

OPTIMAL POWER FLOW FORMULATION BASED ON POWER TRANSFER DISTRIBUTION FACTOR WITH MONITORED LINES

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RESUMO

O objetivo do problema de fluxo de potência ótimo (FPO) é determinar o estado de operação ótimo de um sistema elétrico de potência, otimizando uma função objetivo enquanto satisfaz restrições de operação do sistema, sendo geralmente considerada como função objetivo a minimização dos custos de geração. Este artigo propõe o uso de linhas monitoradas no problema de FPO dc com o objetivo de reduzir o tamanho do problema e, consequentemente, o esforço computacional necessário para resolvê-lo. Para esse fim, a formulação do FPO dc é baseada no fator de distribuição de transferência de potência, a qual não depende dos ângulos de fase de tensões das barras. Três sistemas com diferentes graus de complexidades são testados e analisados com foco na redução da dimensão do problema. Os resultados indicam que a abordagem proposta é eficaz para diminuir as dimensões do problema e o esforço computacional de resolução.

PALAVRAS-CHAVE. Fator de distribuição de transferência de potência. Fluxo de potência ótimo dc. Sistemas de transmissão de energia elétrica.

EN&PG – PO na Área de Energia, Petróleo e Gás

ABSTRACT

The objective of the optimal power flow problem (OPF) is to determine the optimal operating state of an electrical power system, optimizing an objective function while satisfying system operation constraints, generally considering the minimization of generation costs as an objective function. This paper proposes the use of monitored lines in the dc OPF problem in order to reduce the problem size and, consequently, the computational effort required to solve it. For this purpose, the dc OPF formulation is based on the power transfer distribution factor (PTDF), which does not depend on the bus voltage phase angles. Three systems with different degrees of complexity are tested and analyzed with a focus on reducing the size of the problem. The results indicate that the proposed approach is effective in reducing the dimensions of the problem and the computational effort to solve it.

KEYWORDS. DC optimal power flow. Power transfer distribution factor. Power transmission systems.

EN&PG – OR in the Area of Energy, Oil, and Gas

Nomenclature

Indices:

i	Index for buses
ij, ji	Indices for branches

Sets:

Ω_b	Set of buses
Ω_g	Set of generation buses
Ω_l	Set of branches

Function:

v	Objective function
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Parameters:

B	Susceptance matrix
X	Reactance matrix
c_i	Generation cost at bus i
d_i	Active power demand at bus i
$\bar{f}_{ij}$	Maximum active power flow of a line on branch ij
f_{ij}^*	Estimate of the power flow on branch ij
$\bar{g}_i$	Maximum active power generation at bus i
$\underline{g}_i$	Minimum active power generation at bus i
n_{ij}^0	Number of existing lines on branch ij
r_{ij}	Resistance of a transmission line on branch ij
x_{ij}	Reactance of a transmission line on branch ij
$\psi_{ij,k}$	PTDF of branch ij with respect to the power injection/extraction at bus k
$\psi_{s,r,y}$	PTDF of line y when power is transferred from bus s to bus r
σ	Parameter for determining the monitored lines

Variables:

ΔP	Power transfer
Δf_y	Power flow variation on line y
f_{ij}	Power flow on branch ij
f_y^0	Power flow on line y before the power transfer
$\hat{f}_y$	Power flow on line y after power transfer
g_i	Active power generation at bus i
θ_i	Voltage phase angle at bus i

1. Introduction

Efficient management of power systems is essential for ensuring a reliable and cost-effective electricity supply, and optimal power flow (OPF) is vital for achieving this goal. By optimizing a power system's operation, OPF helps minimize power losses, reduce costs, and improve the overall stability and performance of the system.

In recent years, power transmission systems have been under continuous changes, reinforcements, and renewals due to the growth of electricity demand, integration of renewable energy resources, addition of generation, and decommissioning of power plants. Thus, the system must respond to these changes while maintaining optimal operation [Ebeed et al. 2018].

