São José dos Campos - SP
LV SBPO
LV Simpósio Brasileiro de Pesquisa Operacional
São José dos Campos, SP - 6 a 9 de novembro de 2023

Optimizing the runway capacity allocation under uncertain capacity constraints

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DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

RESUMO

No gerenciamento do tráfego aéreo (ATM), a incerteza está em todo lugar, j'a que o transporte aéreo está sujeito a diferentes parâmetros intrinsecamente incertos, como demanda, clima e restrições operacionais imprevisíveis. Assim, a intenção deste estudo é lidar com uma dessas principais restrições: a utilização da capacidade aeroportuária. Este problem tem sido abordado ao longo dos anos geralmente utilizando programação linear, otimização *fuzzy* e programação dinâmica. Assim, uma nova abordagem é proposta, combinando diferentes contribuições da literatura e introduzindo um modelo Minimax Regret para considerar a incerteza da capacidade de pista no problema de otimização da utilização da capacidade aeroportuária. Os resultados são comparados com os cenários de melhor e pior caso para avaliar o desempenho do modelo. Isso mostra que o modelo pode fornecer resultados interessantes e eficientes do ponto de vista de redução do arrepedimento em relação a ambos os cenários em termos de atrasos totais.

PALAVRAS CHAVE. Capacidade aeroportuária, Otimização sob incerteza, Minimax Regret.

Transporte Aéreo, Gerenciamento do Tráfego Aéreo (ATM), Métodos de Otimização sob Incerteza.

ABSTRACT

In the air traffic management (ATM), uncertainty is everywhere, since air transport activities are subject to different intrinsically uncertain parameters, like demand, weather and unpredictable operational constraints impacting the airport capacity. So, this study's intent is to deal with one of these major constraints: the airport capacity utilization. It has been treated in different studies over the years, most of them using linear programming, fuzzy optimization and dynamic programming. So, a new approach is proposed, combining different contributions from the literature and introducing a Minimax Regret model to account for the runway capacity uncertainty in the problem of optimizing the airport capacity utilization. The results are compared to the best-case and the worst-case scenarios to evaluate the model performance. It shows that the model can provide interesting and efficient results from the point of view of reducing the regret in relation to both scenarios in terms of total delays.

KEYWORDS. Airport capacity, Optimization under uncertainty, Minimax regret.

Air Transport, Air Traffic Management (ATM), Methods of Optimization under Uncertainty.

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1. Introduction

Since airport and air traffic operations have multiple uncertainty factors with varied and dynamic uncertainty degrees, any solution to an air transport problem that considers it stationary and deterministic is fated to fail sometime, even if the results seem promising for particular cases. Thus, what is considered to be a successful approach or mathematical formulation may not work when it is most necessary, that is, when the conditions are most chaotic.

So, the main objective of this study is to propose a new approach to the problem of optimizing the airport capacity utilization that takes into account the capacity uncertainty associated with the weather condition, evaluating the results in comparison with the worst-case and best-case scenarios. The problem can be stated as follows: *for a set of periods* $t \in 1, 2, ..., T$*, given the runway capacity constraints for arrivals and departures, the demand of arrivals* $\tilde{a}_t$ *and departures* $\tilde{d}_t$ *at* t *and its accumulated queues* a_t *and* d_t*, respectively, how much arrivals* u_t *and departures* v_t *should we admit within every time step* t *to minimize the total delay?*

The first thing to consider in the problem is the trade-off between arrival and departure capacities. Hence, to express the capacity constraints for arrivals U and departures V, there is a curve named Runway Configuration Capacity Envelope (RCCE), usually expressed by a piece-wise linear function $V = f(U)$, made from historical data. This capacity curve was first presented in Gilbo [1993]. In practice, the airports and the Air Navigation Service Provider (ANSP) define the runway declared capacity based on historical data, workload, and resources available as well as airspace or airport constraints.

So, the capacity is supposed to be predetermined. However, it is not stationary, because there is more than one capacity envelope for different weather scenarios or runway configurations, as later observed by Gilbo [1997], and it changes over time. Usually, two different scenarios are considered for a given runway configuration: an instrumental meteorological condition (IMC) and a visual meteorological condition (VMC), where the latter is less restrictive than the former. These curves are represented in Figure 1. Note that these are the declared capacities, that is, the capacity obtained from historical data, which does not mean that this capacity limit is truly achievable at any time. So, the real capacity is, actually, somewhere between these curves.

