

Vehicle routing under uncertainty via robust optimization

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RESUMO

A dissertação resumida neste artigo aborda o problema de roteamento de veículos com janelas de tempo (PRVJT) sob incertezas na demanda e no tempo de viagem, considerando o paradigma de otimização robusta. Inicialmente, propôs-se um modelo de otimização compacto baseado em restrições de fluxo de commodities para o PRVJT determinístico. Em seguida, estendeu-se o modelo proposto para considerar incertezas nos parâmetros utilizando técnicas de modelagem recentes. Notavelmente, nosso trabalho foi o primeiro a modelar incertezas no tempo de viagem considerando um conjunto de incertezas diferente daquele baseado em cardinalidade restrita, nomeadamente o conjunto de incertezas do tipo mochila. Por fim, os métodos desenvolvidos foram adaptados e aplicados para um caso real de roteamento de aeronaves, considerando diversos aspectos práticos como regras relacionadas ao descanso da tripulação e manutenção de aeronaves.

PALAVRAS CHAVE. Problema de roteamento de veículos, Otimização robusta, Roteamento de aeronaves.

PD - Prêmio de dissertação, L&T - Logística e Transportes

ABSTRACT

The dissertation summarized in this report addresses the vehicle routing problem with time windows (VRPTW) under demand and travel time uncertainties considering the robust optimization paradigm. Initially, we proposed a compact optimization model based on commodity flow constraints for the deterministic VRPTW. Then, we extended the proposed model to consider parameter uncertainties using recently introduced modeling techniques. Notably, our work is the first to model travel time uncertainty considering an uncertainty set different from the cardinality constrained set, namely the knapsack uncertainty set. Finally, the developed methods were adapted and applied to a real case of aircraft routing, considering several practical aspects such as rules related to crew rest and aircraft maintenance.

KEYWORDS. Vehicle Routing Problem, Robust Optimization, Aircraft Routing.

PD - Dissertation Awards, L&T - Logistics and Transportation

1. Introduction

In real-world logistics systems, uncertainties in the input data such as demand and travel time are common challenges. However, most works addressing the vehicle routing problem with time windows (VRPTW) in the literature assume that these parameters are deterministic [Gendreau et al., 2016]. Notably, solutions that ignore the inherent stochastic nature of the parameters are often sensitive to variations, requiring large adjustments once the parameters become known [Oyola et al., 2018]. Over time, various methods have been developed to address the challenge of incorporating uncertainties into solution algorithms, such as the stochastic programming with recourse, stochastic programming with chance-constraints, and robust optimization (RO) [Ordonez, 2010]. Among these, RO offers a key advantage, as it does not require precise knowledge of the underlying distribution of the uncertain parameters, which can be challenging to acquire in practical scenarios. Instead, RO only requires an estimation of the worst-case realization for each parameter, such as the maximum travel time between two nodes.

To control the robustness of the provided solutions, RO relies on *uncertainty sets* to model the uncertain parameter realizations. A solution is only considered robust feasible if it is feasible for any realization within that uncertainty set. Different uncertainty sets often result in solutions with different costs and robustness levels [Subramanyam et al., 2020], a feature that may be explored by decision makers, as they may analyse and select the one most appropriate to their strategy. Despite this advantage, the use of different uncertainty sets for modeling travel time variability has been little explored in the VRP literature. While problems under demand uncertainty were studied with several uncertainty sets [Gounaris et al., 2013; Subramanyam et al., 2020; Wang et al., 2021], works in the robust VRPTW (RVRPTW) considered only the cardinality-constrained uncertainty set [Lee et al., 2012; Agra et al., 2013; Munari et al., 2019], based on limiting the number of parameters attaining their worst-case value.

The candidate's dissertation addresses this gap in the literature by proposing new compact formulations that model demand and travel times considering the knapsack uncertainty set, where the decision maker limits the total deviation of the uncertain parameters. In addition to the proposed compact formulations, we also developed a tailored branch-and-cut (BC) algorithm that uses dynamic programming equations to separate the solutions in polynomial time considering the cardinality-constrained and knapsack uncertainty sets. Another contribution of the dissertation is the proposal of a novel formulation based on commodity flow constraints for the deterministic VRPTW, which was later extended to incorporate uncertainty via RO, as this type of constraints yields a stronger linear relaxation than the Miller-Tucker-Zemlin (MTZ) constraints used in the state-of-the-art model of Munari et al. [2019]. This first part of the candidate's dissertation focuses on the traditional VRPTW under uncertainty, and the computational experiments were based on instances derived from the existing literature.

