MIP model for integrated production and distribution problem with order consideration and cargo arrangement

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RESUMO

Este trabalho apresenta um modelo de programação inteira mista para apoiar decisões integradas de produção e distribuição a nível operacional. São analisados o atendimento de pedidos, as restrições de *carryover* dependentes da sequência e unitização de carga de transporte nas decisões de dimensionamento de lotes e distribuição. Uma análise comparativa do modelo integrado é realizada com três cenários de modelos hierárquicos. Além disso, é apresentado um estudo computacional para avaliar o comportamento do modelo, mostrando que o modelo integrado fornece boas soluçõeses para o planejamento da produção e distribuição.

PALAVRAS CHAVE. Problemas integrados. Produção e distribuição de pedidos. Unitização de cargas.

Logística e transporte. Programação Matemática.

ABSTRACT

This paper presents a mixed-integer programming model to support integrated production and distribution decisions at an operational level. We analyze order fulfillment, carryover setup sequence-dependent constraints, and cargo arrangement in lot sizing and distribution decisions. A comparative analysis of the integrated model is performed with three scenarios of hierarchical models. In addition, a computational study is presented to evaluate the model behavior.

KEYWORDS. Integrated problems. Order production and distribution. Cargo arrangement.

Logistics and transports. Mathematical programming.

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1. Introduction

In a process industry with sequential production, storage, and distribution activities, operations planning is usually performed individually, from predetermined decisions from previous activities. However, production and distribution are two functions that need to be viewed in an integrated way to obtain better operational performance of the organization [Chen, 2010]. Over the years, the concern in integrated production and distribution (IPD) problems has grown. [Chen, 2010], [Fahimnia et al., 2013] and [Darvish et al., 2021] present literature reviews about IPD problems, showing model variations related to the IPD problems applied to supply chain management. However, those surveys do not cover the IPD with order considerations and cargo arrangement.

By covering lot sizing problems integrated with transportation decisions, [Amorim et al., 2013] and [Belo-Filho et al., 2015] explore lot sizing in IPD operational-level problems. The lot sizing decisions must define the amounts of items to be produced and the production sequence in the machines. Thus, a planner must simultaneously decide on production planning and vehicle routing. Recently, [Bank et al., 2022] presented two MIP models and optimality properties for the lot sizing IPD problem. These papers focus on the routing vehicles instead of the loading problem.

Concerning cargo arrangement, [Molina et al., 2016] propose MIP models for lot sizing problems with distribution costs using unit load devices such as pallets and containers. Lot sizing decisions must respect limited capacity and meet the product demand. Therefore, distribution decisions are related to the loading and transporting of products in cargo unitization.

Another important aspect of operations management is order fulfillment [Amorim et al., 2013]. To the best of our knowledge, the grouping of the items in the orders is done for batch delivery, and order fulfillment are few explored in lot sizing problems. A MIP formulation for order production planning problems in the foundry industry is presented by [Furtado et al., 2019].

This paper presents a mixed-integer programming (MIP) model to support integrated production and distribution decisions at an operational level, trying to fill a literature gap. Therefore, a significant contribution of this paper is to incorporate order fulfillment in lot sizing and distribution decisions, carryover setup sequence-dependent constraints, and distribution decisions based on cargo unitization pallets. A comparative analysis of the integrated model is performed with three scenarios of hierarchical models. In addition, a computational study is presented to evaluate the MIP behavior.

The remainder of this paper is organized as follows. Section 2 describes the considered problem and the mathematical formulation for the production and distribution problem with order consideration and cargo arrangement. Then, in Section 3, the results and analysis of the computational study are presented. Finally, the paper is concluded in Section 4 with the main findings and ideas for future work.

2. Problem definition

The problem considered in this paper consists of nonidentical parallel capacitated machines producing multiple items. The items are grouped in customer orders. The big bucket planning horizon is finite, and the dynamic demand is deterministic. Also, the inventory capacity is limited. Multiple items are produced in the same period, but each machine can produce only one type of item per load. The setup changeover (time and cost) between different items on a machine depends on the production sequence. In addition,

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machine configuration is carried out from the end of a previous period until the beginning of the next.

In distribution decisions, items are loaded into pallets without mixing item types. The distribution is performed using several heterogeneous vehicles loaded with pallets. A vehicle serves only one order per period. Orders must be delivered on the programmed due date. Otherwise, the delay incurs costs. Thus, the model must provide a plan with lot sizing and delivery programming with minimum cost to support the production and distribution planning.

Consider the following indices, parameters and decision variables used in the Integrated model.