Another important point to notice is that there are different capacity measures. The main reference for the airport capacity is the runway capacity mentioned before, defined for total arrivals and departures for a given runway configuration. However, there is also a TMA capacity, for the adjacent airspace of an airport, and an airspace or sector capacity, for a given region of the airspace, which can be represented either by movements per hour or simply the number of aircraft at the same time inside an Air Traffic Control (ATC) sector, beyond other airport elements' capacities. Each study can consider a different capacity metric based on the formulation of the problem that is being faced. In Gilbo [1993], the author considered the runway capacity, whereas, in Gilbo [1997] and Teodorovic et al. [2004], the authors analyzed the TMA capacity, separating arrivals and departures through the arrival and departure fixes, which are positions in the adjacent airspace of an airport through which the aircraft must pass to land or depart, respectively. Nonetheless, the most common metric considered in practice is the runway capacity.

On the demand side, it must be noted that the word demand here refers to arrival and departure demand, and not to the passengers' demand for flights. So, there is no uncertainty in the demand considered in the current problem, unlike the demand of passengers for flights, which is completely uncertain and dependent on several parameters and is not included in this problem. When focusing on arrivals and departures, the demand uncertainty is entirely eliminated, because the flight slots are preassigned and the arrivals and departures are known in advance, since they

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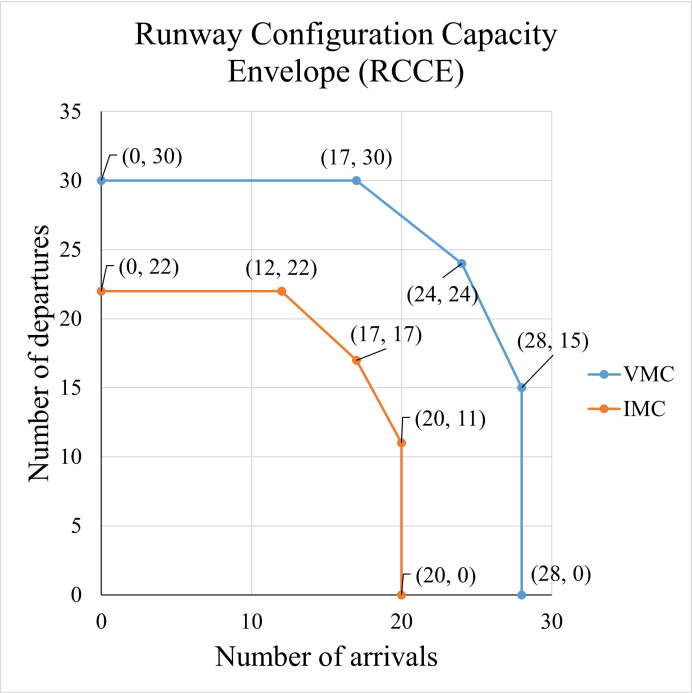


Figure 1: Example of RCCEs for IMC and VMC given a fixed runway configuration [Gilbo, 1997].

enter the queue only when they are confirmed and the flights are closed for changes. Therefore, at this point, the passengers' demand for flights has already been met and does not have any impact on the arrival and departure demand.

There are many studies, from the oldest to the most recent ones, that explore some kind of optimization under uncertainty on the subject [Teodorovic et al., 2004; Jacquillat e Odoni, 2015; Jacquillat et al., 2016; Jacquillat e Odoni, 2018; Shone et al., 2019, 2021]. In Shone et al. [2021], the authors made a wide literature review, highlighting those researches focused on this theme until the moment. The methods for dealing with the problem range from deterministic [Gilbo, 1993, 1997; Dell'Olmo e Lulli, 2003] to stochastic models [Teodorovic et al., 2004; Jacquillat et al., 2016; Shone et al., 2019], and are still under investigation nowadays, indicating that the problem remains of interest. However, even the latest and most advanced methods explored in these studies still struggle to deal with uncertainty and account for every influencing factor over the objective function, because some of them are intangible or hardly measurable, like the cost of switching service rates mentioned by Shone et al. [2019]. Thus, this study intends to propose a way to address these two main concerns: *how to introduce uncertainty in the problem and take other factors into account beyond the delay without the need for a complex objective function.*

2. Literature review

In Gilbo [1993], one can see how a complete deterministic problem is formulated, considering the entire knowledge of demand and capacity constraints at each time step in advance. However, the reality is not like this. So, further efforts were done to improve the problem's approach and solution. There were later attempts to introduce uncertainty into the problem and formulate it as a MDP. The first one regarded to try to formulate the problem as a MDP was Dell'Olmo e Lulli [2003]. Then Jacquillat e Odoni [2015] and Jacquillat et al. [2016] later proposed the formulation of the problem as a discrete-time finite MDP under stochastic queue conditions, using Markov chains, and applied Dynamic Programming to solve it. In Shone et al. [2019], improvements were proposed

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

to that approach. The troubles around uncertainty resulted also in the proposal of the Ground Delay Programs by Hall [1999] and Gilbo e Howard [2000], in order to reduce arrival queues by holding the aircraft in the origin instead of dealing with it in the destination and alleviating the arrivals.