The second part of the dissertation extends the proposed models and methods to a real-world system. More specifically, we conducted a case study for an on-demand air transportation company [Yao et al., 2008] and designed solution methods for the robust aircraft routing problem (RARP). In particular, most papers related to this sector assume that flights can depart and arrive as scheduled [Marla et al., 2018], even though this industry is well known for dealing with time-related variability due to weather conditions, mechanical problems, and traffic at airports, which indicates an interesting contribution of our work. We proposed a BC algorithm that integrates the routing and crew assignment decision process, which is known to provide a better solution in comparison to performing them sequentially. This algorithm starts with an optimization model containing only the routing aspects of the problem, and dynamically introduces cuts related to crew assignment rules

as they are needed. The BC algorithm was originally designed for the deterministic case and then extended to consider deviations in travel times, since even small deviations can make a schedule infeasible in practice [Marla et al., 2018], either because of crew-related rules or excessive delays in departure time.

The remainder of this paper is structured as follows. Section 2 outlines the candidate's journey during the period. Section 3 highlights the main contributions of the Master's dissertation. Finally, Section 4 lists the author's publications related to the dissertation.

2. On the thesis development

The candidate began their Master's program in March 2020. During this period, they successfully completed all the mandatory courses of their Master's program, obtaining the highest grades in all of them. Additionally, they took the following courses: "Advanced Column Generation and Cutting Algorithms", taught by Prof. Dr. Eduardo Uchoa (UFF), "Design and Analysis of Algorithms", taught by Prof. Dr. Diego Furtado Silva (UFSCar), and the summer school "Combinatorial Optimization at Work", organized by Technische Universität Berlin, Germany, achieving high grades in all of them.

The candidate participated and presented in several national and international conferences, namely SBPO 2020 (Online), AIRO 2021 (Naples - Italy, online) - AIRO PhD School & 5th AIRO Young Workshop 2021, EURO 2021 (Athens - Greece, online) - European Conference on Operational Research, ICCL 2021 (Enschede - Netherlands, online) - International Conference on Computational Logistics, and the CNMAC 2021 (online) - Congresso Nacional de Matemática Aplicada e Computacional. At these events, they had the opportunity to share their findings and network with researchers from all over the world.

Additionally, in 2022, the candidate developed two research internships abroad under the supervision of two renowned professors. The first was at the Université Laval (Québec, Canada) under the supervision of Prof. Leandro Coelho. In this internship, the candidate focused on developing solution algorithms for the robust VRPTW, which resulted in the paper "Compact Formulations for the Robust Vehicle Routing Problem with Time Windows under Demand and Travel Time Uncertainty" submitted to the *Computers & Operations Research* (Impact factor 5.159). The second was at the University of Edinburgh (Scotland, United Kingdom) under the guidance of Prof. Douglas Alem, where they focused on developing solution methods for the lot-sizing problem under demand uncertainty via robust optimization for a real veterinary pharmaceutical company. The resulting paper, "Fragility-based lot-sizing in veterinary pharmaceutical plants under demand uncertainty", was submitted to the *International Journal of Production Research* (Impact factor 9.018). It is worth noting that the candidate is the first author on both papers, attesting to their significant involvement in the projects.

The master's defense took place on the 2nd of September, 2022. The examining committee was formed by Prof. Dr. Douglas Alem (University of Edinburgh, Scotland), Prof. Dr. Bruno Petrato Bruck (UFPB, Brazil), Prof. Dr. Walton Pereira Coutinho (UFPE, Brazil), Prof. Dr. Reinaldo Morabito (UFSCar, Brazil) and the supervisor. The committee unanimously approved the candidate and commended the quality of the work.

3. Contributions

The candidate's dissertation is structured in five chapters. The first two chapters contain the introduction and literature review, respectively. The main contributions of the dissertation, which are summarized in this section, are described in Chapter 3, where we presented the models and solution methods developed for the RVRPTW, and in Chapter 4, which focuses on the RARP.

