cesar

thiago.cesar@psr-inc.com

PSR

Praia de Botafogo, 370, 1st floor

Luiz Carlos da Costa Jr

luizcarlos@psr-inc.com

PSR

Praia de Botafogo, 370, 1st floor

ABSTRACT

The planning and operation of large-scale power systems with a significant share of renewable energy sources, such as hydroelectric plants, require the consideration of complex factors, including hydrological uncertainty and operating constraints. In the face of the severe water scarcity in the Brazilian National Interconnected System (SIN) in recent years, the local agencies implemented operating rules to prevent critical reservoir levels and enhance water security, mainly in the Paraíba do Sul and São Francisco basins. This paper introduces a methodology for incorporating these rules as linear constraints through interpolation in computational simulations using the Stochastic Dual Dynamic Programming (SDDP) algorithm. The results in the SIN case study highlight the methodology's effectiveness in maintaining reservoir levels above critical thresholds even in scenarios of low water availability. This approach provides valuable insights for planning and operating power systems with a high share of renewable energy sources and complex operating constraints.

KEYWORDS. Optimization. Integer Programming. Power Systems Planning.

Paper topics: OR in Energy, Oil and Gas. Mathematical Programming.

1. Introduction

The Brazilian National Interconnected System (SIN) is a large-scale power system with the predominance of hydroelectric installed capacity (110 GW) but with significant capacity from other sources: fossil and nuclear thermoelectric plants (26 GW), biomass (15 GW), wind (26GW) and photovoltaic (9 GW) power plants. Although the hydro installed capacity accounts for around 50%, its average contribution to supply the load exceeds 70%.

The hydro system is distributed in cascades with linked plants in which hydrological uncertainty plays an important role in decision-making. This complexity requires using a chain of computational models for planning, programming, and determining short-term market prices.

Furthermore, companies and institutions conduct studies for regulatory impact and project performance. These studies employ computational models to simulate system operations under various scenarios, enhancing the accuracy of commercial and operational forecasts. The widely used SDDP model, developed by PSR, efficiently solves this problem by simulating the system with a detailed physical representation and for different timescales relevant to decision-making.

In recent years, especially in the last decade, the SIN has experienced a severe period of water scarcity, which has led to some reservoirs of hydroelectric plants reaching very critical levels, evidencing several water use conflicts, mainly in the basins of the Paraíba do Sul and São Francisco rivers. This condition led the National Water and Sanitation Agency (ANA) and other local agencies to establish operating rules [ANA, 2017] to prevent these reservoirs from reaching extreme levels and to increase the water security of these basins.

Furthermore, as the participation of non-controllable sources in the matrix and the massive introduction of distributed generation in the system, some operating constraints, hitherto managed within the specific scope of the operation, began to become more evident, making their representation in computer simulations very important, especially for SIN load end service.

For example, we present in figure (1b) the safety curves of the Sobradinho reservoir, on the São Francisco River, from Dec-2022 to Nov-2023. According to this curve, the minimum outflow at the Xingo hydroelectric plant (the last of the cascade) depends on the volume stored in Sobradinho.

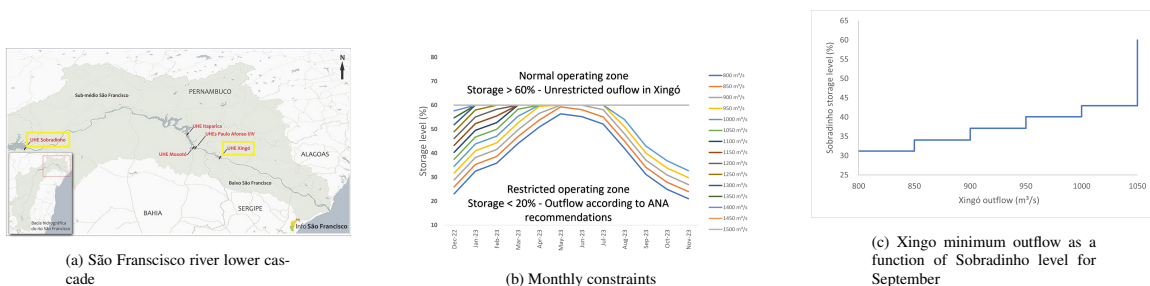


Figura 1: São Francisco cascade and security curve for Sobradinho reservoir and Xingo minimum outflow