As mentioned earlier, the first to examine the problem was Gilbo [1993]. In this study, the authors proposed the following deterministic problem formulation for optimizing the runway capacity allocation, looking for minimizing the total queue composed of arrivals and departures, with some weights of priority for each one.

$$\min_{u^*} \sum_{i=1}^{N} \gamma_i \left[\alpha_i X_{i+1} + (1-\alpha_i) Y_{i+1}\right], \alpha_i \in [0;1]$$

$$\text{Subject to } \begin{cases} X_i \geq 0, & \forall i \in I \\ Y_i \geq 0 & \forall i \in I \\ 0 \leq v_i \leq D_i, & \forall i \in I \\ v_i + g_{ji} u_i \leq b_{ji}, & \forall i \in I; j = 1, ..., n_i \\ 0 \leq u_i \leq B_i, & \forall i \in I \end{cases}$$

with

$$X_{i+1} = X_i + a_i - u_i, \forall i \in I$$

$$Y_{i+1} = Y_i + d_i - v_i, \forall i \in I$$

where X_i is the arrival queue, Y_i is the departure queue, a_i is the arrival demand, d_i is the departure demand, u_i is the arrival capacity allocation, v_i is the departure capacity allocation, and D_i, g_{ji}, b_{ji} and B_i are the parameters of the capacity envelope, all at the i-th time interval or slot. The problem is nonlinear, but it can be easily linearized from the correlation between the queues and the capacity allocations, leading to the following formulation:

$$\max_{u^*} \sum_{i=1}^{N} \sum_{p=i}^{N} \gamma_p \left[\alpha_p u_i + (1-\alpha_p) v_i\right], \alpha_p \in [0;1]$$

$$\text{Subject to } \begin{cases} \sum_{p=1}^{i} u_p \leq X_1 + \sum_{p=1}^{i} a_p, & \forall i \in I \\ \sum_{p=1}^{i} v_p \leq Y_1 + \sum_{p=1}^{i} d_p, & \forall i \in I \\ 0 \leq v_i \leq D_i, & \forall i \in I \\ v_i + g_{ji} u_i \leq b_{ji}, & \forall i \in I; j = 1, ..., n_i \\ 0 \leq u_i \leq B_i, & \forall i \in I \end{cases}$$

In spite of the contribution of this method, which is still in use nowadays, the problem formulation and its solution are limited due to the deterministic approach. In Gilbo [1997], an attempt was made to improve the model by focusing on the TMA capacity, instead of the runway capacity, and distributing arrivals and departures through the fixes in order to minimize the total delay. However, although he had recognized different capacity envelopes, his model did not account for capacity uncertainty and possible unpredictable transitions between IMC and VMC. So, a fuzzy approach was proposed by Teodorovic et al. [2004] to represent the capacity uncertainty. Nonetheless, the authors did not associate the capacity curve with the weather condition and they relied his model on parameters given by experience. Moreover, the TMA capacity is not used in practice very often, since the number of fixes is dynamic and the runway capacity is usually more restrictive.

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

In Dell'Olmo e Lulli [2003], a deterministic dynamic approach with a MDP formulation was proposed. In this study, the states are given by the accumulated delayed demand $z_t = (a_t, d_t)$, such that (adapted):

$$z_{t+1} = z_t + \left(\tilde{a}_{t+1}, \tilde{d}_{t+1}\right) - (u_a(t+1), u_d(t+1))$$

where $z_1 = (\tilde{a}_1, \tilde{d}_1)$, $\left(\tilde{a}_t, \tilde{d}_t\right)$ is the immediate demand at t, $u(t) = (u_a(t), u_d(t))$ is the capacity allocation or served demand, that is, the action taken at t, and z_t represents all (a_t, d_t) with $a_t \in \left[0; \sum_{k=1}^{t} \tilde{a_k}\right]$ and $d_t \in \left[0; \sum_{k=1}^{t} \tilde{d_k}\right]$.