Since the first bases proposed by [Carpentier 1962], the OPF problem has been extensively addressed. Consequently, several models have been developed to formulate different optimization problems through different objective functions, sets of decision variables, and types of constraints. Various methods, such as the Newton method, linear programming (LP), nonlinear programming (NLP), quadratic programming (QP), and the interior point method, have been proposed for the analysis of the OPF [Burchett et al. 1984, Contaxis et al. 1986, Momoh et al. 1999, Sun et al. 1984, Vargas et al. 1993]. However, since the OPF is a nonlinear and nonconvex optimization problem that was classically formulated for symmetric and balanced alternate current (ac) systems, and despite the advances achieved, the ac OPF formulation is not yet widely used and suitable in real-time operations for large-scale power systems. For large systems, methods based on metaheuristics are commonly used [Reddy and Bijwe 2016], although they have the disadvantage of not guaranteeing global optimality.

Instead, a simplified version of the OPF problem based on LP is often used. The prevalent argument for using the LP-based direct current (dc) OPF instead of the NLP-based ac OPF is that LP algorithms are deterministic and always give solutions, but not necessarily the desired ones, while NLP algorithms are less robust and frequently suffer from convergence problems [Wang et al. 2007]. The constraints of the dc OPF problem are linear approximations of the ac nonlinear power flows with respect to voltage magnitudes, voltage angles, admittances, and reactive power. Thus, the linearized version of the OPF has a wide application in electricity markets and power grid analysis and planning. However, the feasibility and applicability of the dc OPF should be further studied.

On the other hand, the linear sensitivity factors are widely addressed to analyze contingency and security constraints [Mehrtash et al. 2020, Thomas and Grijalva 2015]. These factors are derived from the dc power flow and provide a quick calculation to determine the power flow in any line for a system subject to possible interruptions or line outages. Among these factors, the power transfer distribution factor (PTDF) was also addressed in several topics, such as economic dispatch [Hogan 2000], OPF [Christie et al. 2000, Wood et al. 2013], [Giuntoli et al. 2019], transmission expansion planning [Rahmani et al. 2016], and security-constrained generation capacity expansion planning [Hinojosa and Velásquez 2016].

The main feature of the PTDF calculation is that it depends on two aspects: the operational conditions and the network topology [Hogan 2000]. Thus, with any variation of the system topology, there will be different values of PTDF because the network configuration changes. However, it is known that for fixed topology, the PTDF factor remains relatively insensitive to the operating point. Furthermore, it is also known that PTDF calculations depend on the operating point at which they are evaluated. Thus, PTDF factors evaluated at an initial point using the decoupling assumptions are called dc PTDF factors. Then, when the full coupled model is used, and sensitivities are evaluated at the current operating point, ac PTDF factors are obtained. Therefore, for systems with fixed topology, the formulated dc OPF problem based on the PTDF factor has been studied in recent years [Mehrtash et al. 2020, Rahmani et al. 2016, Thomas and Grijalva 2015] since it offers additional advantages, such as a reduction in the complexity of the optimization problem, i.e., a decrease in the number of decision variables.

This work presents an analysis of dc OPF based on PTDF, and one strategy is proposed to reduce the size of the problem. It consists of considering the knowledge of the system operator

