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Hybrid model for portfolio selection using TODIM method and Modern Portfolio Theory

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RESUMO

Este artigo propõe um modelo híbrido para seleção de portfólio de ações, combinando o método TODIM com a Teoria Moderna de Portfólio. O modelo foi aplicado em duas etapas: a primeira ranqueando e selecionando as ações segundo critérios de avaliação de desempenho, e a segunda otimizando o portfólio considerando o conceito de fronteira eficiente. Quatro carteiras foram definidas, duas na primeira etapa (seleção) e duas da segunda etapa (otimização), e comparadas com o benchmarking (S&P500), considerando índices de retorno e risco. Os portfólios resultantes do processo de otimização apresentaram resultados superiores em relação aos originários da etapa primeira etapa e ao benchmark. A combinação das ferramentas mostrou-se eficiente na escolha e composição de carteiras de melhor desempenho.

KEYWORDS. Seleção de Portifólio, Teoria Moderna de Portfólio , Método TODIM.

ABSTRACT

This paper proposes a hybrid model for stock portfolio selection, combining the TODIM method with Modern Portfolio Theory. The model was applied in two stages: the first ranking and selecting stocks according to performance evaluation criteria, and the second optimizing the portfolio considering the concept of efficient frontier. Four portfolios were defined, two in the first stage (selection) and two in the second stage (optimization), and compared with the benchmark (S&P500) considering return and risk indices. The portfolios resulting from the optimization process presented superior results in relation to those originating from the first stage and the benchmark. The combination of tools proved efficient in choosing and composing better performing portfolios.

KEYWORDS. Portfolio selection. Modern Portfolio Theory. TODIM Method

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1. Introduction

All the decisions we make in our daily lives go through some kind of decision process, with a greater or lesser degree of complexity. Problem-solving is a process that aims to determine a desirable outcome, taking into account various relevant aspects. Decisions, when simple, are made almost unconsciously. When we increase the complexity, we need to use tools that can help us in this process, such as mathematical equations, multiple statistics, mathematics, economic theories, and computer devices that help automatically calculate and estimate solutions for decision-making problems [Taherdoost and Madanchian, 2023].

Complex problems are prevalent in all fields where human beings operate, with varying objectives and variables depending on the situation being analyzed. One field of study that is particularly notable for its abundance of highly complex problems requiring the consideration of multiple variables, conflicting objectives, and vast amounts of data is finance.

The financial market is characterized by intense competition, rapid economic, social, and technological changes, and an exponential increase in available data and information due to globalization. These factors have significantly increased the complexity of decision-making in this field. Decisions need to be made accurately and quickly, evaluating several variables and considering an infinity of characteristics and possibilities. For this, financial professionals are increasingly looking for and using tools that can help in this process. The so-called MCDA (multi-criteria decision analysis) are gaining more and more space. These tools are capable of analyzing and evaluating diverse information and criteria, and can be customized to align with the decision-maker's preferences. [Zopounidis and Doumpos, 2002].

These tools are designed to handle large volumes of information, alternatives, and objectives, which are often conflicting and difficult to consider without a structured mathematical process Over the last few decades, several decision-making support tools have been developed, generally based on methodologies or theories that seek to bring the mathematical methodology closer to the model to simulate the human decision-making process.

Based on Prospect Theory (PT), which seeks to approach the human decision-making process, the TODIM Multicriteria Decision Method (acronym in Portuguese for interactive and multicriteria decision making) is a powerful tool to analyze and rank alternatives considering the preferences of one or more decision makers.

A key feature of the model is its ability to approximate the behavior of decision-makers by analyzing gain and loss scenarios differently. Considering the stock market as competitive and dynamic, the use of tools that help decision-makers in their decisions process is increasingly important. Even more so with the exponential increase in information that needs to be analyzed in a short period to maintain the competitiveness of choices.

Given this, the use of combinations of decision support tools combined with optimization tools represents an advantageous option in the preparation of a stock portfolio.