Parameters

$i, j = 1, \cdots, N$: items
$m = 1, \cdots, M$: machines
$k = 1, \cdots, K$: orders
$v = 1, \cdots, V$: vehicles
$t, t' = 1, \cdots, T$: periods
d_{ik} : demand (kg) for item i in order k
p_{im} : processing time to produce 1kg of item i in machine m
s_{ijm} : setup time of a changeover from item i to j in machine m
c_{ijm} : setup cost of a changeover from item i to j in machine m
cap_m : production capacity time of machine m
ch : total storage capacity
b_i : Maximum amount of (kg) items of type i that can be loaded on the same pallet
r_v : Number of vehicles of type v available for delivery
q_v : Maximum number of pallets that can be transported on a vehicle v
dd_{kt} : 1 if the due date of order k is planned to period t, 0 otherwise
h_i : holding cost of 1kg of item i
tr_{vkt} : cost to use vehicle of type v, for delivery the order k, in period t
dc_{kt} : backorder cost of the order k in period t

Decision variables

XO_{kt} : takes 1, if order k is delivered in period t; 0 otherwise
BO_{kt} : takes 1, if order k is delayed in period t; 0 otherwise
X_{imt} : amount (kg) of item i to be produced in machine m at period t
Y_{ijmt} : takes on 1, if there is a changeover from item i to item j in machine m in period t; 0 otherwise
Z_{imt} : takes 1, if item i is prepared to produce into machine m at the beginning of period t; 0 otherwise
V_{imt} : auxiliary variable to design the item i to machine m in period t
I_{it} : amount (kg) of item i held at the end of the period t
W_{ikt} : amount (kg) of item i delivered into order k in period t
A_{ikt} : number of pallets loaded with item i, delivered to meet order k, in period t
F_{ikt} : fraction of uncompleted pallets with item i, delivered to meet order k, in period t
B_{vkt} : number of vehicles of type v used to delivery order k in period t

Integrated model is given by:

$$min \quad \sum_i \sum_t h_i I_{it} + \sum_i \sum_j \sum_m \sum_t c_{ijm} Y_{ijmt} + \sum_v \sum_k \sum_t tr_{vkt} B_{vkt} + \sum_k \sum_t dc_{kt} BO_{kt} \quad (1)$$

S.t:

$$I_{it-1} + \sum_m X_{imt} = \sum_k W_{ikt} + I_{it} \qquad \forall i, \quad \forall t \qquad (2)$$

$$\sum_i I_{it} \leq ch \qquad \forall t \qquad (3)$$

$$\sum_i p_{im} X_{imt} + \sum_i \sum_j s_{ijmt} Y_{ijmt} \leq cap_m \qquad \forall m, \quad \forall t \qquad (4)$$

$$X_{imt} \leq \sum_k d_{ik} (Z_{imt} + \sum_j Y_{jimt}) \qquad \forall i, \quad \forall m, \quad \forall t \qquad (5)$$

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$$\sum_{i} Z_{imt} = 1 \qquad \forall m, \quad \forall t \tag{6}$$

$$Z_{imt} + \sum_{j} Y_{jimt} = Z_{im(t+1)} + \sum_{j} Y_{ijmt} \qquad \forall i, \quad \forall m, \quad \forall t \tag{7}$$

$$V_{imt} + N \cdot Y_{ijmt} - (N-1) - N \cdot Z_{imt} \leq V_{jmt} \qquad \forall i, j \quad i \neq j, \quad \forall m, \quad \forall t \tag{8}$$

$$\sum_{m} \sum_{t} X_{imt} \leq \sum_{k} d_{ik} \qquad \forall i \tag{9}$$

$$\sum_{t} W_{ikt} \leq d_{ik} \qquad \forall i, \quad \forall k, \quad \forall t \tag{10}$$

$$\sum_{t'=1}^{t} W_{ikt'} \geq d_{ik} XO_{kt} \qquad \forall i, \quad \forall k, \quad \forall t \tag{11}$$

$$\sum_{i} W_{ikt} \geq XO_{kt} \qquad \forall k, \quad \forall t \tag{12}$$

$$A_{ikt} = \frac{W_{ikt}}{b_i} + F_{ikt} \qquad \forall i, \quad \forall k, \quad \forall t \tag{13}$$

$$\sum_{v} q_v B_{vkt} = \sum_{i} A_{ikt} \qquad \forall k, \quad \forall t \tag{14}$$

$$\sum_{k} B_{vkt} \leq r_v \qquad \forall v \quad \forall t \tag{15}$$

$$XO_{kt} + BO_{kt} = dd_{kt} + BO_{kt-1} \qquad \forall k \quad \forall t \tag{16}$$

$$X_{imt}, I_{it}, F_{ikt}, W_{ikt}, V_{imt}, \geq 0, I_{i0} = 0 \tag{17}$$

$$A_{ikt}, B_{vkt} \in \mathbb{Z}^{+}, \tag{18}$$

$$Y_{ijmt}, Z_{imt}, XO_{kt}, BO_{kt} \in \{0, 1\} \tag{19}$$

Objective function (1) aims to minimize the total costs of inventory, machine setup, delivery, and delivery delay, respectively. Constraints (2) represent the inventory balancing, establishing that the total amount produced and stored is sufficient to deliver the items in period t. The total capacity to store the items is limited by constraints (3). Constraints (4) are the time capacity constraints of the machines, considering processing and the changeover times of the items.