Differently from Gilbo [1993], the authors have taken into account the fairness principle of first-in-first-out (FIFO) and expressed the objective function in terms of delays and not only queues. Thus, the objective function is the cumulative delay resulting from the sum of the marginal delay increments $\Delta_t(a_t)$ and $\Delta_t(d_t)$, for all t, weighted by a fairness cost coefficient. It is referred to by the authors as a cost function to be minimized:

$$\sum_{t \in \mathcal{T}} \left[\Delta_t\left(a_t\right) + \Delta_t\left(d_t\right)\right]$$

So, the immediate cost function for the MDP to be minimized in the long run can be described as follows:

$$r(z_t, u_t) = \Delta_t\left(a_t\right) + \Delta_t\left(d_t\right)$$

This dynamic programming approach eliminates the need for a piece-wise linear convex capacity envelope. Considering that the queue at $t+1$ is $z_{t+1} = z_t + (\tilde{a}_{t+1}, \tilde{d}_{t+1}) - u(t)$, then the served demand $u(t) = z_t + (\tilde{a}_{t+1}, \tilde{d}_{t+1}) - z_{t+1} \geq 0$ and it must be constrained by the capacity envelope. Thus, with a linear capacity envelope, the problem can also be formulated as a linear programming optimization problem as follows (adapted):

$$\min_{u^*} \sum_{t \in \mathcal{T}} \left[\Delta\left(a_t\right) + \Delta\left(d_t\right)\right]$$

$$\text{Subject to } \begin{cases} \alpha_{i,t}\left(d_{t-1} + \tilde{d}_t - d_t\right) + \beta_{i,t}\left(a_{t-1} + \tilde{a}_t - a_t\right) \leq \gamma_{i,t}, & \forall t \in \mathcal{T}, \forall i \in \mathcal{I}_t \\ d_{t+1} \leq d_t + \tilde{d}_{t+1}, & \forall t \in \mathcal{T} \\ a_{t+1} \leq a_t + \tilde{a}_{t+1}, & \forall t \in \mathcal{T} \\ d_t, a_t \in \mathbb{N}, & \forall t \in \mathcal{T} \end{cases}$$

Nevertheless, despite the MDP representation, the new interpretation of the objective function, and the possibility of nonlinear capacity envelopes, it is still a deterministic formulation of the problem. So, Jacquillat et al. [2016] tried to improve this model, using also a MDP, but associating it with Markov chains and introducing stochastic factors in the environment states. In this work, a simplified quadratic cost function to represent the total delay is considered:

$$\alpha \sum_{t=1}^{T} a_t^2 + \sum_{t=1}^{T} d_t^2$$

with $\alpha \geq 1$, and the weather condition wc_t as well as the wind dynamics ws_t and its associated runway configuration RC_t are modeled with Markov chains.

DOI: 10.59254/sbpo-2023-174932
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This study is considered a benchmark on the subject. The authors basically proposed the representation of the MDP state as a 5-tuple $(a_i, d_i, RC_i, wc_i, ws_i)$, where, at a certain time step i, a_i is the arrival queue, d_i is the departure queue, RC_i is the runway configuration in use, $wc_i \in \{VMC, IMC\}$ is the weather condition and ws_i is the wind state, which affects the set of possible runway configurations. The stochastic behavior of the weather condition and the wind state are introduced by using Markov chains. In Jacquillat e Odoni [2018], an extended analysis of the runway configuration problem was made. In addition, Shone et al. [2019] also presented possible improvements to the model of Jacquillat et al. [2016], and Shone et al. [2021] also made a wide-range literature review on solving air traffic management problems under uncertainty.

In Shone et al. [2019], the authors introduced a new term in the objective function of Jacquillat et al. [2016] and proposed the expression below:

$$\sum_{t=1}^{T} \left(\alpha a_t^2 + d_t^2\right) + \sum_{t=2}^{T} \left[k\Delta t \left(u_t - u_{t-1}\right)^2\right]$$

The term $\sum_{t=2}^{T} \left[k\Delta t \left(u_t - u_{t-1}\right)^2\right]$ accounts for the cost of varying the service rates, given that there is a cost associated with changing the number of resources in use, such as air traffic controllers in position, or keeping them while they are not necessary. So, the optimal or most desirable solution, actually, is not simply the one that produces the minimal delay, but the one that does it with the least variation of resources needed to handle the service rates.