In this work, a hybrid methodology was used, in two stages, combining the multicriteria decision method, TODIM (for stock selection), with (and) the Modern Portfolio Theory Markowitz (for weights optimization) to compose an investment portfolio

The first stage of the application aimed to evaluate and classify the stock to compose the portfolio, through the use of technical indexes. The second stage aimed to indicate the composition, that is, the participation of each of the stock in the portfolio considering the frontier efficiency indicated in the Modern Portfolio Theory.

To the TODIM Model, it is first necessary to identify the alternatives to be ranked. Initially, a total of 502 actions were considered for the application of the model. These stocks make up

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the S&P 500 index, short for Standard and Poor's 500. An American index made up of 500 national publicly traded companies, it's considered one of the best comparison indexes in the American market. Data collected in May of 2023.

These actions were evaluated considering seven performance criteria. These criteria are fundamental indices, based on data from the financial statements of the evaluated companies. These indices serve to diagnose the situation and evaluate the performance of companies. Some of the previously selected alternatives did not have available all data from all indices and were eliminated from the study.

In the end, a decision matrix composed of 485 actions (alternatives) and 7 performance indices (criteria) was obtained. After applying the TODIM Method, shares were evaluated resulting in the ranking of shares. From this ranking, two portfolios were composed (**T25 and T40**) corresponding to the first 25 shares and the first 40 shares, respectively.

In the second stage, the **T25** and **T40** portfolios had their weights optimized using the efficient frontier concept originated from the Modern Portfolio Theory, based on portfolio diversification to reduce risk. Generating two optimized portifolios **M25** and **M40**.

In the results session, 4 portfolios (**T25-T40-M25-M40**) were evaluated for yield and risk performance, and uncovered with benchmarking (S&P500). The 4 portfolios show a better result than the benchmarking in terms of yield and risk. Comparing the **T25-T40-M25-M40** portfolios, the two processed with weight optimization (**M25-M40**) obtained superior results as portfolios before optimization (**T25-T40**).

2. Literature review

2.1. Prospect Theory

The Prospect Theory (PT) is the result of the work of two psychologists, Kahneman and Tversky (1979). They published an article where they questioned the rationality of the decision-maker defended by traditional finance. This theory considers that the decision-maker is not always rational and has different behaviors considering the risk of losses and gains.

In proposing the PT, Kahneman and Tversky, tried to circumvent the series of flaws in the concept of the rational decision-maker. In a series of experiments, they demonstrate that sometimes the investor or decision-maker behaves exactly as he is. A human being who has intrinsic preferences and prejudices and who is influenced by them when making decisions even in the face of contrary facts. Not being able to process and absorb all available information and understand the correlation between factors at the time of decision making [Curtis, 2004].

They demonstrated the individual's tendency to be influenced not only by their logical judgment but also by their behavior at the time of decision-making. In a lottery situation, people were more likely to take risks when faced with the possibility of losing larger amounts at the expense of a certain loss. Whereas in situations where the choice was between larger gains with a certain probability of happening in exchange for a smaller but guaranteed gain. People tend to choose the second option, guaranteeing the gain,called of the reflection effect. This choice is contrary to what is defined by the Utility Theory (UT), as it demonstrates that the same decision-maker can have conflicting behaviors when faced with loss and gain situations, [Silva et al., 2014].

In their work, the two identified other behaviors that conflicted with the idea of a purely rational decision maker. Among these behaviors, the framing effect was identified. Where the decision maker's choice is affected more by how the option is presented than by the option itself. This effect is considered a continuation of the so-called reflection effect described above. Another contradictory behavior refers to the person's tendency to choose different options for the same problem if the options are presented in different ways [Ruggeri et al., 2020].

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2.2. TODIM method

Among the various applications of MCDA in the finance area, we can highlight its application in the investment market. When it comes to financial investments, one of the biggest trade-offs is between risk and return outcomes. Because these are almost always conflicting objectives, that is, the greater the expected return of an asset, the greater the risks involved in the business. In order to find a solution that can best serve the investor, it is necessary that it be structured guided by a decision making method [Mendonça et al., 2020].

