

## **Approaches for Vehicle Routing Problems with Energy Considerations and Selective Backhauls**

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### **RESUMO**

Na tese de doutorado relacionada, foca-se em abordagens de programação linear e inteira para problemas em dois grupos de Problemas de Roteamento de Veículos (VRPs): um considerando consumo de energia e outro que considera backhauls e incerteza. Além de sua relação com custos, minimizar o consumo de energia em roteamento de veículos potencialmente mitiga impactos ambientais, como é o caso do VRP Cumulativo (CMVRP), que minimiza uma medida simplificada de consumo de energia dependente de carga. Para o CMVRP, a tese propõe uma nova formulação cuja relaxação é mais forte que aquelas em que ela é baseada e emprega um algoritmo branch-cut-and-price (BCP) baseado em uma formulação existente a fim de apresentar resultados em estado-da-arte. Na tese são propostas variantes do VRP Capacitado e CMVRP que consideram limites de energia por veículo. Para eles, são elaboradas abordagens BCP onde os subproblemas de pricing usam cargas discretizadas por arco e são apresentados vastos resultados. No melhor do nosso conhecimento, esta é a primeira vez que um consumo de energia dependente de carga é limitado por meio do pricing. Por fim, a tese aborda o VRP com Backhauls Seletivos, no qual, depois de visitar clientes, cada veículo pode visitar um único backhaul para coletar um valor de receita incerto. Para este problema, um modelo de otimização robusta e um algoritmo branch-and-cut são elaborados e testados.

**PALAVRAS CHAVE.** Roteamento de Veículos, Programação Inteira, Otimização Robusta.

**Tópicos.** Logística e Transportes, Otimização Combinatória, Programação Matemática.

### ABSTRACT

In the related PhD thesis, we focus on integer and linear programming approaches to problems in two groups of Vehicle Routing Problems (VRPs): one considering energy consumption and other that considers backhauls and uncertainty. Besides its relation to costs, minimizing energy consumption in vehicle routing potentially mitigate environmental impacts, as is the case of the Cumulative VRP (CMVRP), which minimizes a simplified measure of a load-dependent energy consumption. To the CMVRP, we propose a new formulation whose relaxation is stronger than the ones it is based on and employ a branch-cut-and-price (BCP) algorithm based on an existing formulation to present state-of-the-art results. We propose variants of the Capacitated VRP and CMVRP that consider energy limits per vehicle. To them, we devise BCP approaches where the pricing subproblems rely on discretized loads per arc and present extensive results. To the best of our knowledge, this is the first time a load-dependent energy consumption is limited by the means of the pricing. Finally, we tackle the VRP with Selective Backhauls, in which, after visiting customers, each vehicle may visit a single backhaul to collect an amount of an uncertain revenue. To this problem, a robust optimization model and a branch-and-cut algorithm are devised and experimented.

**KEYWORDS.** Vehicle Routing, Integer Programming, Robust Optimization.

**Topics.** Logistics and Transports, Combinatorial Optimization, Mathematical Programming.

### 1. Introduction

Many NP-hard combinatorial optimization problems can be formulated and solved by mixed-integer linear programming (MILP), as numerous problems in the large category of Vehicle Routing Problems (VRPs) [Toth and Vigo, 2014]. This category originated from the Capacitated VRP (CVRP), which consists in finding the cheapest way to serve a set of customers with a fleet of vehicles of a given capacity, which can be modeled over a digraph. The related PhD thesis employs integer and linear programming approaches to problems in two groups of VRPs: one considering energy consumption and other that considers backhauls and uncertainty. Consider the first group of problems: besides its relation to transportation costs (e.g., fuel, electricity), the interest in the energy consumption of vehicle routing has risen in the last decades due to its potential of mitigating environmental impacts, as the amount of greenhouse gas emissions, and is now an established concern [Kara et al., 2008; Bektaş and Laporte, 2011; Asghari and Mirzapour Al-e-hashem, 2020]. A particular class of problems that consider environment-related aspects is given by the Green VRPs (GVRPs) [Demir et al., 2014; Asghari and Mirzapour Al-e-hashem, 2020].

