A MIXED CONTROL CHART TO MONITOR THE PROCESS VARIANCE: A NEW CASE USING ALTERNATING INSPECTIONS

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RESUMO

Este artigo apresenta o gráfico de controle misto $np_x - S^2$, que emprega inspeções alternadas (por atributos e variáveis) usando sequencialmente amostras de tamanho n_a e n_b. Os itens da amostra de tamanho na são classificadas conforme o procedimento do gráfico np_x, usando um calibrador GO / NO GO e contabilizando o número de itens não aprovados (Y_{n_a}). Os itens da amostra de tamanho nb são medidos e calculada sua variância amostral $S^2_{n_b}$. Se $Y_{n_a} > UCL_{n_a}$ ou $S^2_{n_b} > UCL_{n_b}$ o processo é julgado fora de controle. Após ajustes no processo, a inspeção sempre reinicia com um tamanho de amostra n_a (usando o gráfico np_x), caso contrário, o processo continua. Os parâmetros do gráfico proposto são otimizados por uma busca intensiva, a fim de superar o gráfico S^2 (em termos de ARL_1), restrito a ter tamanho amostral médio o mais próximo do utilizado para o gráfico S^2.

PALAVRAS CHAVE. Controle de Qualidade. Gráficos por atributos e variavéis. Limites Discriminantes.

Tópicos (AD&GP - PO na Administração e Gestão da Produção)

ABSTRACT

This paper presents the $np_x - S^2$ mixed control chart , which employs alternating inspections (by attributes and variables) using sequentially samples of size n_a and n_b. The items of sample of size n_a are classified according to the np_x chart procedure, using a GO / NO GO gauge and counting the number of non-approved items (Y_{n_a}). The items of sample of size n_b are measured and calculated its sample variance $S^2_{n_b}$. If $Y_{n_a} > UCL_{n_a}$ or $S^2_{n_b} > UCL_{n_b}$ the process is judged

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out of control. After adjustments in the process, the inspection always restarts with a sample size n_a (using the np_x chart), otherwise, the process continues. The parameters of the proposed chart are optimized by an intensive search, in order to outperform the S^2 chart (in terms of ARL_1), restricted to have average sample size closer to the used for S^2 chart.

KEYWORDS. Quality Control. Attribute and Variable Control Charts. Discriminant Limits.

Paper topics (AD&GP - OR in Administration and Production Management)

1. Introduction

It is common that processes suffer influence of some sort of variation and such fact affects their outcomes. Even if it is undesirable its presence, it is a natural event in any random processes. Therefore, many researchers and quality improvement practitioners search ways to monitor it.

Among the techniques employed in Statistical Process Control (SPC), we have the control charts, which can be by variables or attributes. In cases where the measurement of the quality characteristic of interest is expensive and/or time consuming, a reasonable alternative is the adoption of control charts by attribute (that is, the items are classified as conforming or non-conforming and these quality characteristics are called attributes) in monitoring the process mean or process variance. Generally inspections by attribute are cheaper and simpler than by variables, although presents poor performance in comparison.

Examples of these charts are the p and np attribute control charts. In the traditional p and np control charts, an item is classified as conforming when it attains a stated criterion by the engineering team like the specification limits, otherwise, the item is classified as non-conforming. However, the speed of an abnormal case of these charts (p and np charts) to monitor process mean is slow and does not present a good performance needing a sample nine times larger than the used for an $\overline{X}$ control chart as stated in Montgomery [2009].

Several proposals about attribute control charts have been discussed in the literature to monitor process mean, however these studies are not recent. For example we can cite some relevant earlier contributions as Tippett [1944], Stevens [1948], Mace [1952]. It is worth mentioning an interesting attribute control chart, called np_x proposed by Wu et al. [2009], to monitor the process mean, that presents a good performance. Its procedure consists of classifying each item of a fixed sample size n as approved or not according to the warning limits w_u using a typical gauge "GO / NO GO" . Figure 1 shows an example of this type of tool. Others contributors dedicated to propose new attribute control charts Ho e Costa [2011]; Quinino et al. [2015, 2020], as well as new control charts that are mixture of attribute and variable charts Ho e Aparisi [2016], all of them to monitor a process mean.