In this context, MCDA tools have been gaining more and more prominence in applications in the area. Being widely used in processes where one seeks to maximize liquidity, profitability and financial value, while seeking to minimize risks and investment costs, objectives that often conflict with each other [Almeida-Filho et al., 2021].

Considering the advances that Prospect Theory has brought in relation to the understanding of human behavior in the face of risk decisions, the TODIM method (a Portuguese acronym for interactive and multicriteria decision-making) was developed with the objective of modeling preference patterns regarding the decision-maker's behavior in the face of risk [Gomes and Lima, 1992].

For these reasons, the method attracted the interest of researchers, being one of the first MCDMs to incorporate in its modeling the concepts of individual behavior based on Prospect Theory. The method is capable of working with complex problems with several alternatives and criteria [Wang and Liu, 2017].

As seen in session 3.1, the method has different functions when performing pairwise comparisons. When the alternative has a superior result in a criterion (gain) the method uses a specific function. In cases when the alternative has a lower result (loss) the model uses another function. Thus, the model weights the losses and gains differently [Rangel and Gomes, 2007].

Lu et al.(2020) indicated in his work some advantages of using the method for selection problems. First, the method is already a popular tool for solving performance appraisal problems. The second advantage is that in addition to taking into account the advantage that one alternative has over the other, it also considers the superiority of the alternative concerning all the others. Third, as it is based on Prospect Theory, the method addresses the psychological behavior of the decision-maker taking into account. Which seeks to consider the behavioral component in the selection methodology.

Each calculated number should be interpreted as the measure of overall desirability value or utility, or simply as the value of a specific alternative. The order of alternatives is a consequence of the order of their respective values. Therefore, by ordering all the alternatives, TODIM method ranks the alternatives by using the preferences expressed by an individual or a group of decision makers. By changing the set of preferences, we can eventually come up with a new result through sensitivity analysis.

2.3. Modern Portfolio Theory

We can classify as portfolio theory the problems of resource allocation between different asset options, this problem is faced by almost all institutions that need to include in their selection analysis the characteristics of their liabilities [Elton and Gruber, 1997].

The Modern Portfolio Theory (MPT), proposed by Markowitz in 1952, brought a new investment strategy, which was widely used over the decades because it was able to optimize the trade-off between return and risk. Applied in diversified portfolios, helping investors to identify applications with better returns in the long term and with better levels of risk in the short term [Liu, 2023].

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For Haddad and Sanders(2018), risk is an uncertain condition or event that may or may not occur and, if it does, will have a negative or positive effect on at least one of the objectives. As risks arise from uncertainty, all decision processes involve, to some degree, an element of risk. When we talk about a non-theoretical environment, we have several internal and external factors that influence the results of decisions and many of them are unknown.

Markowitz worked on the variance percentage of the asset portfolio. He proved that keeping the variance constant results in the maximization of the expected return. Whereas keeping the expected return constant results in the minimization of the variance. Based on this, he formulated the so-called efficient frontier that allows the decision maker to identify the best option considering his preference in relation to risk [Elton and Gruber, 1997].

MPT is a useful tool that allows the projection of the ideal portfolio, based on returns and risk. As it uses the key concept of diversification, that is, to choose the ideal portfolio investors must choose assets that are inversely correlated. This means that when the value of a stock goes down, another stock goes up, with this it is possible to reduce portfolio risks [Liu, 2023].

2.4. Portfolio size

The risk level of a stock portfolio is intrinsically linked to the proportions of individual stocks, as well as their variances and covariances. Any change in one of these variables can influence the risk of the portfolio as a whole. However, in general, it is observed that when stocks are randomly selected and combined in equal proportions within a portfolio, the portfolio risk tends to decrease as the number of different stocks in it increases [Statman, 1987]. Several works aim to parameterize the ideal size of a portfolio aiming at its diversification. However, a consensus regarding this value still seems difficult to obtain.