However, even though these approaches took uncertainty into account with a transition probability between the VMC and the IMC in the Markov chain formulation, the transition probability between states or scenarios or the behavior of the queues are unknown and can be only estimated, since the weather prediction is very limited and so is the probability of occurrence of some weather scenario, which is itself a random variable. Thus, any stochastic optimization model that tries to formulate the problem introducing some probability measure or distribution for the transition between states or scenarios or for the occurrence of different scenarios will also fall in limited results, because it is trying to predict what is unpredictable by nature using uncertain probability estimates. It is also difficult, not to say infeasible, to model the queues' behavior reliably.

Therefore, the proposed approach is to assume that one cannot predict what is going to happen, that it is completely random, and to find the solution that will result in the least regret in both scenarios, that is, the solution that, even though may not be optimal for the observed conditions, is good enough, efficient and flexible to be adjusted when the conditions in place don't match those that were considered in the model. It corresponds to the Minimax Regret method [Aissi et al., 2007; Thipwiwatpotjana e Lodwick, 2013], which someway relaxes the constraints and allows solutions inside the feasible region that may be better than the optimal if the constraints are not deterministic or if the scenarios that actually take place are not those that were considered in the model. Beyond considering dynamic and uncertain weather conditions, this approach also leads to a less demanding solution and with a lower cost of switching the capacity allocation, because the service rates are not in extreme positions in the varying feasible region, achieving the goal defined by Shone et al. [2019] with a simpler objective function and fewer assumptions about the environment's behavior.

3. Modeling procedures

The Minimax Regret method seeks to minimize the cost incurred when the hypotheses considered when making a decision are proven to be wrong. In the problem studied here, there are basically two possible scenarios for each time period: VMC or IMC. So, according to the Minimax

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

Regret solution, for each time period, one is going to choose the scenario that will lead to the least error if it proves to be wrong.

For instance, if one chooses to consider that the scenario is going to be VMC and actually it happens to be IMC, then the capacity constraints considered to obtain the optimal solution are less restrictive than the real available capacity. In this case, greater queues and delays than predicted are going to be generated, producing real costs to the air transport stakeholders. On the other hand, if one chooses to consider that it is going to be IMC and, in fact, it occurs a VMC, then the arrivals and departures were unnecessarily delayed, implying a cost of opportunity, because unnecessary delays and queues were generated as well as resources were unnecessarily employed. Both cases are financially and operationally harmful to the airlines and the airport administration. So, given that one cannot predict which scenario is going to happen and that the costs of being wrong are undesirable, the Minimax Regret approach seems to be reasonable and justifiable.

Thus, the idea of this study is to combine some of the contributions described before and to propose a new one, using the Minimax Regret method to introduce the capacity uncertainty, in order to relax the constraints and search for a solution with fewer variations in an unstable and uncertain situation, which can be found inside the region between the capacity curves and not over one of them. For the objective function, the one proposed by Jacquillat et al. [2016] is a good representation of the queues' impact on air traffic, because it associates, in a simple but effective way, queues with delays, what really matters in practice. Using the Minimax Regret method is a way of introducing the cost of switching the service rates mentioned by Shone et al. [2019] in the function of Jacquillat et al. [2016] without trying to express it explicitly. Thus, the evaluation function to be considered in this investigation is the following:

$$z(x) = \sum_{t=1}^{T} \left[\gamma_t \left(\alpha_t a_t^2 + d_t^2\right)\right]$$

with

$$x = \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \\ \dots & \dots \\ u_T & v_T \end{pmatrix}$$

$$a_t = a_{t-1} + \tilde{a}_t - u_t$$

$$d_t = d_{t-1} + \tilde{d}_t - v_t$$

where, for each time period $t \in \{1, 2, ..., T\}$, α_t is the weight assigned for the arrival priority, γ_t is the weight assigned for each time step t, a_t and d_t are, respectively, the arrival and departure queues at the end of t, $\tilde{a}_t$ and $\tilde{d}_t$ are the arrival and departure demand during t, and u_t and v_t are the decision variables corresponding to the arrival and departure capacity allocation at t.

Look that it is not being called an objective function, because the Minimax Regret method has a more complex objective function, composed of a variation of this function. By this method, for each scenario ξ_k, $k = 1, 2, ..., n$, the first step is to calculate its isolated optimal solution $z\left(x^*(\xi_k)\right)$, given by:

$$z\left(x^*(\xi_k)\right) = \min_x z(x)$$

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

subject to

$$\begin{cases} v_t \leq f_t^k(u_t) & t = 1, 2, ..., T \quad (i) \\ u_t \leq a_{t-1} + \tilde{a}_t & t = 1, 2, ..., T \quad (ii) \\ v_t \leq d_{t-1} + \tilde{d}_t & t = 1, 2, ..., T \quad (iii) \\ u_t, v_t \geq 0 & t = 1, 2, ..., T \quad (iv) \\ u_t, v_t \text{ integers} & t = 1, 2, ..., T \quad (v) \end{cases}$$

where f_t^k in the constraint (i) is the capacity envelope at each time step t in the scenario ξ_k, (ii) and (iii) ensures that $a_t, d_t \geq 0$, (iv) is a non-negativity condition and (v) is the integer condition.