Constraints(5) guarantee that the production of an item i can only occur if machine m is prepared for this item, while constraints (6) - (8) determine the sequence of items to be produced on each machine in each period. Constraints (6) assure that each machine can only be set up for a single item at the beginning of each period. Constraints (7) ensure a balanced network flow of the machine configuration states and carry the setup configuration state of the machine to the next period. Constraints (8) avoid sub-tours from occurring in item preparation sequence.

Constraints (9) ensure that the total produced is at most the amount demanded by each item. Constraints (10) ensure that the amount of items delivered for each order does not exceed the quantity demanded per item. Constraints (11) determine that an order is only considered complete when all items in the order are delivered. Constraints (12) impose that at least one delivery is made on the period the order is considered complete. The number of pallets loaded with item i, delivered to meet order k, in period t is defined by constraints (13). If pallets are not fully loaded $F_{ikt} > 0$.

The entire vehicle capacity, in pallets, will be transported. Such condition is

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ensured by constraints (14). Constraints (14) define the number of vehicles needed to perform the deliveries for each order, and the total number of vehicles is bounded by constraints (15).

Constraints (16) represent the flow balance in the delivery of orders. When an order k is delivered in period t, $XO_{kt} = 1$. Note that this constraint only allows the order to be complete on the planned due date or it is delayed. After the expected due date, as long as the order is not delivered, $BO_{kt} = 1$. Finally, the non-negativity and variable dominance conditions are represented in constraints (17) - (19).

2.1. Example problem

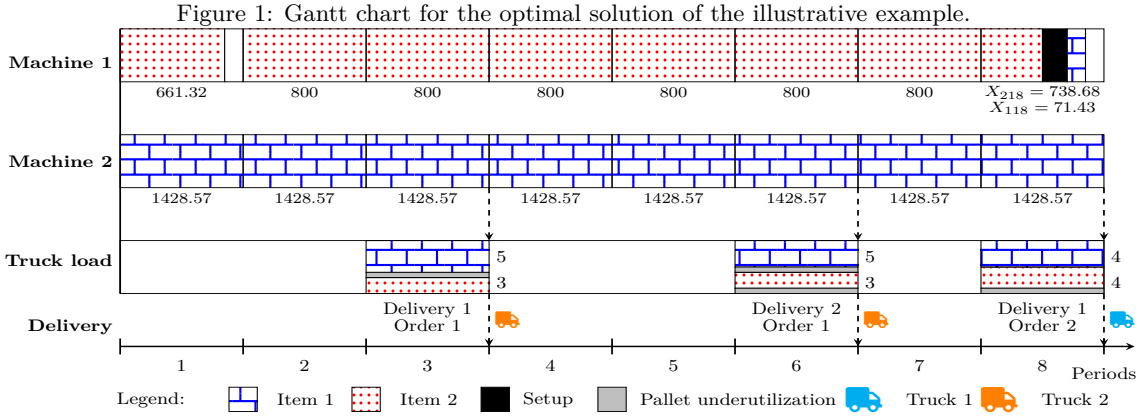
In this section, a small data set is considered, with two machines with capacities of 2000 and 3000 time units, respectively. The planning horizon has $T = 8$ periods, and the storage capacity is not bound. Order $k = 1$ has due date at $t = 3$ and order $k = 2$ at $t = 8$. If orders are not delivered on time, costs of $dc_{kt} = 15000$ occur per delayed period. When an order is not delayed, $dc_{kt} = 100000$ (prohibitive delay cost). The rest of the example data is given in Table 1, s_{ijm}/c_{ijm} is valid for all m and tr_{vkt} for all t.

Table 1: Data for illustrative example with two items, two production machines, and two vehicles.

	d_{ik}		s_{ijm}/c_{ijm}			
	$k=1$	$k=2$	$j=1$	$j=2$	h_i	b_i
$i=1$	8000	3500	0/0	1.3/2	0.001	880
$i=2$	4000	2200	22	0/0	0.001	704
			tr_{vkt}			
	q_v	r_v	$k=1$	$k=2$		
$v=1$	8	1	1000	1100		
$v=2$	8	1	800	1200		

Using a commercial solver to solve the problem, the optimal solution found is shown in Figure 1. The initial configuration of each machine at the beginning of period t is given by $Z_{12t} = 1$ and $Z_{21t} = 1$. There was idle time on machine 1 at the end of periods $t = 1$ and $t = 8$ and a configuration changeover from item $i = 2$ to $j = 1$ on machine 1 at $t = 8$ ($Y_{2118} = 1$). Deliveries occur at periods $t = 3$ and $t = 6$ to meet order $k = 1$ and at period $t = 8$ to meet order $k = 2$. In other periods, production quantities are stored to delivery the orders in due date. Table 2 displays the values of variable I_{it} in the optimal solution of the example.

Figure 1: Gantt chart for the optimal solution of the illustrative example.