According to Kryzanowski and Singh (2010), the ideal portfolio size is related to the investor's objectives. Considering which metrics he wants to evaluate the performance of the portfolio. Taking into account traditional risk metrics, such as variance, semivariance, and NV (Number of Variables), it was found that portfolios composed of 20 - 25 stocks were able to capture approximately 90% of the potential benefits of diversification. If the investor is looking for the same efficiency when talking about risk indices such as Sharpe and Calmar, the recommendation is for a portfolio comprising 40 - 100 stocks.

3. Application

The work proposed a combination of two methods in a two-step model for composing a stock portfolio. The first step use the TODIM method to rank the alternatives (stocks that will compose the portfolio). Then Markowitz optimization was applied, seeking to find the best composition for the portfolio considering the previously selected stocks.

At the end of each stage, the results of the portfolio were compared considering the return, cumulative return over the 5-year period and risk indices, Sharpe and Calmar, considering the S & P500 index as benchmarking.

3.1. Application of the TODIM Method

To apply the TODIM method, the alternatives (n) to composing the model were first selected. It was decided to use the stocks that make up the Index of S&P 500.

To compose the decision matrix, the indices chosen to compose the evaluation criteria (m) of the alternatives were collected on the Statusinvest website [1], which provides a series of fundamental and technical indices referring to companies listed on the stock exchange. Initially, 13 performance criteria listed under the nomenclature C1-C13 were selected, as indicated in Table 1.

[1] https://statusinvest.com.br/

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To meet one of the assumptions of the model, which determines that the criteria are independent. An analysis of the covariance of the indices was performed as indicated in the matrix shown in Figure 1 with the aim of reducing the dependence of the model variables. The matrix graphically indicates the positive or negative covariance of the indices. Based on these results, 5 indices were excluded , leaving 7 evaluation criteria as indicated in Table 1. Figure 1 indicates the covariance matrix considering the 7 selected criteria.

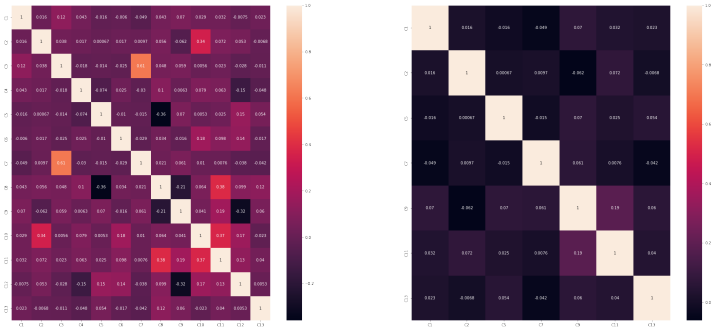


Figure 1: Covariance matrix of performance evaluation criteria
Source: authors 2023

Reference	Initial Code	Criteria	Final Code
C1	P/E	Price to Earnings Ratio	C1
C2	P/BV	Price-to-Book Ratio	C2
C3	P/EBIT	Price-to-Earnings Before Interest and Taxes Ratio	X
C4	VPA	Net Asset Value per Share	X
C5	P/B	Price-to-Assets Ratio	C5
C6	LPA	Earnings per Share	X
C7	ND/E	Net Debt-to-Earnings Before Interest and Taxes Ratio	C7
C8	PL/ATI	Equity-to-Assets Ratio	X
C9	GM	Gross Margin	C9
C10	ROE	Return on Equity	X
C11	ROA	Return on Assets	C11
C12	TURN	Asset Turnover Ratio	X
C13	CAGR	Compound Annual Growth Rate of Revenues over 5 Years	C13

Table 1: Codes and description of the stock performance evaluation indices (criteria)
Source: authors 2023

C1 - (P/E): This ratio measures the relationship between the share price and the earnings per share of the company represented in the stock.

C2 - (P/BV): This ratio measures the relationship between the share price and the equity value per share of the company.

C5 - (P/B) :Indicates whether the asset is being overvalued or undervalued.