Note that, for each time period t, there are two possible capacity envelopes or meteorological conditions, IMC and VMC. So, there are not only two scenarios in the problem, corresponding to IMC and VMC, but two possible scenarios for each time step $t = 1, 2, ..., T$, which means that there are $n = 2^T$ scenarios. It is a computational limitation of this method, as a combinatory optimization problem Aissi et al. [2007], because one single problem is turned into 2^T non-linear optimization problems only in the first step of the Minimax Regret method, without speaking of the following procedures.

Before going on into the Minimax Regret formulation of the problem, three different approaches for this first step are analyzed: (a) incremental and real-time solution, (b) simplified time-block solution, and (c) full solution. Each one of them has its benefits and drawbacks, as follows.

In the incremental and real-time solution (a), as one can see, the problem is recursively defined, that is, it can be structured as a Markov Decision Process, following the Markov property, which means that each state, corresponding to the tuple (a_t, d_t, wc_t), where $wc_t \in \{IMC, VMC\}$ is the weather condition at t, depends only on the previous state $(a_{t-1}, d_{t-1}, wc_{t-1})$ and the interaction of the agent and its actions (u_t, v_t) with the environment. It reduces the problem to a single time step, that is, to $T = 1$, considering that the previous state is already defined. So, in this case, there are only two scenarios in total, IMC and VMC, and two decision variables u_1 and v_1. It is a feasible and strong approach, but it operates in real time and requires to be constantly running, being fed, and calculating the decision variables, what represents an operational limitation of this approach.

In the simplified time-block solution (b), a dimensionality reduction is done considering that the weather condition does not change suddenly or many times in a small time frame or even in a day. In other words, one can say that, for example, in a three-hour period, if the weather changes from VMC to IMC, it is not going to change back to VMC during this time or it will do it only once, because the weather changes are not abrupt or frequent. In this case, only two scenarios, VMC or IMC, are considered for a given time block. It is a reasonable assumption and reduces significantly the dimensionality of the problem. However, it can be wrong, because weather in fact can change more rapidly than considered and the time blocks must be small enough to guarantee that it is not going to happen or that it is sufficiently unlike to happen.

In spite of that, although it may not provide the most *efficient* solution in terms of its ability to really minimize the regret, it can provide, overall, a solution efficient enough to outperform deterministic methods. Moreover, the time block can be reduced to only one time step $T = 1$, incurring in the incremental real-time solution. Therefore, this approach is more general than the previous, with a lower computational cost compared to the full solution (c) and it is able to provide efficient-enough solutions that consider the uncertainty and introduce some balance in the possibility of being wrong, that is, that reduces the regret.

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

Finally, the full solution (c) corresponds to the full implementation of the problem, with all of the possible scenarios, without any other simplifying assumption or operational effort to reduce the problem dimensionality. It is obviously the best option in terms of efficiency of its results, but it can be significantly less efficient regarding its computational complexity, being extremely time-consuming or even operationally unfeasible due to the curse of dimensionality produced by the exponential growth of scenarios over time. For each time period t introduced in the model, the number of scenarios doubles. So, the limitation of this approach is basically its computational cost, though it could provide the most efficient solution. Therefore, this approach is discarded, because it does not bring a balance between its time cost and solution efficiency.

Proceeding to the next and final step of the Minimax Regret method, adopting the approach (b) for the first, given the optimal solutions $z(x^*(\xi_{VMC}))$ and $z(x^*(\xi_{IMC}))$ of VMC and IMC, respectively, for the time frame, one must solve the following optimization problem:

$$\min R(x) = \max\left\{\left|z(x) - z\left(x^*(\xi_{IMC})\right)\right|, \left|z(x) - z\left(x^*(\xi_{VMC})\right)\right|\right\}$$

subject to

$$\begin{cases} v_t \leq f_t^k(u_t) & t = 1, 2, ..., T \quad (i) \\ u_t \leq a_{t-1} + \tilde{a}_t & t = 1, 2, ..., T \quad (ii) \\ v_t \leq d_{t-1} + \tilde{d}_t & t = 1, 2, ..., T \quad (iii) \\ u_t, v_t \geq 0 & t = 1, 2, ..., T \quad (iv) \\ u_t, v_t \text{ integers} & t = 1, 2, ..., T \quad (v) \end{cases}$$

where $R(x)$ is the regret function and k, in the capacity envelope f_t^k, is IMC when $R(x) = |z(x) - z\left(x^*(\xi_{IMC})\right)|$ and VMC otherwise, that is, when $R(x) = |z(x) - z\left(x^*(\xi_{VMC})\right)|$.