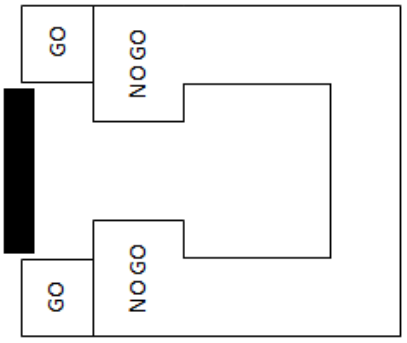


Figure 1: Ring Gauge GO/NO GO.

Silva et al. [2021] proposed the "Improved np_x" version of the attribute chart to monitor a process mean. This approach also used in Silva et al. provides a significant reduction in terms of ARL. It consists of the use of a same sequence in the selection of the sample sizes to be inspected (n_a, n_b) with $n_a > n_b$. Each item is classified as approved or not according to a discriminant limit

(w_i, $i = a,\ b$) using a GO/NO GO gauge. In the end of inspection we'll have Y_a and Y_b items classified as disapproved. Whenever $Y_a > UCL_{n_a}$ or $Y_b > UCL_{n_b}$, the process is judged to be out of control and after adjustments in the process, the inspection is always restarted with the larger sample size of n_a units. The parameters used in the construction of the "Improved np_x" control chart were obtained through a search for values that optimize their performance, such that can compete with the standard attribute np_x chart and the traditional $\overline{X}$ control chart. The performances were compared in terms of average run length (ARL) in scenarios of shift sizes (δ) in the process mean. This same scheme of inspection was previously studied by Ho et al. [2020] to improve the performance of $\overline{X}$ control chart to monitor a Weibull mean. Such design simplifies the decision-making, because it is a predetermined fixed sequence, facilitates its operation, minimizes the need for specialized training, as well as can reduce the possibility of errors in the elaboration of the control chart and, consequently, the decision-making in the long term.

In the literature, fewer attribute control charts to monitor a variance are found Ho e Quinino [2013, 2016]; Bezerra et al. [2018]. Most recently Silva et al. [2022b] proposed new attribute control charts named as $np_x^{\sigma^2}$ and $np_{x(I)}^{\sigma^2}$, both to monitor the process variance. In the $np_x^{\sigma^2}$ control chart the inspection procedure of np_x control chart above described is extended in monitoring the process variance, (instead of the process mean). In the $np_{x(I)}^{\sigma^2}$ attribute control chart, the scheme using sequentially two samples of sizes n_a and n_b in the inspection procedure proposed by Ho et al. [2020] and Silva et al. [2022a] is extended in monitoring the variance.

The main contribution of this paper is to present the mixed $np_x - S^2$ control chart. It consists of the application of alternate inspections (by attribute and variable) using sequentially samples of sizes n_a and n_b (n_a ¿ n_b) to monitor changes in the variance of a random variable (X), such that can compete with the traditional variable S^2 control chart used to monitor the variance.

Some assumptions are made in this study as the quality characteristic of interest follows a Normal distribution, with known mean μ_0 (it remains unchanged) and standard deviation σ_0, and independent observations are collected. Due to special causes, the variance value σ_0^2 may change to $\sigma_1^2 = \delta^2\sigma_0^2$, where δ expresses the magnitude of the shift over the standard deviation (σ_0). The rest of this paper is organized as follows: the mixed $np_x - S^2$ control chart applied in monitoring the process variance is presented in Section 2; in Section 3 its performance is compared with the traditional S^2 variable control chart; an empirical example is presented in Section 4 and Section 5 provides the conclusions of this study.