To meet the demand of order $k = 1$ two deliveries are planned using vehicle $v = 2$. The first delivery is made on the due date $t = 3$, but the order will only be considered complete at $t = 6$, when the entire amount demanded is fully delivered, so

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Table 2: Amount (kg) of items held at the end of each period.

Item	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8
1	1428.57	2857.14	0.00	1428.57	2857.14	571.43	2000.00	0.00
2	661.32	1461.32	149.32	949.32	1749.32	661.32	1461.32	0.00

Table 3: Amount of each item delivered to meet each order.

			Pallets	
		Weight (kg)	**Delivered**	**Not completed**
$k = 1$		(W_{ikt})	(A_{ikt})	(F_{ikt})
Delivery 1	Item 1	4285.71	5	0.13
	Item 2	2112.00	3	0.00
Delivery 2	Item 1	3714.29	5	0.78
	Item 2	1888.00	3	0.32
$k = 2$				
Delivery 1	Item 1	3500.00	4	0.02
	Item 2	2200.00	4	0.88

$BO_{13} = BO_{14} = BO_{15} = 1$ and $XO_{16} = 1$. For order $k = 2$, a single delivery with vehicle $v = 1$ is planned on the due date at $t = 8$, incurring no delay delivery cost, i.e. $XO_{28} = 1$ and $BO_{2t} = 0$ for all t.

In each delivery planned, the items are unitized on pallets. The pallets are arranged inside the trucks according to the values shown in Table 3. Variables A_{ikt} determine the number of pallets needed to transport the load W_{ikt} given in kilograms. For example, for order $k = 1$, $4285.71kg$ of item 1 must be delivered over 5 pallets. These 5 pallets are not complete, leaving empty $F_{113} = 0.13$ of the pallets. Also, 2112 kg of item 2 is transported in this same delivery on exactly $A_{123} = 3$ full pallets. The total cost of this example in the optimal solution is 47720.24 units.

It is common for companies to solve the production and distribution problems hierarchically. First, the amount produced on each machine and other production decisions are taken, and then the delivery loads are determined. In this sense, we analyzed the integrated model with three scenarios of non-integrated models. In the first scenario, we use the classical capacitated lot sizing (CLSP) and distribution models (DP). In the second and third scenarios, we consider orders in the lot sizing stage based on the model proposed by [Furtado et al., 2019]. The difference between the second and third scenarios is the distribution stage. In the second scenario, the inventory of the items is available for delivery at any time. In the third scenario, the inventory of the items is available only when the production stage completes an order for these items.

i) First scenario of non-integrated production and distribution problem (CLSP + DP1)

CLSP

$$min \sum_{i=1}^{N}\sum_{t}^{T} h_i I_{it} + h_i^- I_{it}^- + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} c_{ijm} Y_{ijmt} \quad (20)$$

S.t:

$$I_{it-1} - I_{it-1}^- + \sum_m X_{imt} = dem_{it} + I_{it} - I_{it}^- \quad \forall i, \forall t \quad (21)$$

$$(3) - (8)$$

$$X_{imt}, I_{it}, V_{imt}, \geq 0, \quad (22)$$

$$Y_{ijmt}, Z_{imt} \in \{0, 1\} \quad (23)$$

DP1

$$min \quad \sum_{v=1}^{V}\sum_{k=1}^{K}\sum_{t}^{T} tr_{vkt}B_{vkt} + \sum_{k=1}^{K}\sum_{t}^{T} dc_{kt}BO_{kt} \tag{24}$$

S.t:

$$\sum_{k} W_{ikt} \leq \sum_{m} X_{imt}, \forall i, \forall t \tag{25}$$

$$(10) - (16)$$

$$F_{ikt}, W_{ikt} \geq 0, \tag{26}$$

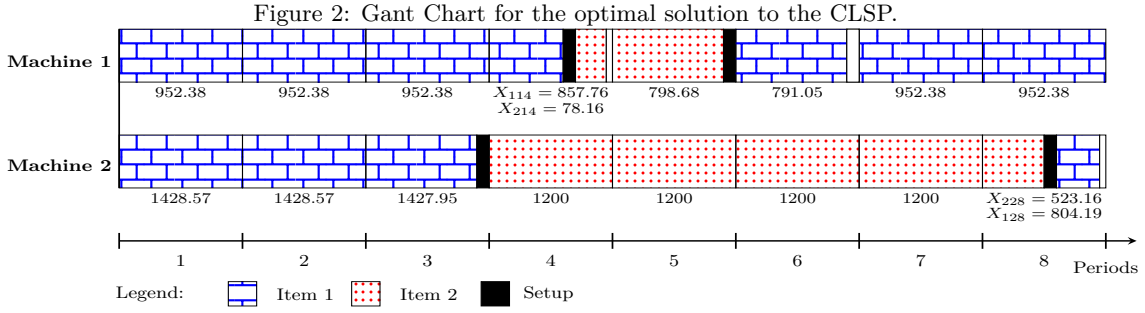
$$A_{ikt}, B_{vkt} \in \mathbb{Z}^{+}, \tag{27}$$

$$XO_{kt}, BO_{kt} \in \{0, 1\} \tag{28}$$

Considering that the model is no longer integrated with delivery decisions, constraint (2) is changed to a common inventory balance constraint represented in (21). The decision variable I_{it}^{-} counts the amount (kg) of item i that is delayed in period t.