One can observe, thus, that the constraints are dynamic. For each solution x tested, a different scenario, IMC or VMC is considered, corresponding to the one that leads to the greatest regret. Therefore, the final solution of the Minimax Regret method will correspond to a specific weather condition. However, the solution x^e of service rates is not optimal for this scenario k, which is given by $x^*(\xi_k)$, but the most regret-reducing given the possible scenarios taken into account. This solution is then called efficient, not to be confused with the optimal solution.

4. Results and discussions

For the implementation of the methodology described previously, the data provided by Gilbo [1997] is used. For the purpose of this study, only the arrival and departure total demand are of interest, and not each fix demand, resulting in the demand information in Table 4. For the analyses, the factors $\gamma_t = 1$ and $\alpha_t = 2$, for all $t = 1, 2, ..., T$, were arbitrarily considered, because these are values commonly used. The RCCEs considered in the study were also those provided by Gilbo [1997] and presented in Figure 1.

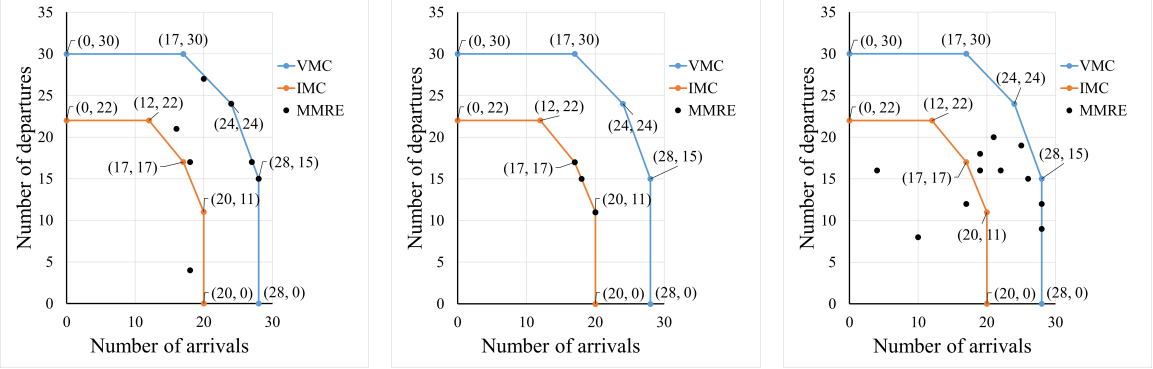
As mentioned in Section 3, the first step of the Minimax Regret method is to solve the optimization problem for each one of the scenarios, obtaining the optimal solutions $z(x^*(\xi_{VMC}))$ and $z(x^*(\xi_{IMC}))$, adopting the approach (b) of the time-block solution, with two scenarios, VMC or IMC, respectively, for the entire set of time periods. Doing so, the results shown in Figures 2a and 2b, for VMC and IMC, respectively, are obtained, with $z(x^*(\xi_{VMC})) = 7037$ and $z(x^*(\xi_{IMC})) = 86953$.

Running then the Minimax Regret final step with dynamic constraints, the results of Table 1 are obtained, corresponding to the Minimax Regret Efficient (MMRE) solution. It is obtained

DOI: 10.59254/sbpo-2023-174932
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t	Time	Demand		Capacity (int)		Queue (≥ 0)	
		Arrival ($\tilde{a}_t$)	Departure ($\tilde{d}_t$)	Arrival (u_t)	Departure (v_t)	Arrival (a_t)	Departure (d_t)
0	16:30-16:45	-	-	-	-	0	0
1	16:45-17:00	26	36	26	15	0	21
2	17:00-17:15	38	32	28	9	10	44
3	17:15-17:30	42	9	28	12	24	41
4	17:30-17:45	29	15	4	16	49	40
5	17:45-18:00	6	7	19	16	36	31
6	18:00-18:15	13	10	19	16	30	25
7	18:15-18:30	14	17	19	18	25	24
8	18:30-18:45	20	33	21	20	24	37
9	18:45-19:00	40	34	25	19	39	52
10	19:00-19:15	25	22	22	16	42	58
11	19:15-19:30	13	13	17	12	38	59
12	19:30-19:45	12	1	10	8	40	52
	Total	278	229	238	177	357	484

Table 1: Efficient solution for the Minimax Regret method.