One way to take energy consumption into account is by considering it in the objective function. While costs in the CVRP are measured in terms of distance, the Cumulative VRP (CMVRP) is a variant that aims to minimize a simplified measure of a load-dependent energy consumption [Kara et al., 2008]. In this problem, each arc's energy consumption is defined as the product of the arc distance by the accumulated weight of the vehicle's curb weight and all the items picked since the beginning of the route. We explore MILP formulations for the CMVRP, using the concepts of two previous formulations to propose a new one, whose linear programming (LP) relaxation is stronger than both LP relaxations of the original ones. Furthermore, we solve the CMVRP with a branch-cut-and-price (BCP) algorithm based on a formulation from the literature, for which we use the generic BCP framework proposed by Pessoa et al. [2020], the so-called VRPSOLVER. The results of our approach improve upon the state-of-art results of Fukasawa et al. [2016]. Additionally, we slightly improve the analysis of an approximation algorithm for the version of the

CMVRP that does not fix the number of vehicles, going from a factor 4 [Gaur et al., 2013] to a factor  $4 - \frac{4}{3s^3Q^2}$ , where  $Q$  is the capacity and  $s$  is a scaling factor.

Another way to deal with energy consumption is by limiting the energy consumption of each vehicle. This is particularly relevant when vehicles need to be restricted by a certain range since recharging or refueling may not be easily done, which is the case of electrical vehicles [Conrad and Figliozzi, 2011]. In this topic, we propose the CVRP with Limited Energy (CVRP-LE) and CMVRP with Limited Energy (CMVRP-LE), which are, respectively, similar to the CVRP and CMVRP with the difference that the new ones have limited energy per vehicle. The CMVRP-LE can be seen as a special case of other problem in the literature [Goeke and Schneider, 2015], whereas, surprisingly, we did not find in the GVRP literature a problem that can accommodate the CVRP-LE as a special case. Other works in the literature consider branch-and-price (BP) or BCP methods to approach GVRPs and take into account energy consumption, energy limits, and load capacity (e.g., Desaulniers et al. [2016]), however, their energy consumption do not depend on the accumulated load. We devise, with the VRPSOLVER, BCP approaches to the CMVRP-LE and CVRP-LE, that are modeled by a set-partitioning-based master problem formulation and a pricing subproblem, where the latter is solved as a Resource Constrained Shortest Path Problem (RCSPP). Our master problems are formulated with either arc-load or arc variables, while the subproblems rely on arc-loads. An arc-load, by its turn, denotes how much load is carried through an arc, and its concept is pivotal to track and limit the load-dependent energy consumption in our modeling of the pricing subproblems. BCP methods with arc-load-based master and pricing problems were used before for the CVRP [Pessoa et al., 2008] and CMVRP [Fukasawa et al., 2016], however, to the best of our knowledge, this is the first time a load-dependent energy consumption is limited by the pricing subproblem. The thesis presents extensive computational results for the CMVRP-LE and CVRP-LE with the models we propose, comparing them with their versions without energy limit.

For the second group of problems approached, consider the VRP with Backhauls (VRPB), that has two different types of customer: the linehaul customers, which receive goods from the depot, and the backhaul customers, which send goods back to the depot [Toth and Vigo, 2014]. In the line of the VRPB, we study the VRP with Selective Backhauls (VRPSB) [Baldacci et al., 2010], which aims to minimize the routing costs minus the revenues collected at backhaul customers optionally selected, where each route may visit a single backhaul. In the VRPSB, selecting one backhaul rather than another depends on the revenue it provides, however, it might be hard to exactly know the quality attributes beforehand, as is the case of moisture of raw-materials in the wood-base supply chain, that can only be roughly estimated [Alvarez and Vera, 2014]. We explore the VRPSB under uncertain revenues, where the solutions are required to achieve a minimum revenue from the raw-materials collected at the backhauls. A deterministic version of the problem is given by a MILP formulation and a robust optimization (RO) counterpart model is derived. In this robust approach, the revenue uncertainty is modeled as an uncertainty set, where it is possible to adjust the budget of uncertainty [Bertsimas and Sim, 2004]. We devise a branch-and-cut (BC) algorithm to solve the deterministic and RO formulations, and we also compare them with a heuristic approach.