The optimal solution to the CLSP is illustrated in Figure 2. The initial configuration of each machine at the beginning of period $t = 1, 2, 3$ is given by $Z_{11t} = 1$ and $Z_{12t} = 1$. There is a configuration changeover from item $i = 1$ to item $j = 2$ on machine $m = 1$ in period $t = 4$ ($Y_{1214} = 1$). Another changeover is made at the end of period $t = 5$ ($Y_{2115} = 1$) that remain the configuration in periods $t = 6$ to $t = 8$. On machine $m = 2$, there is a configuration changeover from $i = 1$ to $j = 2$ at the end of period $t = 3$ ($Y_{1223} = 1$), so the item $i = 2$ is produced on machine 2 in periods $t = 4$ to $t = 8$. In period $t = 8$ a changeover occurs ($Y_{2128} = 1$) and item $i = 1$ is produced on both production machines.

Figure 2: Gant Chart for the optimal solution to the CLSP.



The results of the inventory variables are shown in Table 4. Comparing the results obtained in Table 2, we can see that fewer items are stored, reducing inventory costs. The non-integrated model cannot meet the entire demand of order $k = 1$, in period $t = 3$, similar to the integrated model. Thus $I_{13}^{-} = 857.76$ kg, $I_{23}^{-} = 4000$ kg, $I_{24}^{-} = 2721.84$ kg and $I_{25}^{-} = 723.16$ kg represent the backlog items by the model.

Table 4: Amount (kg) of items held at the end of each period for CLSP.

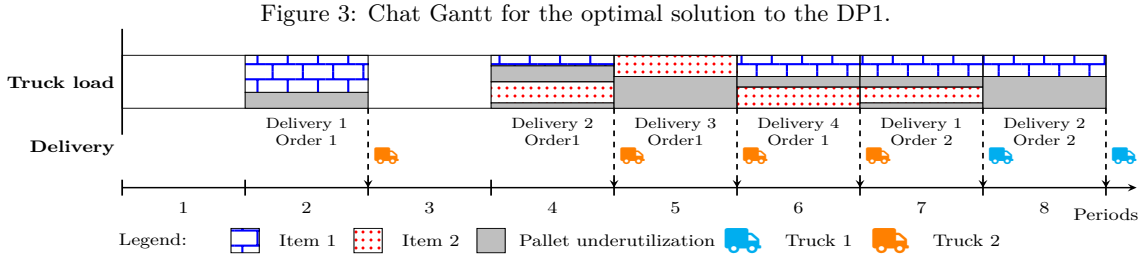
Item	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
1	2380.95	4761.90	0.00	0.00	0.00	791.05	1743.43	0.00
2	0.00	0.00	0.00	0.00	0.00	476.84	1676.84	0.00

After solving the CLSP, the values of the variables X_{imt} are input parameters for the amount that should be unitized on pallets and delivered to customers in the DP1 model. The optimal solution for DP1 is represented in Figure 3. It can be seen that more deliveries are necessary to fulfill the orders compared to the solution obtained by integrated model.

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For example, for order $k = 1$, four deliveries were planned with vehicle $v = 2$, while the integrated model needed 2 deliveries with vehicle $v = 2$. In addition, for order $k = 2$, two deliveries were planned with vehicle $v = 1$, and the integrated model fulfilled this order with one delivery vehicle $v = 1$.

Figure 3: Chat Gantt for the optimal solution to the DP1.



Similar to integrated model order $k = 1$ is complete at $t = 6$, thus $XO_{16} = 1$, this order is delayed by $BO_{13} = BO_{14} = BO_{15} = 1$. Order $k = 2$ is complete at $t = 8$, on the expected due date, so $XO_{28} = 1$ and $BO_{2t} = 0$ for all t.

The objective function for the non-integrated transportation model is 50400.00 cost units, 5.61% higher than the value found by the integrated model, considering the costs related to the production and distribution of the items. Moreover, CLSP does not address order fulfillment and distribution decisions in the production stage. As a result, fewer items are stored, and we have less than the truckload in the distribution stage. For this reason, more trips are needed to fulfill the same order compared to the results obtained by the integrated model.

ii) Second scenario of non-integrated production and distribution problem

OPP

$$min \quad \sum_i \sum_t h_i I_{it} + \sum_i \sum_j \sum_m \sum_t c_{ijm} Y_{ijmt} + \sum_k \sum_t dc_{kt} BO^P_{kt}$$

S.t:

$$I_{it-1} + \sum_m X_{imt} - I_{it} = \sum_k d_{ik} \cdot XO^P_{kt}, \forall i, \forall t \quad (29)$$

$$XO^P_{kt} + BO^P_{kt} = dd_{kt} + BO^P_{kt-1} \forall k, \forall t \quad (30)$$