2. The $np_x - S^2$, a mixed control chart to monitor the variance

In this paper alternating inspections by attribute and variable are used. The use of gauges provides greater operational simplicity in inspection by attributes, thus its use is widely employed in checking whether the discriminant limits have been attained or not Kennedy et al. [1987]. Considering that the main target of an attribute control chart is in detecting deterioration in quality Chiu [1975], only the upper control limit is considered. If the calculated statistic exceeds the upper control limit, the process is judged as out of control and must be stopped to search the special causes, then adjusted to the state of in control again.

This paper presents the $np_x - S^2$, a mixed control chart for monitoring the variance employing sequential samples of sizes n_a and n_b ($n_a > n_b$). In an attribute inspection, let X_{a_i} be the observed quality characteristic of $i-th$ item in the sample size n_a. If $\dfrac{X_{a_i} - \mu_0}{\sigma_0} > |z_a|$ then the item is classified (using a tool) as not approved, otherwise it is classified as approved with $z_a = \dfrac{W_a - \mu_0}{\sigma_0}$, the standardized discriminant limit, (W_a, the non-standardized discriminant limit). At the end of the inspection we will have Y_a, the number of not approved items. In a variable

inspection, the items are measured and obtained its sample variance, $S^2_{n_b} = \frac{\sum_{i=1}^{n_b}(X_{bi} - \overline{X}_b)^2}{n_b - 1}$, $\overline{X}_b = \frac{\sum_{i=1}^{n_b} X_{bi}}{n_b}$, X_{b_i} be the observed quality characteristic of $i-th$ item in the sample size n_b.

The inspection procedure can be described considering the following steps:

1. Take a sample of size n_a and obtains Y_a using a ring gauge for example. If $Y_a \leq UCL_{n_a}$ then the process is judged to be in control and go to step 2, otherwise, the process is judged to be out of control (a search for the special causes begins) and go to the step 3;
2. Take a sample of size n_b, measure the items and obtain $S^2_{n_b}$. If $S^2_{n_b} \leq UCL_{n_b}$ then the process is judged to be in control, otherwise, the process is judged to be out of control (and a search for the special causes begins) and go to step 3;
3. Repeat the step 1.

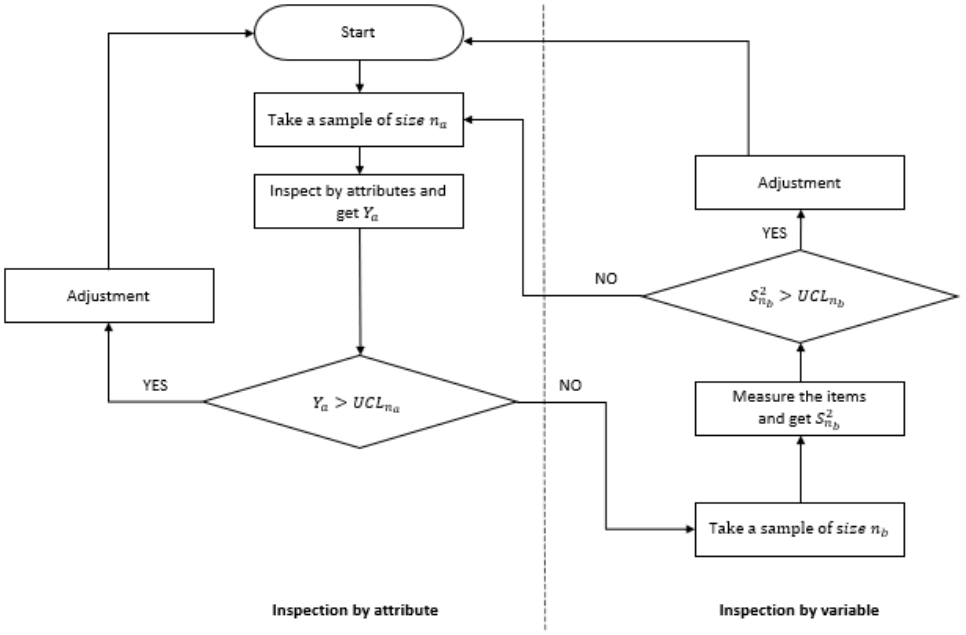


Figure 2: Inspection procedure of the $np_x - S^2$ mixed control chart.