3.1. Part I - Solution methods for the VRPTW

In the first part of the dissertation, we studied the traditional RVRPTW. Thus far, the most effective compact model for the RVRPTW was based on MTZ constraints [Munari et al., 2019]. Hence, we proposed a novel compact formulation for the deterministic VRPTW based on commodity-flow constraints, which are known for having stronger linear relaxations in comparison to models relying on MTZ constraints [Letchford e Salazar-González, 2015], and then extended them to incorporate uncertainty following via RO. It is worth noting that, to the best of our knowledge, no other work in literature models the deterministic VRPTW with commodity flow constraints, let alone using them in a RO formulation. Hereafter, we consider a graph $G = (N, A)$, where $N = \{0, n + 1\} \cup N^*$ is the set of nodes such that N^* represent the customers and $\{0, n + 1\}$ represent the depot. Set A is the set of arcs in the graph.

The proposed deterministic model uses a new decision variable g_{ij} , which represents the time when a vehicle completes serving customer i and becomes available to travel to customer j if the arc (i, j) is traversed. Let $\bar{t}_{ij}$, s_i , a_i , and b_i be parameters representing, respectively, the nominal travel time on arc (i, j) , the service duration, and the opening and closing time windows of node i ; and let x_{ij} be a binary decision variable that assumes the value of 1 if and only if arc (i, j) is traversed. The deterministic commodity flow constraints for time windows can be defined as:

$$\sum_{j=1}^{n+1} g_{ij} \geq s_i + \sum_{h=0}^n (g_{hi} + t_{hi}x_{hi}), \quad i \in N^*, \quad (1)$$

$$(a_i + s_i)x_{ij} \leq g_{ij} \leq (b_i + s_i)x_{ij}, \quad (i, j) \in A. \quad (2)$$

Constraints (1) compute the time flow through the visited nodes, and constraints (2) enforce the time windows. Similar constraints were defined to control vehicle load and capacity.

To extend the previous model to incorporate demand and travel time uncertainty, we are the first to use a recent modeling technique based on the linearization of dynamic programming equations [Munari et al., 2019], originally developed for the cardinality-constrained uncertainty set. Additionally, we adapted this technique to consider the knapsack uncertainty set and used it to obtain new formulations based on the commodity flow and MTZ constraints. Notably, our work is also the first to explicitly incorporate the knapsack uncertainty set into a compact formulation for a problem with travel time uncertainty. In this technique, we introduce an additional index δ that stores the accumulated deviation in a route from the load variables $(g_{r_i r_j \delta})$. This is based on the computation of the accumulated time after serving customer r_j in route $R = (0, r_1, \dots, r_n)$ considering a budget of deviation δ , that can be performed using the following recursive equation:

$$g_{r_j r_{j+1} \delta} = \begin{cases} a_{r_0}, & \text{if } j = 0, \\ \max\{a_{r_j}, w_{r_{j-1} \delta} + \bar{t}_{r_{j-1} r_j} + s_{r_{j-1}}\}, & \text{if } \delta = 0, \\ \max\{a_{r_j}, w_{r_{j-1} \delta} + \bar{t}_{r_{j-1} r_j} + s_{r_{j-1}}, \\ \quad w_{r_{j-1}(\delta-\lambda)} + \bar{t}_{r_{j-1} r_j} + s_{r_{j-1}} + \lambda\}, & \text{if } \lambda \leq \hat{t}_{r_{j-1} r_j}, \lambda \leq \delta \leq \Delta^t, \end{cases} \quad (3)$$

where Δ^t is the budget of the knapsack uncertainty set. The first line is a boundary condition that sets the start time for the vehicle at the first node in the route. The second line is another boundary condition that calculates the total elapsed time in the deterministic case, considering the opening time windows. The third line determine the maximum elapsed time by evaluating whether it is more advantageous to disregard the deviation in a specific arc or to allow a deviation level (λ) ranging from 1 unit up to the maximum deviation of the arc ($\hat{t}_{r_{j-1} r_j}$ or δ , whichever is larger). It is

then possible to linearize (3) and derive the robust counterpart of constraints (1)-(2) given by:

$$\sum_{j \in N} g_{ij\delta} \geq \sum_{\substack{h \in N \\ \lambda \leq \bar{t}_{hi}}} (g_{hi\delta-\lambda} + (\bar{t}_{hi} + \lambda)x_{hi}) + s_i, \quad i \in N, \delta \leq \Delta^t, \lambda \leq \delta, \quad (4)$$

$$(a_j + s_j)x_{ij} \leq g_{ij\delta} \leq (b_j + s_j)x_{ij}, \quad (i, j) \in A, \delta \leq \Delta^t. \quad (5)$$

Similar expressions were obtained to control vehicle load and capacity, which were further improved as the total load does not have constraints similar to the time windows opening time, thus the deviation can be considered sequentially at the nodes of a route until the knapsack is filled or the vehicle arrives at the end of the route. They were also used to model the problem using MTZ-based formulations. To our knowledge, we are the first to propose these constraints and models.