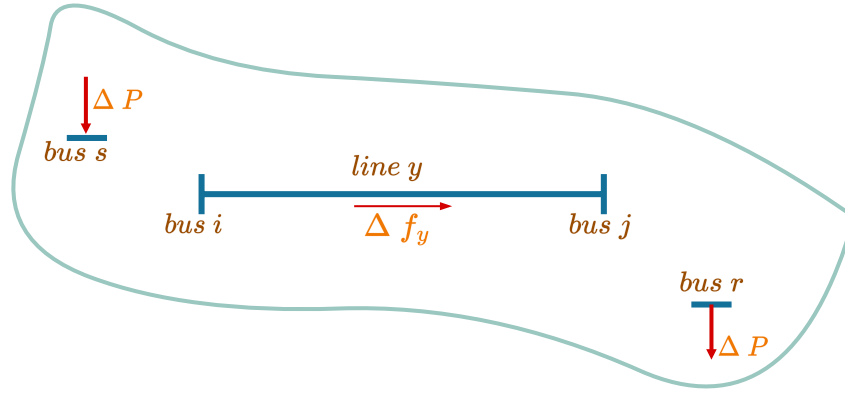


Fig. 1: Effect of flow on transmission line y due to a power transfer between buses s and r

about the lines' loading. Therefore, these lines are considered more relevant in analyzing the dc OPF problem, being called *monitored lines*.

The organization of this paper is as follows: Section 2 presents the concept of linear sensitivity factors and PTDF; Section 3 presents the traditional dc OPF formulation and the formulation based on PTDF; Section 4 presents the strategy proposed for reducing the size of the dc OPF model based on PTDF. The results and discussions obtained with the proposed approach for three systems with different complexity are shown in Section 5. Finally, Section 6 presents the conclusions of this article.

2. Linear Sensitivity Factors

Linear sensitivity factors are a part of sensitivity analysis used to determine the change in power flow on a specific line with respect to some event in the power system. These factors are derived from the dc power flow and can simplify the calculation in some critical problems, such as the OPF problem. Among these sensitivity factors, the PTDF stands out [Chychykina et al. 2016, Mehrtash et al. 2020]. In this section, the PTDF will be discussed in more detail to be used later in the OPF problem.

2.1 Power transfer distribution factor

In general, the PTDF determines a relationship between changes in the power flow of a given transmission line and changes in power injections for a given network configuration. In Fig. 1, the line y is connected between buses i and j , and an amount of power, ΔP , is transferred from bus s to bus r .

The PTDF estimates the portion of transferred power flowing through a transmission line y [Wood et al. 2013]. Then, the PTDF factor can be defined as shown in (1).

$$\psi_{s,r,y} = \frac{\Delta f_y}{\Delta P} \quad (1)$$

where $\psi_{s,r,y} \in [-1,1]$, Δf_y is a variation of the power flow of line y , and $\psi_{s,r,y}$ is the PTDF of line y when a power variation is made from bus s to bus r . The new power flow of line y after the transfer occurs will be $\hat{f}_y = f_y^0 + \psi_{s,r,y}\Delta P$, where f_y^0 is the initial power flow, before the power transfer.

The PTDF factors can be calculated based on the line parameters. For such, the system is modeled based on the linear dc power flow. Then, the linear dc power flow equations can be used to obtain the PTDF. The variation of the bus phase angles is given by (2).