2. Methodology

The Stochastic Dual Dynamic Programming (SDDP) algorithm is a widely used method for operation planning problems, particularly in countries like Brazil with a significant reliance on hydro plants [Pereira e Pinto, 1991]. SDDP employs Benders decomposition to break down the multistage stochastic problem into smaller sub-problems for each stage. It creates a cost function for each stage to represent the total cost from that stage until the end of the horizon. Problem (1) illustrates a simplified version of the one-stage operation planning problem.

$$Q_t(\hat{v}_{t-1}) = \text{minimize} \quad \sum_{j \in J} c_j g_{t,j} + \sigma r_t + \tilde{Q}_{t+1}(v_t) \quad (1a)$$

$$\text{subject to} \quad v_{t,i} = \hat{v}_{t-1,i} + \hat{a}_{t,i} - u_{t,i} - s_{t,i} + \sum_{m \in M_i} (u_{t,m} + s_{t,m}) \quad \forall i \in I \quad (1b)$$

$$\sum_{i \in I} \rho_i u_{t,i} + \sum_{j \in J} g_{t,j} + r_t = \hat{d}_t \quad (1c)$$

$$\{v_t, u_t, s_t, g_t, r_t\} \in \Phi \quad (1d)$$

The objective function (1a) minimizes the total cost of thermal generation $c_j g_{t,j}$ for each plant j in J , along with the costs of energy not supplied σr_t and future costs $\tilde{Q}_{t+1}(v_t)$. Constraints (1b) represent the hydro water balance for each plant i in I , where the current stage's final storage $v_{t,i}$ equals the sum of the final storage $v_{t-1,i}$, water inflow $\hat{a}_{t,i}$, and turbinning $u_{t,m}$ and spilling $s_{t,m}$ from upstream plants, minus the plant's own turbinning $u_{t,i}$ and spilling $s_{t,i}$. The load balance constraint (1c) ensures that the sum of hydro generation $\rho_i u_{t,i}$, thermal generation $g_{t,j}$, and energy not supplied r_t equals the net demand $\hat{d}_t$. Equation (1d) defines the set Φ , containing additional constraints like maximum generation, unit commitment, and water usage limitations.

We propose a special ordered set of type 2 (SOS2) [Beale, 1963] to model the hydro operating constraints using piecewise linear functions. Let x and y be decision variables from the traditional operation planning problem (e.g., final storage and minimum total outflow of a hydro plant). The set K consists of points $(\hat{x}_k, \hat{y}_k)$ defining the piecewise linear function. Derived variables λ_k and z_k are employed in the SOS2 modeling. The formulation is presented in (2).

$$\begin{pmatrix} x \\ y \end{pmatrix} = \sum_{k \in K} \lambda_k \begin{pmatrix} \hat{x}_k \\ \hat{y}_k \end{pmatrix} \quad (2a)$$

$$\sum_{k \in K} \lambda_k = 1 \quad (2b)$$

$$\lambda_k \leq z_{k-1} + z_k \quad \forall k \in K \quad (2c)$$

$$\sum_{k \in K} z_k = 1 \quad (2d)$$

$$z_0 = 0, z_K = 0 \quad (2e)$$

$$z_k \in \{0, 1\}, \lambda_k \geq 0 \quad \forall k \in K \quad (2f)$$

Constraints (2a) and (2b) define variables x and y as a convex combination of the input points. Constraints (2c), (2d) and (2e) ensure the solution is the combination of at most two consecutive points. Constraints (2f) define the variables z_k as binary and the variables λ_k as non-negative.