The main chapters of the thesis are based on papers we produced for scientific international indexed journals, where two of them are published and one is in preparation. In the remainder of this work, each of the sections 2, 3, and 4 present more information on the contributions and results of these chapters/papers, given by, respectively, Mulati et al. [2022a], Mulati et al. [2022b], and Santos et al. [2020]. The referred sections have the same title of the chapters/papers. Subsequently, Section 5 provides some remarks.

## 2. The Arc-Item-Load and Related Formulations for the Cumulative Vehicle Routing Problem [Mulati et al., 2022a]

In the related chapter/paper, we propose several different formulations for the CMVRP and study their LP relaxations. In particular, we study formulations based on combining the Arc-Item (AI) formulation (that keeps track of whether a given customer has already been visited when traversing a specific arc) with another one from the recent literature, the Arc-Load (AL) formulation (that determines how much load goes through an arc). Both formulations have been studied independently before – the AI is very similar to a multi-commodity-flow formulation in Letchford and Salazar-González [2015] and the AL has been studied in Fukasawa et al. [2016] – and their LP relaxations are incomparable. Nonetheless, we show that a formulation combining the two, called Arc-Item-Load (AIL), may lead to a significantly stronger LP relaxation, thereby indicating that the two formulations capture complementary aspects of the problem. We state that AIL dominates the formulations it is based on. In addition, we study how set-partitioning-based formulations can be combined with these formulations. In this sense, we propose the set-partitioning AIL<sub>qR</sub> formulation, that have a pricing subproblem solvable in pseudo-polynomial time, and prove that AIL and AIL<sub>qR</sub> are equivalent. We also contribute by proposing a somewhat natural way to combine AI with a set-partitioning formulation, namely the AI<sub>qR</sub> formulation, which dominates AL. However, such a natural formulation is not likely to be useful in practice since we conclude that its pricing subproblem is strongly NP-hard.

We present computational experiments over 100 well-known benchmark instances of the CVRP adapted to the CMVRP, that highlight the advantages and drawbacks of the LP relaxations of AIL, AL, and AI formulations. In Figure 1, we present a plot of the cumulative frequency of (integrality) gaps of the instances. To report these results, we present only those tests where optimal solutions were found for the LP relaxations of all the three models and also have integer solutions available, leaving us with 71 instances up to 67 vertices. A point  $(x, y)$  in this plot indicates that there are  $y$  instances with a gap of at most  $x\%$ . One can see that the AIL relaxed formulation reaches a gap of at most 5% in 64 instances. For the other two formulations, AL and AI, respectively, only 50 and 11 instances have a gap of at most 5%. Despite having presented a promising experimental gap, the main drawback of AIL is the computational time it requires to have its relaxation solved. While AI and AL took an average of 16.8 s and 19.6 s in those 71 instances, the AIL computational time average is almost 5 h, with some peaks of more than 21.5 h.

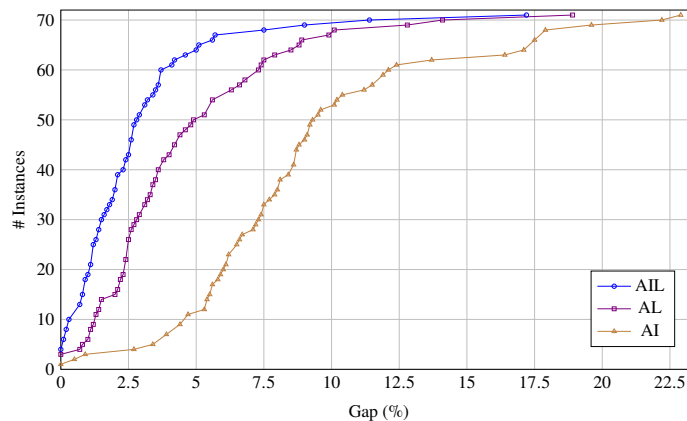


Figure 1: Gap cumulative frequency

Therefore, our experiments show a significant gain in the LP relaxation gap of AIL. The gap advantage of the AIL may be attributed to the fact that each base formulation alone captures different and complementary aspects of the CMVRP, and putting them together tends to produce a stronger formulation that is worth exploring. The downside is the large amount of time spent in solving the tighter formulation.