$$\Delta\theta = [X]\Delta P \quad (2)$$

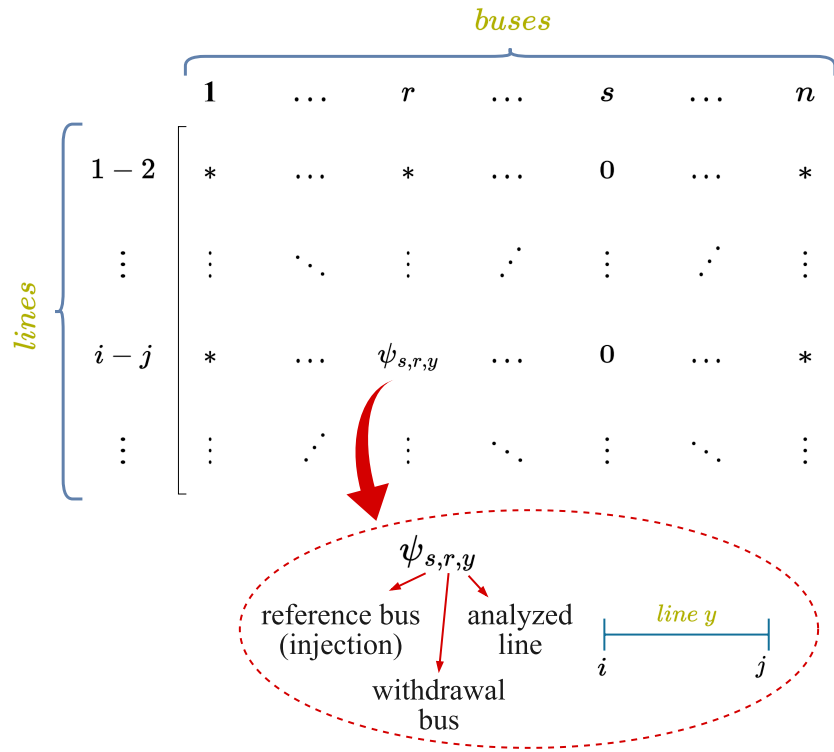


Fig. 2: PTDF matrix

To simplify the calculations, it is assumed that the power transfer from bus s to bus r is equal to 1 MW. Then, the power variation would be as shown in (3).

$$\Delta P = \begin{bmatrix} 0 \\ \vdots \\ +1 \\ -1 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} s \\ r \end{matrix} \quad (3)$$

Hence, from (2) and (3), the variations in phase angles would be as shown in (4).

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_n \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & & X_{2n} \\ \vdots & & \ddots & \vdots \\ X_{n1} & & \cdots & X_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ +1 \\ -1 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} s \\ r \end{matrix} \quad (4)$$

Further, the variations in phase angles of bus i and bus j can be expressed as $\Delta\theta_i = X_{is} - X_{ir}$ and $\Delta\theta_j = X_{js} - X_{jr}$ respectively, and the change in the power flow of line y is $\Delta f_y = 1/x_y (\Delta\theta_i - \Delta\theta_j)$. Therefore, the PTDF would be as shown in (5).

$$\psi_{s,r,y} = \frac{1}{x_y} [(X_{is} - X_{ir}) - (X_{js} - X_{jr})] \quad (5)$$

where X_{is} , X_{ir} , X_{js} , and X_{jr} are the terms of the reactance matrix X . The matrix X can be obtained as the inverse of a matrix which is a modification of the susceptance matrix B since this matrix is singular. Matrix B can be written as $B_{kk} = \sum_{m \in \Omega_k} x_{km}^{-1}$ and $B_{km} = B_{mk} = -x_{km}^{-1}$, then its modification consists of replacing a very large value on the diagonal element corresponding to the reference bus while the other elements in the row and column corresponding to the reference bus remain

equal. Hence, the matrix with the PTDF factors is filled in, as shown in Fig. 2, where the rows and columns correspond to the system lines and power draw buses, respectively. Also, it is essential to mention that by the superposition property, when the outage of a large generating unit occurs, it is considered that all power changes would be made up by the reference generation. Thus, the variations of the power flow in any bus will be analyzed based on the reference bus.

3. OPF FORMULATIONS

The OPF problem focuses on determining the optimal operating state of an electrical power system subject to technical and physical constraints. In this work, the main objective is the minimization of generation costs. The dc OPF formulation is based on LP, which is fast and ensures global optimality when the model is adequately defined [Thomas and Grijalva 2015].

3.1 Traditional dc OPF formulation

The traditional dc OPF formulation is an approximation of the nonlinear ac OPF problem [Kile et al. 2014], where it is assumed that the lines' resistances are much smaller than their reactances, i.e., $r_{ij} \ll x_{ij}$. Then, the line susceptance can be approximated by $-1/x_{ij}$. Additionally, the following considerations are taken into account:

- The voltage magnitudes at the buses are close to 1 p.u.;
- The differences in the voltage angles between buses are small;
- The $Q \times V$ relationship is neglected. Thus, the reactive power is eliminated from the problem.