C7 - (ND/E): Is a measure of financial leverage and debt repayment time.

C9 - (GM) : Indicates the profit percentage of the company.

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C11 - ROA : Demonstrates the profitability of an asset.

C13 - CAGR : Is a measure of compound annual growth.

To be used in the TODIM calculations the criteria P/BV, ND/E, P/B , and P/E had their values multiplied by (-1), so that the goal is to maximize its result, instead of minimizing it.These criteria started to be referenced in the work with the addition of the suffix: max.The other criteria had his results used as they are.

The selection of alternatives and collection of performance criteria values (indexes) was carried out in May 2023.

Alternatives	P/E max	ROA	GM	CAGR	P/BV max	ND/E max	P/B max
AAL	0.001934	265.0	5909.0	282.0	-0.006289	0.001904	0.071428
AAP	0.000728	442.0	4449.0	354.0	0.003676	0.006756	0.016393
AAPL	0.000348	2745.0	4306.0	1146.0	0.000207	0.018867	0.001270
...	...	...	...	...	...	...	...
ZBH	0.000157	212.0	7158.0	-232.0	0.004255	0.001394	0.007462
ZBRA	0.000338	615.0	4539.0	921.0	0.001996	0.00257	0.005494
ZION	0.002645	98.0	0.0	405.0	0.014705	-0.000723	0.25
ZTS	0.000243	1506.0	6945.0	877.0	0.000528	0.006060	0.001615

Figure 2: TODIM Decision Matrix (485 alternatives X 7 Criteria)
Source: authors 2023

After defining the criteria, they were weighted according to their importance in the final decision according to the opinion of experts (Table 2).

The TODIM Method compares the performance of each alternative (n) in relation to each of the criteria (m) in a pairwise comparison. For each outcome, the model uses a different formulation to quantify the overall value of that alternative. To incorporate Prospect Theory, the method uses an attenuation factor (θ), which is used in cases where the alternative has a lower result than the compared one, that is, it has a loss. The calibration of this theta allows adjusting the model to suit a decision-maker with a greater or lesser risk propensity[Puppo et al., 2022].

In this work, we used θ value equal to 5 to simulate the preferences of a person with less aversion to risk. Consequently, the penalty imposed on alternatives with suboptimal performance is reduced, thereby emulating the preferences of an individual with a higher tolerance for risk.

To access the TODIM model in its entirety, consult the article by Leles et al.(2020).

	Criteria						
	P/E	ROA	GM	CAGR	P/BV	ND/E	P/B
w_{rc}	0.4	0.4	0.4	0.7	0.7	0.7	0.7

Table 2: Weight of importance of the criteria

The decision matrix composed of 7 criteria and 485 alternatives (stocks), resulted in the ranking of 1 to 485 stocks to indicate the composition of the portfolio (Table 3). Based on the study by Kryzanowski and Singh (2010), two portfolio sizes were defined. The first portfolio consists of 25 stocks, selecting the 25 first placed in the first stage (**T25**). The second portfolio comprises 40 shares (**T40**), following the same selection logic. In order to determine the composition of

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the portfolios from the initial stage, the desirability values of the respective results for each of the portfolios (**T25** and **T40**) were summed, and the value of each stock was then divided by the sum value. This resulted in normalized values with a sum of 1. The obtained value represents the participation of each share within the portfolio.

Position	Index	Alternative	Desirability Value (ξ_i)
1	216	HON	1.0
2	453	VRSN	0.9954210227425768
3	446	URI	0.991924775577946
4	333	ODFL	0.975304895700805
5	436	TXN	0.9748240330843794
...	...	...	...
482	92	CI	0.39722407978746427
483	162	ES	0.39722407978746427
484	306	MS	0.08253958521768588
485	193	GD	0.0

Table 3: Result of the TODIM method ranking the alternatives by the global desirability value
Source: authors 2023

3.2. Application of the Markowitz Method

In the second step of the model, using modern Markowitz portfolio Theory across the efficient frontier. Portfolios **T25** and **T40** had their composition optimized, thus generating two more portfolios M25 and M40. Figure 3 indicates the position of the optimal portfolios, highlighted in red, indicated by the method considering the efficient frontier. Considering the volatility and expected return of the portfolio for **M25** and **M40**.