Figure 2 illustrates inspection procedure of the $np_x - S^2$ control chart. Figure 3 shows an example of the $np_x - S^2$ control chart, with the control limits set at $UCL_{n_a} = 3$, (the control limit used for attribute inspection, represented by squares) and $UCL_{n_b} = 11.6$, (the control limit for variable inspection, represented by diamonds). Note that the inspection at the 5$^{\text{th}}$ sample is an attribute one (represented by a square) presents $Y_a > UCL_{n_a}$, then the process is halted for an adjustment. The inspection procedure is restarted at 6$^{\text{th}}$ sample using a sample size n_a, followed by a sample size n_b. Note that the 7$^{\text{th}}$ sample (represented by a diamond - sample size n_b) shows $S^2_{n_b} > UCL_{n_b}$ as in the previous case the process is halted for adjustment and the inspection procedure is restarted at 8$^{\text{th}}$ sample using a sample size n_a. That is, the inspection always restarts with sample size n_a (using the np_x chart). Such strategy is interesting, taking into account that attribute inspections are faster, cheaper and economically feasible.

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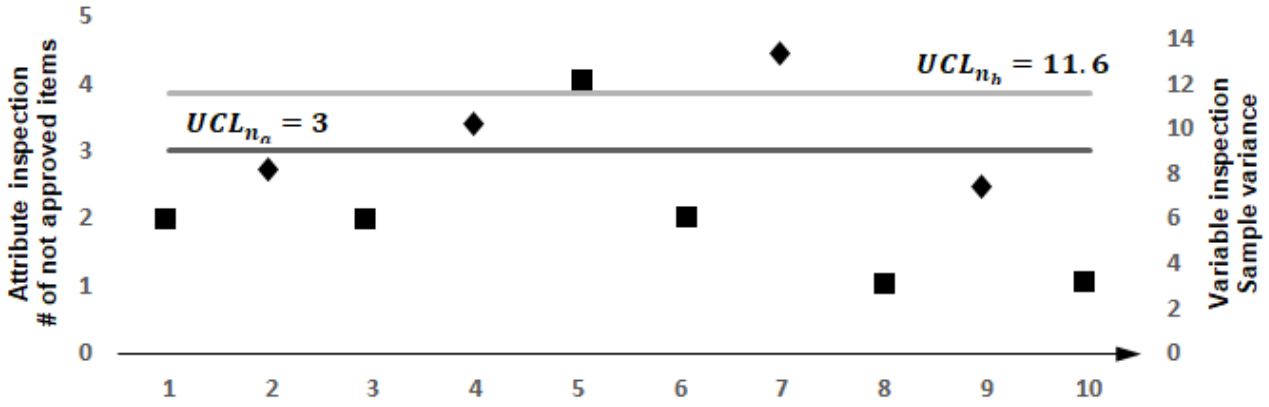


Figure 3: An example of the $np_x - S^2$ mixed control chart to monitor the variance.

The procedure described above is applied for cases of increases in the variance, that is: the alternative hypothesis as $H_1 : \sigma^2 > \sigma_0^2$. For other alternative hypothesis as $H_1 : \sigma^2 < \sigma_0^2$ or $H_1 : \sigma^2 \neq \sigma_0^2$ few adjustments of the described procedure are necessary.