The traditional dc OPF model, taking into account the previous considerations, is present in (6)–(11).

$$\text{minimize } v = \sum_{i \in \Omega_g} c_i g_i \quad (6)$$

subject to

$$\sum_{j \in \Omega_l} f_{ji} - \sum_{ij \in \Omega_l} f_{ij} + g_i = d_i \quad \forall i \in \Omega_b \quad (7)$$

$$f_{ij} = n_{ij}^0 \frac{\theta_i - \theta_j}{x_{ij}} \quad \forall ij \in \Omega_l \quad (8)$$

$$|f_{ij}| \leq n_{ij}^0 \bar{f}_{ij} \quad \forall ij \in \Omega_l \quad (9)$$

$$\underline{g}_i \leq g_i \leq \bar{g}_i \quad \forall i \in \Omega_g \quad (10)$$

$$\theta_{ref} = 0 \quad (11)$$

where (6) is the objective function to be optimized and represents the system operation cost. Constraint (7) is the active power balance at each bus of the transmission system, representing the application of Kirchhoff's current law to the system, (8) represents the application of Kirchhoff's voltage law to each fundamental loop of the system. Constraint (9) is the transmission capacity of the branches of the system, while (10) is the generation capacity limit of the buses, and (11) represents the angular reference of the system. At the buses without generation, $g_i = 0$.

Since voltage angles, power generations, and power flows on the branches are variables in this model, there are $|\Omega_b| + |\Omega_g| + |\Omega_l| - 1$ decision variables, of which $|\Omega_l| + |\Omega_g|$ are bounded variables, and $|\Omega_b| + |\Omega_l|$ constraints in this model, which may cause high computational effort to solve the problem. However, in recent years, dc OPF formulations based on PTDF have been studied since they offer additional advantages, such as reducing the complexity of the optimization problem by reducing the number of decision variables and constraints.

3.2 DC OPF formulation based on PTDF

The PTDF has been used as an alternative to eliminate the voltage phase angle from the OPF model, which means that the formulation based on PTDF solves the problem by considering only the active power generations as the decision variables. In this part, for the formulation of the dc OPF problem, it is adopted the notation for the PTDF factor on branch ij as $\psi_{ij,k}$, where k represents the power injection/extraction bus. The PTDF-based OPF formulation is presented in (12)–(15).

$$\text{minimize } v = \sum_{i \in \Omega_g} c_i g_i \quad (12)$$

subject to

$$\sum_{i \in \Omega_g} g_i - \sum_{i \in \Omega_b} d_i = 0 \quad (13)$$

$$\left| \sum_{k \in \Omega_b} \psi_{ij,k} (g_k - d_k) \right| \leq \bar{f}_{ij} \quad \forall ij \in \Omega_l \quad (14)$$

$$\underline{g}_i \leq g_i \leq \bar{g}_i \quad \forall i \in \Omega_g \quad (15)$$

where constraint (13) is the global active power balance of the network, constraint (14) represents the limits of power flows on the branches, and constraint (15) is the generation limit and represents the limits of variables g_i , which are the only variables in the problem. At the buses without generation, $g_i = 0$.

Note that, in contrast to constraint (7), constraint (13) represents the global balance of active power in the system, with only one constraint of this type in the model. Therefore, the resulting model presents only $|\Omega_g|$ bounded variables and $|\Omega_l| + 1$ constraints. Moreover, constraint (14) does not depend on the phase angles of the bus voltages and only requires the values of the matrix with the PTDF given in (5).

4. DC OPF Based on PTDF Considering Monitoring Lines

As discussed, the PTDF-based OPF formulation presents a reduction in the number of variables and constraints with respect to the traditional OPF formulation. However, constraint (14) implies that for each branch ij , there will be one constraint that depends on $\psi_{ij,k}$ and all generation values. The size of the problem can be further reduced if the load factor information of each line is considered. If the system operator knows the loading of the lines, constraint (14) can be disregarded for lightly loaded lines with no impact on the optimal solution of the problem since this constraint will only be active for the lines that are operating at their maximum capacity. Therefore, only heavily loaded lines should be monitored and considered in the OPF model.