$(3) - (8)$

$X_{imt}, I_{it}, V_{imt}, \geq 0,$

$Y_{ijmt}, Z_{imt}, XO^P_{kt}, BO^P_{kt} \in \{0, 1\}$

DP2

$$min \quad \sum_{v=1}^{V} \sum_{k=1}^{K} \sum_t tr_{vkt} B_{vkt} + \sum_{k=1}^{K} \sum_t dc_{kt} BO^D_{kt}$$

S.t:

$$\sum_m X_{imt} + I^D_{t-1} - I^D_t = \sum_k W_{ikt} \quad \forall i, \forall t \quad (31)$$

$(10) - (16)$

$F_{ikt}, W_{ikt} \geq 0, A_{ikt}, B_{vkt} \in \mathbb{Z}^+,$

$XO^D_{kt}, BO^D_{kt} \in \{0, 1\}$

In this scenario, we consider that items are grouped into orders in the production stage. For clarity, we classified variables XO_{kt} and BO_{kt} with superscripts P (production) and D(distribution) to represent this situation. The production stage is represented by the order planning problem (OPP) model. Furthermore, the items are available for delivery after production in the distribution stage. Thus, the distribution model decides on delivering or keeping the items in inventory.

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The optimal solution to the OPP is illustrated in Figure 4. The initial configuration of each machine at the beginning of period t is given by $Z_{12t} = 1$ and $Z_{21t} = 1$. There was idle time on machine $m = 1$ at the end of periods $t = 8$ and on machine $m = 2$ at the end of period $t = 1$. There is a configuration changeover from item $i = 2$ to $j = 1$ on machine 1 at $t = 8$ ($Y_{2118} = 1$).

Figure 4: Gant Chart for the optimal solution to the OPP.

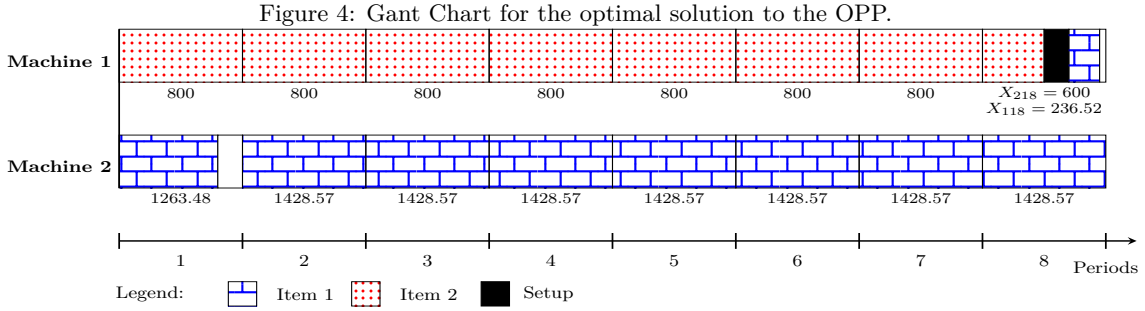


Table 5: Amount (kg) of items held at the end of each period for OPP.

Item	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
1	1263.48	2692.05	4120.62	5549.19	6977.76	406.33	1834.90	0.00
2	800.00	1600.00	2400.00	3200.00	4000.00	800.00	1600.00	0.00

The results of the inventory variables are shown in Table 5. In this scenario, the items are stored until the order is considered complete by production, increasing holding costs. Similarly to the integrated model, order $k = 1$ is completed in $t = 6$, and order $k = 2$ is completed in $t = 8$. If all items are available for distribution after production, the distribution obtains the same outcome as the integrated model. As a result, there is a 0.04% cost saving by the integrated model.

iii) Third scenario of non-integrated production and distribution problem (OPP + DP3)

DP3

$$min \sum_{v=1}^{V}\sum_{k=1}^{K}\sum_{t}^{T} tr_{vkt}B_{vkt} + \sum_{k=1}^{K}\sum_{t}^{T} dc_{kt}BO^{D}_{kt}$$

S.t:

(31)

$$W_{ikt} \leq d_{ik}XO^{P}_{kt}, \forall i, \forall k, \forall t \qquad (32)$$

(10) − (16)