The inspection scheme can be described through a Markov chain with four transition states:

- State A, if Y_a is within control limits;
- State B, if $S_{n_b}^2$ is within control limits;
- State C, if $S_{n_b}^2$ is out of control limits and
- State D, if Y_a is out of control limits

with the transition matrix **P**:

$$\mathbf{P} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & P_B & 1-P_B & 0 \\ P_A & 0 & 0 & 1-P_A \\ P_A & 0 & 0 & 1-P_A \\ P_A & 0 & 0 & 1-P_A \end{pmatrix} \end{array} \tag{1}$$

with the probabilities P_A (the probability of being in State A) and P_B (the probability of being in State B) calculated as follows:

$$P_A = \sum_{Y_a=0}^{UCL_{n_a}} \binom{n_a}{Y_a} {p_0}_a{}^{Y_a} (1 - {p_0}_a)^{n_a - Y_a} \tag{2}$$

$$P_B = \frac{UCL_{n_b} \times (n_b - 1)}{\sigma_0^2} \tag{3}$$

when the process is in-control. The value of p_{0_a} in (2) is the probability of an item to be not approved, in the sample of size n_a, expressed by $p_{0_a} = P\left(\frac{X_{a_i} - \mu_0}{\sigma_0} > |z_a|\right)$. The value of UCL_{n_b} in (3) is calculated by $UCL_{n_b} = \left(\frac{\sigma_0^2\delta^2}{n_b - 1}\chi^2_{z_b,n_b-1}\right)$, where $\chi^2_{z_b,n_b-1}$ denotes the upper percentage points $|z_b|$ of the chi-square distribution with $n_b - 1$ degrees of freedom, and σ_0^2, the variance. When the process is in control $\delta^2 = 1$.

The transition matrix **P** is irreducible, aperiodic, and $\mathbf{P}^{\,t}$ converges to a matrix in which all rows are equal to the vector π when $t \to \infty$. The vector $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ is a stationary distribution and each element π_i, $i = 1, 2, 3, 4$ is associated with one state of matrix **P**. It can be obtained as the solution of the linear system of equations $\pi = \pi \times \mathbf{P}$, subject to the restriction $\sum_{i=1}^{4} \pi_i = 1$. By solving the mentioned system, we obtained $\pi_1 = \frac{P_A}{(1 + P_A)}$; $\pi_2 = \frac{(P_A \times P_B)}{(1 + P_A)}$; $\pi_3 = \frac{(P_A - P_A \times P_B)}{(1 + P_A)}$ and $\pi_4 = \frac{(1 - P_A)}{(1 + P_A)}$. The values π_3 and π_4 indicate the long-term probability of the inspection signaling that the process is out of control, using the samples n_a and n_b, respectively. The Average Sample Size (ASS) used in this procedure is calculated as $ASS = n_a(\pi_1 + \pi_4) + n_b(\pi_2 + \pi_3)$.

Many authors have used the average run length (ARL) to summarize the characteristics of the control charts , which expresses the average number of samples plotted before of an indication of an out of control condition Montgomery [2009]. In this study, the average number of samples up to the signal of an out of control condition, considering the matrix **P** above mentioned can be expressed by $ARL = \frac{1}{\pi_3 + \pi_4}$. When the process is in control:

$$ARL_0 = \frac{1 + P_A}{1 - (P_A \times P_B)} \tag{4}$$

If the process presents status out of control, that is, the variance has changed from σ_0^2 to $\sigma_1^2 = \delta^2\sigma_0^2$, $(\delta > 1)$, the values of the probabilities of being in the states A and B consequently change, assuming new values P_{A_1} and P_{B_1}, according to expressions (5) and (6):

$$P_{A_1} = \sum_{Y_a=0}^{UCL_{n_a}} \binom{n_a}{Y_a} {p_{1_a}}^{Y_a}(1 - p_{1_a})^{n_a - Y_a}, \text{with } p_{1_a} = P\left(\frac{X_{a_i} - \mu_0}{\sigma_1} > |z_a|\right) \tag{5}$$

$$P_{B_1} = \frac{UCL_{n_b} \times (n_b - 1)}{\sigma_1^2} \tag{6}$$

When σ_0^2 changes to σ_1^2, we have two possible events: the change can occur before or after the inspection performed with the sample of size n_a. Considering the equal probability of occurrence of these two events, when the process is out of control, the performance of the control chart proposed in this paper is evaluated through the value of ARL_1, as follows:

$$\begin{aligned} ARL_1 &= 0.5\left[(1 - P_{B_1}) + \left(1 + \frac{1}{\pi_3 + \pi_4}\right)P_{B_1}\right] + 0.5\left(\frac{1}{\pi_3 + \pi_4}\right) \\ &= \frac{2 + P_{A_1} + P_{B_1}}{2 - 2P_{A_1}P_{B_1}} \end{aligned} \tag{7}$$