Fig. 3 presents a flowchart of the proposed approach for efficiently solving the PTDF-based dc OPF formulation. It is shown that, first, the parameters of the network topology are read and the PTDF matrix of the network is obtained. It should be noted that the PTDF matrix does not change for a given system, even if its demand and generation change. Then, the parameters of the OPF problem are read. Next, the OPF problem formulation based on PTDF is built. Based on the knowledge of the system operator, those lines that usually operate a limit σ of their capacity will be considered monitored lines. Finally, the problem is solved, and the results are printed.

To consider monitored lines in the problem (12)–(15), constraint (14) is redefined to be considered for $\forall ij \in \Omega_l |f_{ij}| \leq \sigma \bar{f}_{ij}^*$, in which $\bar{f}_{ij}^*$ is an estimate of the power flow on branch ij based on the system's operator knowledge. With this change, constraint (14) will be considered only for heavily loaded lines.

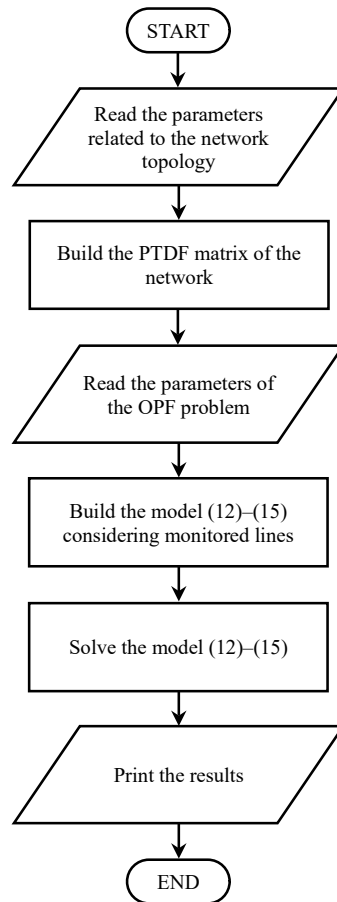


Fig. 3: Flowchart of strategy considering monitoring lines

The consideration of monitored lines will significantly impact the PTDF-based dc OPF model size for large power systems. For instance, in a system with 12,000 buses and 15,000 branches in which only 50 lines operate close to or at their maximum capacities, the dc OPF model based on PTDF considering monitored lines would have only 51 general constraints, besides $|\Omega_g|$ bounded variables. In contrast, the traditional dc OPF model would present 12,000 power balance constraints and 15,000 constraints related to power flows on branches.

5. Tests and Results

Three transmission systems were used to test the proposed approach for PTDF-based dc OPF considering monitored lines: the IEEE 30-bus, the IEEE 588-bus, and the IEEE 10,000-bus transmission systems.

The calculations of the PTDF matrices were performed in MATLAB®, and the time of this process was insignificant to be considered in the computational efficiency, while the traditional dc OPF and the PTDF-based dc OPF formulations were implemented in AMPL and solved using the solver CPLEX v20.1. The tests were conducted using a computer with a 3.20 GHz Intel® Core™ i7-8700 processor and 32 GB of RAM. The lines with loading higher than $\sigma = 90\%$ of their capacity were considered monitored lines.

5.1. IEEE 30-bus transmission system

Table 1 shows the results for the IEEE 30-bus transmission system, which was tested using the traditional dc OPF formulation, the formulation based on PTDF, and the formulation based on PTDF considering monitored lines. Note that the traditional formulation presents 31 decision variables, higher than the number verified in the formulations based on PTDF; also, the number of constraints and nonzero elements is reduced considerably.

Moreover, the PTDF formulation considering monitored lines presents the same number of variables, constraints, and nonzero elements with respect to when all lines are considered. In this case, the formulation presents the same dimension as when all lines are monitored due to *pre-solve* being able to reduce the size of the original problem.