$F_{ikt}, W_{ikt} \geq 0, A_{ikt}, B_{vkt} \in \mathbb{Z}^{+}$,

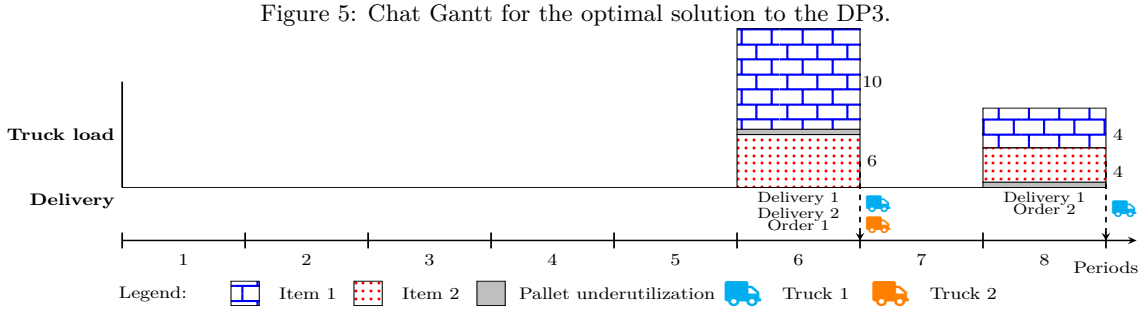
$XO^{D}_{kt}, BO^{D}_{kt} \in \{0, 1\}$

In this scenario, we consider variable XO^{P}_{kt} from the OPP model as a parameter for DP3. $XO^{P}_{kt} = 1$ for all periods starting when the order was completed by production. Constraints (32) assure that distribution occurs when all order items are made. The optimal solution for OPP is shown in Figure 4 and Table 5. Figure 5 exhibits the optimal solution obtained for DP3.

Due to production only releasing order $k = 1$ at period $t = 6$, distribution can only be performed from $t = 6$ to $t = 8$. Two vehicles are available ($v = 1$ and $v = 2$) at $t = 6$; both are used to deliver order $k = 1$. Compared to the integrated model, this solution is more costly since a more expensive vehicle is needed for distribution. On the other hand,

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Figure 5: Chat Gantt for the optimal solution to the DP3.



if one vehicle is unavailable, the order would only be complete at period $t > 6$, resulting in higher penalties for backorder. In this sense, the integrated model provides better vehicle allocation for distribution. Also, order $k = 2$ is delivered at $t = 8$ without delay. The integrated model achieves 0.46% savings in total costs compared to the third hierarchical scenario.

3. Computational study

The objective of the computational tests is to assess how the number of machines and vehicles affects the solution quality by considering integrated production and distribution decisions with cargo unitization and customer orders. For this purpose, a set of instances have been systematically generated with different parameters. The process of data generation is documented as follows.

3.1. Data generation

The instance generator is based on works by [Amorim et al., 2013], [Molina et al., 2016], and Furtado et al. [2019], and real data collected from a textile company that presents characteristics analyzed in this paper. There are $N = \{5, 6, 7\}$ items, $K = \{3, 4, 5\}$ orders, $M = \{2, 3\}$ machines, $V = \{2, 3\}$ vehicles and $T = 30$ periods. A total of $3 \cdot 3 \cdot 2 \cdot 2 = 36$ instances were generated.

The fixed parameters are $h_{it} = 3$ and $r_v = 1$. The uniform distribution is used to generate the follow parameters: $b_i = U[200, 900]$, $p_{im} = U[6, 45]$, $q_v = U[4, 8]$. Demand for items is calculated by: 75% of the demand $d_{ik} = U[1000; 10000]$, and the remaining 25% is set to 0. The setup time and cost are computed as: $s_{ijm} = U[720, 3600]$ and $c_{ijm} = (66.67/2160) \cdot s_{ijm}$ for all $i \neq j$, , where 2160 is the average between 720 and 3600, and 66.67 is a fixed value based on [Amorim et al., 2013].

The machine capacity is determined according to $cap_m = \frac{\sum_{ik} dem_{ik} p_{im}}{0.6T}$. We define the capacity utilization to be around 60% with this expression. For the computation of travel costs tr_{vk}, all customers are positioned randomly in a square of locations from $(0, 0)$ to $(100, 100)$, fixing the depot at the point (50,50). The Euclidean distance ($dist_k$) is then calculated between all pairs of customers. The fixed cost of using vehicle v is set to $f_v = U[200, 250]$. Finally, the travel costs are computed by: $tr_{vk} = f_v \cdot dist_k$

To calculate the order delay costs, we first determine the due date for each order. For that, we determine the time needed to produce the order k: $OT_k = \frac{\sum_{im} p_{im} d_{ik}}{M} \forall k$. The time needed to deliver each order k is $DT_k = dist_k/vel$, where vel is the vehicle's speed set as $100km/h$. So the minimum due date is $t_1 = DT_k + OT_k$, and the due date is set as $dd_{kt} = U[t_1, T]$. The cost for delaying order k in period t is $dc_{kt} = OT_k \cdot \beta_k \cdot (t + 1)$ where

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β_k is the initial delay of order k (in periods). If order k is not delayed, $dc_{kt} = 100000$ is a large number.

3.2. Computational results

The integrated model was coded using Visual Studio 2019 with C++ language and the CPLEX 12.10 optimization software, with the default settings and a time limit of 3600s. The tests were performed on an Intel Core i7 with 2.9 GHz and 32GB of RAM on a Windows 10 operational system. Table 6 shows the value of solution cost, optimality gap (calculated by the optimization software), computational time, and fulfilled orders.