### 3. Branch-Cut-and-Price Approaches for Vehicle Routing Problems with Load-Dependent Energy Consumption [Mulati et al., 2022b]

In the related chapter/paper, we employ, using the VRPSOLVER, BCP approaches to tackle the CMVRP, which minimize a load-dependent energy consumption, and the CMVRP-LE and CVRP-LE, whose load-dependent energy consumption is limited by the pricing subproblem. Particularly, we propose the CMVRP-LE and CVRP-LE and devise set-partitioning-based master formulations and correspondent pricing subproblems for the three ones. Our master problems are formulated with either arc-load or arc (vehicle-flow) variables, which are coupled to resource constrained path variables. The pricing subproblems are modeled by RCSPs with digraphs based on arc-loads, in which the arc-loads are pivotal in tracking and limiting the energy consumption. In the subproblems, we keep track of the load resource and/or the energy resource. According to the VRPSOLVER framework, resources are classified in main or secondary and they are also defined as disposable or non-disposable. In this work, we focus on resource disposability: in the disposable case, the accumulated resource consumption is allowed to be arbitrarily raised (an amount of it might be disposed/dropped); otherwise the resource is non-disposable. Typically, disposable resources favor the procedure to check dominance among labels, but may generate more labels than otherwise. We propose several configurations and present extensive results. Besides, we tailor the use of several features provided by the VRPSOLVER, as *ng*-paths, path enumeration, rounded capacity cuts, limited memory rank-1 cuts, primal heuristics, and strong branching.

The computational experiments, all in the same computer, are carried out over 100 well-known benchmark instances of the CVRP adapted to the problems we deal with. To all the problems, we employ a preprocessing to only keep the arc-loads corresponding to packageable demands, which led, in average, to 4.16% less arc-loads than a previous approach (we decrease over 10% in seven instances). For the CMVRP, the master formulation and subproblem are modeled with arc-loads and the best results are yielded with the resource configuration that solely uses the disposable load resource. Comparing this approach to the BCP of Fukasawa et al. [2016] in similar conditions, over the 78 instances where both found feasible solutions, we are able to increase the number of optimal solutions found from 63 to 78 and improve the average optimality gap from 0.55% to 0% in a time average of 232.7 s – at least 7 times lower. When we do not enforce similar conditions nor a filter tied to other experiment, our approach is able to find 95 optimal solutions in the average time of 428.2 s. Thus, we improve upon the state-of-the-art exact results for this problem.

The master formulation used for the CMVRP-LE is similar to the one of the CMVRP, while, in addition to the disposable load, the best resource configuration contains the non-disposable load and the disposable energy resources. The presence of the non-disposable load means that the best performance was achieved by the approach that forbids the creation of labels in which the load resource would be disposed, despite having a less aggressive label dominance rule. Due to the fact that the CVRP-LE's objective function does not consider energy consumption, we are able to also use a master formulation given only by arcs. With this master formulation, the best resource configuration for the CVRP-LE has disposable load, non-disposable load, and disposable energy, where the presence of the non-disposable load avoids, in this case, an important performance issue intrinsic to this problem: the load excess effect.

The best average times to process the CMVRP, CMVRP-LE, and CVRP-LE are, respectively, 428.2 s, 1029.5 s, and 1335.7 s over the set of 100 instances; which probably indicates a reverse order of hardness among these problems. It is also worth noting that the instances solved in each of these problems has up to 101 vertices, and, for the majority of the instances, the variants with limited energy resulted in optimal solutions different from their counterparts without such a limit. We recall that, amid the VRPs that consider load-dependent energy consumption, the enforcing of a limit on the energy consumption of the vehicles by the pricing subproblem constitutes a novelty in the BCP/BP literature.

#### 4. A Robust Optimization Approach for the Vehicle Routing Problem with Selective Backhauls [Santos et al., 2020]

In the related chapter, our main contribution on the respective paper regards the devising of the MILP formulation and the BC algorithm applied to the VRPSB with uncertain revenues tackled by RO, as well as its deterministic version. The VRPSB also considers that a minimum required revenue must be attained. In the RO approach, the revenues of the backhauls are considered to be bounded by a polyhedron set of uncertainty and a robust parameter, called budget of uncertainty, is employed to control the size of the uncertainty set [Bertsimas and Sim, 2004]. Besides, probabilistic bounds of constraint violation (on the required revenue) for the RO formulation are estimated. In this sense, the method that estimates the probability *a posteriori* is the most rigorous one, but it requires a computed solution. On the other hand, a method based on the Irwin-Hall distribution provided tighter probabilistic bounds than the other *a priori* methods, noting that this method has the advantage of only depend on some instance data and the budget of uncertainty.