3. Case study

To verify the implementation of this methodology, simulations were carried out with the SDDP model considering the interpolation constraints to represent the safety curve of the São Francisco basin. These simulations considered the database of studies of the monthly operation program (PMO) prepared by ONS. In this study, the initial conditions of the January 2018 PMO were considered but were updated according to the recorded history of the evolution of the system configuration. Simulated hydrological scenario corresponds to the period from Jan-18 to Dec-2022.

Initially, it is important to verify whether the outflow values of the Xingo hydro plant were limited according to the safety curve. In this check, the limits of the curve were calculated externally along the horizon and compared with the values reached by the optimal dispatch model.

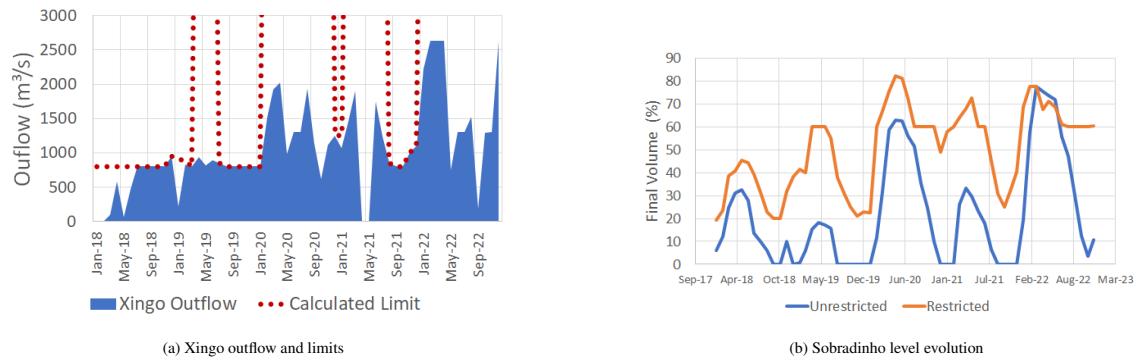


Figura 2: Results for Xingo outflow and Sobradinho reservoir level

As shown in figure (2a), the Xingo outflow respects the limits established in the safety curve, confirming that the proposed methodology worked properly. At this point, it is important to recover the objectives of this safety curve, basically, to guarantee a parsimonious use of the water resources of the São Francisco river basin, allowing that, even in scenarios of low inflows, the reservoirs do not reach such critical levels. In this way, it is important to contrast the levels of the Sobradinho reservoir before and after the implementation of the safety curve, shown in figure (2b).

As can be seen from the curves, the initial objective was achieved by incorporating the safety curve in the SDDP model. The Sobradinho reservoir did not reach levels below 20% of storage, even considering very adverse events recorded in recent history.

4. Conclusions

This paper presents a methodology for incorporating linear constraints by interpolation in simulations of electrical power systems using the SDDP stochastic optimization algorithm.

The methodology employed was applied to the context of the Brazilian electrical system (SIN) in which these constrained have been introduced, especially for operating constraints conditioning variables among themselves. The presented case study applied the methodology to represent the safety curves of the reservoirs of the São Francisco River, whose main objective is to guarantee a more parsimonious use of the main reservoir of the basin (Sobradinho), associating the minimum outflows in the cascade to the levels of this reservoir.

In the case study, databases from the National Electric System Operator (ONS) were used and the methodology was able to correctly represent these constraints and the objectives of the most parsimonious use of the reservoir were achieved. In future works, it is expected to apply the same methodology to other SIN basins and study strategies for applying the methodology to problems with hourly granularity, in which the number of solved problems increases substantially.

Referências

ANA (2017). Resolution n. 2081/2017.

Beale, E. (1963). Two transportation problems. In *Proceedings of the Third International Conference on Operational Research*, p. 780–788, London. Dunod, Paris and English Universities Press.

Pereira, M. V. e Pinto, L. M. (1991). Multi-stage stochastic optimization applied to energy planning. *Mathematical programming*, 52:359–375.