In addition to running the compact formulations in a general-purpose commercial solver, we also designed a tailored BC algorithm to improve computational performance. The algorithm is based on heuristic separation algorithms designed for the deterministic and robust CVRP [Lysgaard et al., 2004; Gounaris et al., 2013] and exact separation algorithms based on the dynamic programming equations (3). To evaluate the computational performance of the proposed algorithm, computational experiments were performed using benchmark instances from the literature [Munari et al., 2019], using different deviation levels and uncertainty budgets for different uncertainty sets. Table 1 shows the average results of the standalone formulations with MTZ and commodity flow constraints and the tailored BC methods based on them, considering all instances with up to 50% deviation on travel time. For each value of budget, it presents the risk of infeasibility (Risk) determined by a Monte Carlo simulation with 10,000 scenarios with; the price of robustness (PoR), which is the increase of the solution value with respect to the deterministic model, as a percentage; the average quality of the LP relaxation as a percentage (QLR); and the percentage of all instances solved to optimality by each algorithm. In the dissertation, the candidate presents solutions with $\Gamma = 10$ and $\Delta = 60$, but for this set of instances, these budgets provided solutions that are strictly worse than those obtained with $\Gamma = 5$ and $\Delta = 40$, respectively, since they had the same risk level but higher cost.

Tabela 1: Average results of the formulations with the MTZ and commodity flow constraints and the BC methods based on them, for instances with deviation of 50% on travel time.

	Budget	Risk(%)	PoR(%)	QLR(%)	MTZ		Commodity Flow		
					Opt (%)		QLR(%)	Opt(%)	
					Model	BC		Model	BC
Deterministic	-	63.9	0.0	53.5	85.7	87.5	80.7	92.9	92.9
Cardinality- Constrained (Γ)	1	32.1	4.1	53.1	79.7	87.5	76.3	85.7	86.9
	5	0.0	12.2	50.6	73.1	81.5	73.9	60.1	75.0
Knapsack (Δ)	20	32.1	0.9	52.3	64.8	85.1	81.3	35.1	68.4
	40	0.0	3.6	50.6	47.6	79.1	74.6	20.2	57.1

The results show that the robust optimization approach offers attractive solutions that provide a trade-off between solution risk and cost. For example, decision makers can choose a deterministic solution with a risk level of 63.9%. Alternatively, they can choose to implement the solution with a deviation of $\Delta = 20$, which reduces the infeasible risk by nearly half at a negligible cost increase of 0.9%. Another option is to eliminate all risks associated with the solution by paying an additional 2.5% for the solution with $\Delta = 40$. Notably, the knapsack uncertainty set proved to be more suitable for solving these instances, providing strictly better solutions at the same risk level and

lower cost than the cardinality constrained uncertainty set. In terms of computational performance, the commodity flow formulation outperformed the MTZ-based formulations in the deterministic case and for instances with $\Gamma = 1$, as well as provided a numerically stronger linear relaxation, but the overall performance of the MTZ formulation was better. The BC algorithm improved the solver performance for both optimization models, especially for the MTZ formulation, which has faster node exploration, resulting in the best overall results.

3.2. Part II - Case-study for the RARP

In the second part of the dissertation, the candidate conducted a case study of an on-demand air transportation company. In this type of company, the customer chooses the itinerary and the type of aircraft that will serve them. The main challenge is to serve all customers with minimal operating costs, while complying with several regulations related to aircraft maintenance and crew working hours [Khaled et al., 2017]. Due to these regulations, designing efficient aircraft routes for a set of demands can be quite a challenging task.