The model found optimal solutions for 5 of 36 instances and solutions near the optimality (very low gaps) for 8 instances. However, the solver used the maximum resolution time and did not provide reasonable solutions to 4 instances (i27, i29, i33, and i35) that presented a gap greater than 30% and higher costs caused by delayed orders.

The difference between instances i1 and i2 is the number of vehicles. Instance i1 has two vehicles, and i2 has three, resulting in a reduction in total costs because it is possible to plan cheaper deliveries a cheaper delivery. The same result is seen when all parameters are identical, except for the number of vehicles. Furthermore, in i8, when we add a vehicle, we make it possible to fulfill all orders, unlike in i7. In this sense, it is possible to infer that the number of vehicles is a bottleneck identified by the model.

Table 6: Results obtained by CPLEX: objective function value, computational time, gap and fulfill orders.

								Orders			
Instance	N	M	K	V	Obj. Function	Gap (%)	Time (s)	Met on time	Delayed	Unmet	Shipment
i1	5	2	3	2	207953.88	0.00	139.70	3	0	0	26
i2	5	2	3	3	179521.34	0.00	210.20	3	0	0	26
i3	5	2	4	2	314374.69	0.02	3606.55	4	0	0	36
i4	5	2	4	3	274341.28	0.00	954.99	4	0	0	36
i5	5	2	5	2	379055.06	0.05	3602.87	5	0	0	43
i6	5	2	5	3	336869.56	0.02	3600.44	5	0	0	43
i7	5	3	3	2	207848.78	0.00	217.77	3	0	0	26
i8	5	3	3	3	179430.96	0.00	92.25	3	0	0	26
i9	5	3	4	2	314295.49	0.03	3607.33	4	0	0	36
i10	5	3	4	3	274376.04	7.04	3603.09	4	0	0	36
i11	5	3	5	2	378971.35	5.39	3603.26	5	0	0	43
i12	5	3	5	3	336807.36	9.66	3603.51	5	0	0	43
i13	6	2	3	2	233927.77	0.14	3603.40	3	0	0	30
i14	6	2	3	3	200714.80	0.11	3601.95	3	0	0	29
i15	6	2	4	2	350842.46	4.34	3603.86	4	0	0	42
i16	6	2	4	3	297412.72	3.33	3603.05	4	0	0	39
i17	6	2	5	2	426615.08	3.39	3603.55	5	0	0	50
i18	6	2	5	3	361330.99	3.74	3604.05	5	0	0	46
i19	6	3	3	2	233611.92	0.53	3604.54	3	0	0	30
i20	6	3	3	3	200452.66	0.03	3603.53	3	0	0	29
i21	6	3	4	2	350507.44	4.25	3609.89	4	0	0	42
i22	6	3	4	3	297010.30	3.56	3606.20	4	0	0	39
i23	6	3	5	2	426101.35	3.24	3611.35	5	0	0	50
i24	6	3	5	3	360908.07	3.63	3617.72	5	0	0	46
i25	7	2	3	2	323769.67	2.84	3603.60	3	0	0	43
i26	7	2	3	3	271878.22	0.10	3601.57	3	0	0	40
i27	7	2	4	2	105360049.36	51.40	3612.16	3	0	1	43
i28	7	2	4	3	416435.85	2.60	3605.76	4	0	0	54
i29	7	2	5	2	191237501.01	36.59	3607.42	4	1	1	50
i30	7	2	5	3	483194.49	2.90	3607.71	5	0	0	63
i31	7	3	3	2	323303.41	2.51	3622.43	3	0	0	43
i32	7	3	3	3	271963.10	0.94	3621.66	3	0	0	40
i33	7	3	4	2	78015850.71	34.37	3608.96	4	2	0	59
i34	7	3	4	3	415915.05	2.41	3610.01	4	0	0	54
i35	7	3	5	2	175377759.12	40.16	3606.21	4	2	1	51
i36	7	3	5	3	483509.13	2.97	3609.03	5	0	0	63

Another bottleneck is the number of machines. When the number of orders

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increases and the number of machines remains at two, CPLEX produces lower quality solutions (column Gap) than three machine instances (e.g., i27 and i33). As expected, the number of shipments grew with increasing orders (e.g., i1 and i3). Nevertheless, the model provided operational planning for integrated lot sizing and distribution, capable of meeting all scheduled orders for 91,66% of tested data sets.

4. Conclusions

This study provides a MIP model for integrated lot sizing and distribution problems with customer orders and cargo arrangement, considering production capacity and carryover setup sequence-dependent constraints and distribution decisions based on cargo unitization pallets. The problem is solved using CPLEX, and the results show that this model can provide better solutions than the non-integrated models. Furthermore, a helpful result is the relationship between the number of vehicles and machines and the orders met on time.

A set of instances was generated to evaluate the behavior of the integrated model. Some optimal solutions were found for small instances, and good solutions were provided for a reasonable number of tested instances. However, CPLEX found it challenging to solve instances with increasing numbers of orders and items when the number of machines and vehicles remained low. Hence, a solution method based on metaheuristics should be analyzed for future work to deal with large instances.

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