The performance of the BC algorithm is compared with an Adaptive Large Neighborhood Search (ALNS) heuristic algorithm. Our experiments are conducted over 30 benchmark VRPB instances adapted to the VRPSB. These instances have, respectively, at most 100, 75, and 11 linehaul customers, backhaul customers, and vehicles. For each instance, the budget of uncertainty and the minimum required revenue are varied, yielding 90 combinations. It was possible to observe that, as expected, the cost of robust solutions may be higher than the cost of nominal solutions, denoting the so-called *price of robustness*. The BC is able to provide optimal solutions for almost all the combinations with up to 45 linehaul customers. On the other hand, the exact method is unable to find feasible solutions in 18 out of the 90 combinations. The ALNS obtained optimal or good solutions for all instances in a short time. In average, the BC takes 1616.7 s and yields an integrality gap of 3.24%. The ALNS, by turn, takes an average time of 18.5 s (considering 30 runs on each combination) and shows an average integrality gap of 3.27% (over the best runs of each combination). A strong point of the BC is its ability to find 51 optimal solutions, while the ALNS obtains 31 optimal ones. It is also evidenced that the RO approach created a tractable robust model, where solutions may be obtained with similar computing effort as with the deterministic model.

#### 5. Remarks

The thesis approaches three variants of the CVRP that minimize the amount of load-dependent energy consumption and/or impose energy limits per route. It also solves the VRPSB with uncertain revenues. These problems are listed in the following, alongside their main points.

- **CMVRP:** We propose, among others, a new MILP formulation whose LP relaxation is stronger than the ones it is based on, and, based on a previous formulation, we solved it with a BCP method to present state-of-the-art results.
- **CMVRP-LE and CVRP-LE:** We proposed these problems and employed BCP approaches to them, where we devise master formulations and corresponding arc-load-based pricing sub-

problems. Among problems with load-dependent energy consumption, our enforcing of energy limit via pricing subproblem constitutes a novelty. We present extensive results.

- VRPSB with uncertain revenues: We approach it by RO using a BC algorithm, which includes a MILP formulation. We present exact results and compare them with a heuristic approach.

The research in the PhD thesis resulted in two papers published in international journals, one paper being prepared for submission to an also international journal, and a short paper published in a conference. These works are listed in the following.

1. Journal paper entitled “*The Arc-Item-Load and Related Formulations for the Cumulative Vehicle Routing Problem*”, published in the *Discrete Optimization* journal [Mulati et al., 2022a].
2. Journal paper entitled “*A Robust Optimization Approach for the Vehicle Routing Problem with Selective Backhauls*”, published in the journal *Transportation Research Part E: Logistics and Transportation Review* [Santos et al., 2020] – a joint work with Maria J. Santos, Eduardo Curcio, and Pedro Amorim of the Faculty of Engineering of the University of Porto, Portugal.
3. Working paper entitled “*Branch-Cut-and-Price Approaches for Vehicle Routing Problems with Load-Dependent Energy Consumption*” [Mulati et al., 2022b], to be submitted to an international indexed journal on the topics of the research.
4. Conference short paper entitled “*Tighter Analysis of an Approximation for the Cumulative VRP*”, published in the proceedings of the *II Encontro de Teoria da Computação – XXXVII Congresso da Sociedade Brasileira de Computação* [Mulati and Miyazawa, 2017].

To confer the degree of PhD in Computer Science, this thesis was defended in December/2022 at the Institute of Computing of UNICAMP. This thesis was evaluated by professors that have been involved in high quality international research works in the thesis’ topics. Besides the advisor, the PhD thesis committee was composed by: Eduardo Uchoa (TEP/UFF), Rafael Martinelli (DEI/PUC-Rio), Kelly Poldi (IMECC/UNICAMP), and Reinaldo Morabito Neto (DEP/UFSCar).

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