We addressed this problem by proposing a BC algorithm that starts with the routing- and maintenance-related constraints and dynamically introduces cuts related to crew assignment rules as they are violated. The rules evaluated include maximum work and flight time in a shift, minimum rest periods, and split duty rules, which are rules that allow the company to extend the duration of a shift based on the amount of time the crew remains on the ground. For the separation procedure, we proposed and implemented a labeling algorithm based on extensions that resort to one of six different crew rest strategies: no rest between serving two consecutive calls; wait until the latest possible time to start serving a call; rest before a positioning flight; rest after a positioning flight; rest before and after a rest period; and let the crew rest during two consecutive maintenance periods. A label is generated only if it is feasible in terms of crew rules and customer time windows. To control the growth of generated labels, we use a dominance rule based on total elapsed time and accumulated duty time to quickly eliminate unpromising labels.

The proposed algorithm was then extended to consider travel time uncertainty via RO, for both the cardinality-constrained and the knapsack uncertainty sets, relying on the dynamic programming equations strategy discussed in Section 3. Considering time uncertainty is essential in the airline industry, as delays are fairly common and may render a solution infeasible due to crew-related regulations or customer time window constraints. Computational experiments were conducted using instances generated with 4 months of the company's data. Each instance contains three consecutive days of demand, which is the planning horizon used by the company. Table 2 summarizes the results obtained in the instances of the second month of data, which contains an average of 40.75 requests and 29.88 planes per instance, considering a maximum deviation level of 25% over nominal travel times. The table shows the average computation time (CPUt), the risk of infeasibility (Risk) as determined by a Monte Carlo simulation, and the price of robustness (PoR) for each budget and uncertainty set. The deterministic case is identified with the budget 0 of each uncertainty set.

Tabela 2: Aggregated results for the RARP instances considering a deviation level of 25%

Cardinality-Constrained (Γ)				Knapsack (Δ)			
Budget	CPUt(s)	Risk(%)	PoR(%)	Budget	CPUt(s)	Risk(%)	PoR(%)
0	0.16	14.36%	-	0	0.16	14.36%	-
1	0.15	7.06%	1.27%	20	0.71	14.36%	0.35%
5	0.27	2.63%	2.21%	40	0.85	7.06%	1.27%
10	0.32	2.63%	2.21%	60	1.36	7.04%	1.57%

Notably, the proposed algorithm performed exceptionally well on the real-world instances, taking less than a second to solve most of them. In addition, using RO we are able to provide valuable solutions showing an interesting trade-off between solution cost and infeasibility risk. The decision maker can choose the deterministic solution and accept an average infeasibility risk of 14.36%, or choose solutions that are only 1.27% more expensive ($\Gamma = 1$, $\Delta = 20$) to halve this risk. Additionally, they can choose a solution that is 2.21% more expensive than the deterministic one to further reduce the risk to only 2.63%. Thus, the RO approach is able to provide several interesting options, allowing the decision maker to make an informed decision that better suits their strategy.

4. List of Publications

In the following, we present a list of the papers resulting as part of the candidate activities during their Masters:

- **Book chapter 1.** *A Branch-and-Cut Algorithm for Aircraft Routing with Crew Assignment for On-Demand Air transportation* (2021) - published in *Lecture Notes in Computer Science, volume 13004*.
- **Submitted paper 1.** *Compact Formulations for the Robust Vehicle Routing Problem with Time Windows under Demand and Travel Time Uncertainty* - submitted to *Computers & Operations Research* on January 2023.
- **Conference paper 1.** *Otimização robusta para o problema de roteamento de veículos com janelas de tempo: uma nova formulação compacta baseada em fluxo de commodities* (2020) - published in the proceedings of the *LII Simpósio Brasileiro de Pesquisa Operacional*.
- **Conference poster 1.** *Uma formulação para o problema de roteamento de veículos robusto usando o conjunto de incertezas do tipo mochila* (2021) - published in the proceedings of the *XL Congresso Nacional de Matemática Aplicada e Computacional*.
- **Extended abstract 1.** *The robust vehicle routing problem: a novel commodity flow formulation based on the linearization of recursive equations* (2021) - published in the proceedings of the *XXXI European Conference on Operational Research*.
- **Extended abstract 2.** *A novel commodity flow formulation for the robust vehicle routing problem* (2021) - published in the proceedings of the *AIRO PhD School & 5th AIRO Young Workshop 2021*.
- **Working paper 1.** *Robust aircraft routing problem under travel-time uncertainty* - to be submitted to a top-tier international journal in August.
- **Working paper 2.** *Polynomial-time separation algorithms based on closed-form equations for robust vehicle routing under travel time uncertainty* - to be submitted to a top-tier international journal in August.

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