Table 1: Results for the IEEE 30-bus transmission system

Formulation	Number of variables	Number of constraints	Nonzero elements	Objective function (US\$/h)	Time (%)	Number of monitored lines
Traditional	31	107	265	7,506.48	100.00	41
Based on PTDF	2	1	2	7,506.48	51.61	41
Based on PTDF considering monitored lines	2	1	2	7,506.48	51.61	1

In all cases, the same objective function was obtained, but the PTDF-based formulations were able to solve the problem in 51.61% of the traditional formulation, thus demonstrating the effectiveness and efficiency of the PTDF-based dc OPF model.

5.2. IEEE 588-bus transmission system

Table 2 shows the results for the IEEE 588-bus transmission system. Note that the traditional formulation presents 710 decision variables for this system, which is more than five times higher than the number of variables in the PTDF-based formulations. Also, the number of constraints is reduced considerably. However, the number of nonzero elements is higher in the PTDF-based formulation without considering monitored lines in relation to the traditional formulation because (14) presents a great number of variables, thus increasing the number of nonzero elements in the constraint. On the other hand, in the PTDF-based formulation considering monitored lines, the number of constraints and nonzero elements are reduced considerably. Regarding the objective function, the PTDF-based formulations present a value that is only 0.05% higher than the traditional formulation, obtained in 67.39% of the time when considering 26 monitored lines.

Table 2: Results for the IEEE 588-bus transmission system

Formulation	Number of variables	Number of constraints	Nonzero elements	Objective function (US\$/h)	Time (%)	Number of monitored lines
Traditional	710	1,947	4,778	246,596.99	100.00	686
Based on PTDF	124	295	35,296	246,706.37	100.00	686
Based on PTDF considering monitored lines	124	24	2,713	246,706.37	67.39	26

5.3. IEEE 10,000-bus transmission system

Table 3 shows the results for the IEEE 10,000-bus system, which is a large-sized transmission system. Note that, again, the number of variables and constraints are reduced considerably in the PTDF-based formulations. However, the number of nonzero elements in the problem is more

than 65 higher in the PTDF-based formulation without considering monitored lines than in the traditional formulation. In contrast, when considering monitored lines, the number of constraints and nonzero elements is reduced considerably; thus, the computational time obtained with six monitored lines in the PTDF-based formulation is only 4.49% of the time required to solve the traditional model. The objective function obtained with the PTDF-based dc OPF formulation considering monitored lines is only 0.15% higher than the objective function of the solution of the traditional formulation.

Table 3: Results for the IEEE 10000-bus transmission system

Formulation	Number of variables	Number of constraints	Nonzero elements	Objective function (US\$/h)	Time (%)	Number of monitored lines
Traditional	11,355	36,335	89,508	676,471.48	100.00	13,193
Based on PTDF	1,356	4,335	5,873,813	685,068.38	1,013.02	13,193
Based on PTDF considering monitored lines	1,356	7	9,485	677,456.72	4.49	6

6. Conclusions

This work presented an approach for reducing the size of the dc optimal power flow (OPF) problem formulation based on the power transfer distribution factor (PTDF). The knowledge of the lines' loading is considered, and the heavily loaded lines are monitored.

Traditional dc OPF and PTDF-based dc OPF formulations were presented and compared with the proposed strategy of considering monitored lines. Three systems, with different degrees of complexity, were used to evaluate the proposal. The results indicate that the proposal effectively reduces the problem's size and the computational effort required to solve it. Although for large systems there is a slight difference in the objective function of the solution obtained with the PTDF-based formulation considering monitored lines with respect to the traditional formulation, this is compensated by the high reduction in the computational effort to solve the problem. This slight difference can be explained by numerical errors when constructing the PTDF matrix.

For future works, this strategy could be applied to the transmission system expansion planning problem and extended using other linear sensitivity factors in the security-constrained OPF problem.

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