The parameters of the current proposal are: the sample sizes n_a and n_b, the control chart limits for each sample size UCL_{n_a} and UCL_{n_b}, the standardized discriminant limit z_a . After chosen the sample sizes n_a and n_b, the control chart limits UCL_{n_a} and UCL_{n_b}, the standardized discriminant limit z_a are searched to meet a target value of ARL_{0_T}, (say, 370.4) and other restrictions. Explicitly it consists of finding the vector $\mathbf{\Theta}$:

$$\begin{aligned} \mathbf{\Theta} = (z_a^o, UCL_{n_a}^o, UCL_{n_b}^o) &= arg(min\ ARL_1) \\ \textit{Subject to the constraints} &: 0 \leq z_a \leq 3.5 \\ & 0 \leq UCL_{n_a} \leq n_a \\ & |ARL_{0_c} - ARL_{0_T}| \leq \varepsilon, \quad \varepsilon = 0.15 \end{aligned} \tag{8}$$

3. The performance of the $np_x - S^2$ control chart to monitor the variance

In this section the performance of the $np_x - S^2$, the mixed control chart is evaluated and compared with the performance of S^2 control chart, both control charts set to have an $ARL_{0_T} = 370.4$. To get the parameters of the $np_x - S^2$ control chart a specific program was developed for this aim using the software R due to the high number of configurations (of sample sizes n_a and n_b, control limits, discriminating limit, etc). The search provided several cases that meets the condition of the ARL_0 value of approximately 370.4. Among them some configurations that compete with the S^2 control chart were selected. The designs selected to evaluate the proposed approach present the following features: 1- the sample size n_a is approximately (or equal) twice the sample size used for the S^2 chart to yield an average sample size (ASS) closer to the sample size used for S^2, 2 - the performance to detect small shifts should be at least similar to the S^2 chart.

Regarding the inspection cost, if we consider the unitary inspection costs for attributes and measurement equal to \$1.00 and \$4.00, respectively. The inspection cost for sample size equal three for S^2 control chart will be equal \$12.00, while in P1 plan are inspected seven units (total) and its cumulative inspection cost will be \$13.00, five units inspected by attributes (\$5.00) and two unit inspected by variable (\$8.00), with mean unitary inspection cost about \$1.86. Nevertheless, the average sample size for P1 plan is 3.503, which provides the average inspection cost (AIC) equal \$6.51 for P1 plan, for example. Its performance is better even if compared with the $np_{x(I)}^{\sigma^2}$, but it does not present the lower cost among the options. Similarly the inspection cost for sample size equal four for S^2 control chart will be \$16.00, while the average inspection cost for P2 plan will be \$9.51, taking account their mean unitary cost (\$1.90) and average sample size ($ASS = 5.004$ for P2 plan).