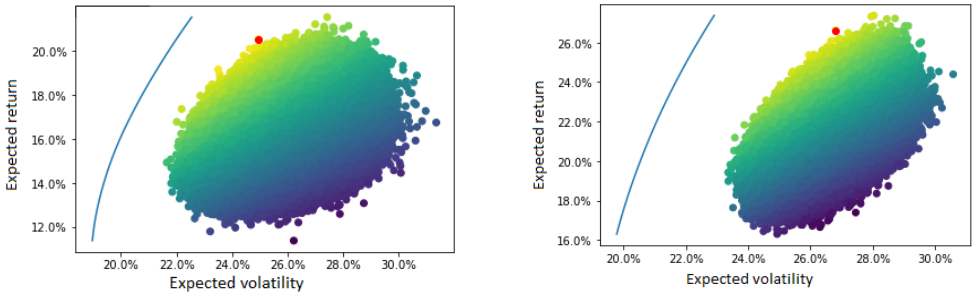


Figure 3: Result of Markowitz optimization efficient frontier for M25 and M40
Source: authors 2023

3.3. Results

At the end of each stage, the portfolio performance evaluation process was carried out. After the first stage (TODIM method), we obtained two tables with the indications of the stocks and the result of the model.

The second stage resulted in two other portfolios (**M25** and **M40**) consisting respectively of 25 and 40 stocks. The results of both portfolios were compared with market indice, S&P500. Fig 5 indicates the performance of the 4 indices over the last 5 years. From the graph, it is possible

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Composition of the portfolio with 25 stocks (TODIM and Markowitz)

Position	Stocks	T25	M25
1	HON	0.04121738	0.10685372
2	VRSN	0.04102865	0.00749256
3	URI	0.04088454	0.05631129
4	ODFL	0.04019952	0.01676352
5	TXN	0.0401797	0.01686692
...	...	...	...
20	NUE	0.03966212	0.06087740
21	EXPD	0.03964236	0.00552155
22	ULTA	0.03963316	0.02783855
23	AMAT	0.03962508	0.01685743
24	EOG	0.0396019	0.01614732
25	MAS	0.03959501	0.01948939

Composition of the portfolio with 40 stocks (TODIM and Markowitz)

Position	Stocks	T40	M40
1	HON	0.02592494	0.06064049
2	VRSN	0.02580623	0.01134365
3	URI	0.02571559	0.05646218
4	ODFL	0.02528470	0.06006561
5	TXN	0.02527225	0.00476567
...	...	...	
35	TT	0.02468587	0.00430603
36	LEN	0.02468013	0.01989849
37	EQT	0.02466397	0.00153565
38	MRNA	0.02466102	0.04165321
39	PXD	0.02463627	0.00020050
40	META	0.02462095	0.02246486

Figure 4: Stock portfolio with two sizes, results of the application of the first and second stages of the model.
Source: authors 2023

to identify that the optimized portfolio (**M25** and **M40**) performed better than the others. We can compare the performance ratios of the TODIM portfolio and the Markowitz portfolio as indicated in Table 4. The performance comparison of the portfolios was carried out considering an interval from 01/02/2018 to 04/28/2023, a total of 63 months of evaluation.