(a) Optimistic solution, corresponding to the optimal solution of VMC.

(b) Pessimistic/robust solution, corresponding to the optimal solution of IMC.

(c) Balanced and least-regret solution, corresponding to the Minimax Regret Efficient (MMRE) solution.

Figure 2: Optimal solution for VMC (a), for IMC (b) and MMRE (c).

for VMC, resulting in a total delay of $R(x^e) = 47048$, almost the average between the optimal solutions for each isolated scenario.

To evaluate this solution, it is necessary to compare it with the optimal solutions for each individual scenario. So, there are the following solutions: (1) VMC optimal solution (optimistic solution), (2) IMC optimal solution (robust or pessimistic solution), and (3) MMRE solution (least-regret solution). It is interesting to observe how the solution points are spread in a scatter graph to evaluate how it would perform if the constraints were relaxed and not two fixed deterministic curves, like if it was a curve between the VMC and IMC curves, as well as if we had switching curves. In Figures 2a, 2b and 2c there are the solutions (1), (2), and (3), respectively.

One can see that, as expected, the optimal solutions for each individual problem have their decision variable points placed over the capacity curve, as the Simplex Method does since it considers that the optimal solutions are on the vertices of the convex feasible region, over its boundaries. However, it is extremely dependent on the constraint. Slightly changing the feasible region, by changing the capacity curve, imply in turning the optimal solution into an unfeasible or bad solution and requiring running the model again. It is even worse, considering that the constraints are not really known or that they are constantly changing.

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On the other hand, looking at the MMRE solution, one can notice that the decision variable points are significantly more spread through the feasible region between the capacity curves. Thus, this solution balances the risk of taking the optimistic solution and leading to an unfeasible solution in practice or in taking the pessimistic solution and incurring a huge cost of opportunity at the same time that it produces low costs of switching the capacity allocation over time, as the solutions points are close to both capacity envelopes.

5. Conclusions and further research

This study explored the application of the Minimax Regret method of Thipwiwatpotjana e Lodwick [2013] and Aissi et al. [2007] in the problem of optimizing the airport runway capacity utilization under uncertainty. Different studies and their approaches were analyzed and detailed and some of their contributions were introduced in the model presented here. The problem was first faced by Gilbo [1993] and the data used in the implementation of the proposed model was obtained from Gilbo [1997], which dealt with a similar problem.

The limitations of the method developed by this study were presented and analyzed. It considered a simplified time-block solution, with only two scenarios for all time periods $t = 1, 2, ..., T$ under analysis. This approach can be particularized to an incremental real-time solution, considering a single time step $T = 1$. It was also concluded that the full solution, with two scenarios for each time step, would lead to $n = 2^T$ scenarios, being computationally unfeasible for the Minimax Regret method.

The Minimax Regret Efficient solution obtained, which is a solution that minimizes the maximum regret, has shown to be a balanced solution for the problem, behaving as if the constraints were relaxed and the capacity envelope were placed somewhere in between the VMC and IMC curves, spreading the solution points through the feasible region and between the capacity curves, what also leads to a solution less susceptible to costs of switching the capacity allocation over time. This behavior is more powerful than the optimal deterministic solutions for each individual scenario because these solutions are positioned over the capacity curves, that are uncertain and dynamic.

Thus, the results and discussions drawn in this investigation seem to contribute to how to introduce the capacity uncertainty into the problem without making assumptions about the transitions between weather conditions, without trying to model the queues' behavior, like in the dynamic programming approach of Jacquillat et al. [2016] and Shone et al. [2019], and without speculate about the real capacity values, like in the fuzzy approach of Teodorovic et al. [2004]. However, it is far from being a definite solution to the problem but offers a new point of view. Further studies could focus on different methods for introducing uncertainty into the problem, like the use of reinforcement learning, which is also based on a MDP formulation but can be stronger than the dynamic programming approach, because it deals better with uncertainty and it is able to reduce significantly the dimensionality of the problem, eliminating the need for a model of the environment, as the dynamic programming does. Moreover, the analyzes developed here should undergo tests in a real application to allow ensuring that it is really applicable in practice.

DOI: 10.59254/sbpo-2023-174932
http://live-proceedings.pantheonsite.io/p/174932

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DOI: 10.59254/sbpo-2023-174932
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