Index	S&P500	T25	T40	M25	M40
Annual return	8.5%	13.8 %	14.2%	16.9 %	16.1%
Cumulative returns	54.7%	98.9 %	102.6%	129.5 %	120.9%
Annual volatility	21.5%	28.9 %	28.1%	28.7 %	26.5%
Sharpe ratio	0.49	0.59	0.61	0.69	0.70
Calmar ratio	0.25	0.34	0.36	0.41	0.45
Stability	0.76	0.81	0.83	0.86	0.84
Max drawdown	-33.9%	-40.5 %	-39.9%	-41.6 %	-36.1%
Omega ratio	1.10	1.12	1.12	1.14	1.14
Sortino ratio	0.68	0.85	0.88	0.99	0.99
Tail ratio	0.87	0.98	0.94	0.97	0.94
Daily value at risk	-2.7%	-3.6 %	-3.5%	-3.5 %	-3.3%
Alpha	-	0.06	0.06	0.08	0.07
Beta	-	1.08	1.08	1.18	1.11

Table 4: Comparison of stock portfolio performance results against S&P500 benchmarks
Source: authors 2023

The performance of the 4 portfolios (**T25-T40-M25-M40**) was compared, considering the yield: annual and accumulated return (5 years), and the risk metrics: Sharpe and Calmar.

The results are summarized in Table 4, where we can see that the performance of the portfolios from the second stage (**M25** and **M40**), which underwent optimization, had a better result than those from the first stage (**T25** and **T40**).

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Figure 5: Yield performance of the 4 equity portfolios compared to the S&P500 benchmark.
Source: authors 2023

Index	T25	T40	M25	M40
Annual return	5.3%	5.7%	8.4%	7.6%
Cumulative returns	44.2%	47.9%	74.8%	66.2%
Sharpe ratio	20.4%	24.4%	40.8%	42.8%
Calmar ratio	36.0%	44.0%	64.0%	80.0%

Table 5: Detailed comparison of portfolios in relation to yield and risk indices (sharpe and calm) of the S&P500 benchmarks
Source: authors 2023

These results were observed both concerning the return indices (the higher the better) and the risk indices Sharte and Calmar (the higher the better). Table 5 details the difference in performance of each of the 4 portfolios against the S&P500 performance indices. We can see that the portfolio with the best performance was the **M25** with the best performance results, with an accumulated gain in the period of 74.8%, compared to the benchmarking.

Regarding the variation resulting from the increase in the shares that make up the portfolio. Considering the **T25** and **T40** portfolios no major variations were observed in the results of the portfolio of 25 shares to those composed of 40 shares (Figure 5).

In relation to the **M25** and **M40** portfolios, the results of the first one are superior. One of the possible causes is that with the increase of variables (stocks), it would be necessary to increase the number of simulations to reach the optimal value. As the number of simulations was maintained in the experiment, this may have reflected in the performance of the larger portfolio.

Comparing the execution time of the model, the **T25** and **T40** portfolios required a much smaller calculation time (146.00 seconds), compared to (3758.00 seconds) for the calculation of **M25** and **M40**.

4. Conclusion

This article proposed a hybrid model for investment portfolio selection. Combining the TODIM method and Modern Portfolio Theory. Four investment portfolios were selected using the stocks that make up the S&P 500 index as a basis, which were evaluated considering seven performance criteria. As a result of applying each of the methods, four portfolios with different sizes (**T25 - T40 - M25 - M40**) were generated. The portfolios were compared with each other

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and with the S&P500 benchmarks, considering yield and risk indices. The portfolios (**M25-M40**) resulting from the combination of the two methods, TODIM and MPT, obtained superior results to the S&P500 index, and the portfolios resulting only from the TODIM method, (**T25 - T40**).

The results indicate that the combination of the two methods presents advantages in the selection of the stock portfolio, with the increase of the return and the reduction of the risk indices when compared with the portfolio obtained only with the application of the TODIM method.

Despite the combination of the two methods presenting interesting results, the second step, which consists of optimizing the weight of the shares, has a computational demand 25 times greater than the first. This results in increasing the total process time.

The comparison of results between the portfolios of the same method with size variation of 25 stocks and 40 stocks showed no significant variations, neither concerning the performance results of the portfolios, nor in terms of computational time.

As future work, there are some lines to be explored, such as changing the calibration of the TODIM model by changing the weight values, to make the model more sensitive to one or another criterion. Increase the stock size difference amplitude to see if portfolio size can change portfolio performance and computational demand.

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