In Table 1 are presented some designs of the $np_x - S^2$, the mixed control chart that could compete (that is, lower ARL_1 values) with S^2 control chart for sample sizes $n_{S^2} = \{3, 4\}$, used for S^2 control chart, considering shifts of size $\delta = 1.10$ to 2.00 by 0.1. The four plans shown in Table 1 are ones (among several plans) that provide the lowest sample size (or average sample size) besides lower ARL_1 values than those obtained with S^2 chart. An additional comparison is made with the results obtained by the $np_{x(I)}^{\sigma^2}$ control chart, in this case the proposal also offers better performance in terms of ARL_1, however with higher average inspection cost. For average sample sizes $ASS \geq 6$ its performance are similar to the $np_{x(I)}^{\sigma^2}$ control chart or the $np_{x(I)}^{\sigma^2}$ control chart becomes a better alternative (such results are not showed in this paper), taking a count the cost and its operational procedure. Many interesting results can be point out from this table:

- For small sample ($n_{S^2} \leq 4$), the $np_x - S^2$ mixed control chart has better performance than S^2 control chart in all shift sizes in the range $\delta = 1.10$ to 2.00 (by 0.1), that is, it provides lower

Table 1: ARL values of control charts: S^2 versus $np^{\sigma^2}_{x(I)}$ versus $np_x - S^2$ mixed control chart - $n_{S^2} = 3, 4$.

	ARL_1					
δ	S^2	$np^{\sigma^2}_{x(I)}$	P1: $np_x - S^2$	S^2	$np^{\sigma^2}_{x(I)}$	P2: $np_x - S^2$
1.0	370.398	370.367	370.299	370.398	370.544	370.392
1.1	132.699	131.388	131.415	117.832	112.876	116.938
1.2	60.785	59.186	59.195	49.85	46.145	48.692
1.3	33.107	31.620	31.626	25.748	23.261	24.719
1.4	20.443	19.149	19.155	15.354	13.65	14.509
1.5	13.856	12.744	12.752	10.178	8.964	9.492
1.6	10.078	9.120	9.13	7.307	6.409	6.744
1.7	7.741	6.907	6.921	5.574	4.891	5.108
1.8	6.206	5.472	5.489	4.459	3.928	4.067
1.9	5.147	4.495	4.517	3.703	3.284	3.37
2.0	4.387	3.803	3.829	3.167	2.835	2.883
Sample Size	$n_{S^2} = 3$	$ASS = 3.002$ $n_a = 4$ $n_b = 2$	$ASS = 3.503$ $n_a = 5$ $n_b = 2$	$n_{S^2} = 4$	$ASS = 5.010$ $n_a = 9$ $n_b = 1$	$ASS = 5.004$ $n_a = 7$ $n_b = 3$
Estimated Inspection Cost	$c_{S^2} = 12$	$AIC = 3.00$	$AIC = 6.51$	$c_{S^2} = 16$	$AIC = 5.01$	$AIC = 9.51$
UCL	$UCL_{S^2} = 5.915$	$UCL_{n_a} = 0$ $UCL_{n_b} = 0$	$UCL_{n_a} = 0$ $UCL_{n_b} = 11.022$	$UCL_{S^2} = 4.719$	$UCL_{n_a} = 2$ $UCL_{n_b} = 0$	$UCL_{n_a} = 1$ $UCL_{n_b} = 6.729$
Discriminant limit	-	$z_a = 3.325$ $z_b = 3.31$	$z_a = 3.32$	-	$z_a = 2.04$ $z_b = 3.5$	$z_a = 2.445$

ARL_1 value. A small increase over the average sample size is necessary. For $n_{S^2} = 3, 4$ the ASS are, respectively around 20%, 25% higher, such increases are acceptable against the gains in an achieved performance.

- Considering that the the inspection procedure always starts (or restarts) with the sample of size n_a (by attribute) which inspection is simpler operationally, faster, cheaper and fully feasible, even if there is a small increase over the average sample size, it is worth applying the $np_x - S^2$ mixed control chart.

- Additionally, if we consider the unitary inspection costs for attributes and measurement equal to \$1.00 and \$4.00, respectively. We will have in P1 and P2 plans the average inspection costs (AIC) equal \$6.51 and \$9.51, respectively. Thus, an inspection by variable with sample size equal three and four ($n_{S^2} = 3$ and $n_{S^2} = 4$) for the S^2 control chart could be suitably replaced for plans P1 and P2, respectively.

4. Empirical Example

In this section a numerical example is presented to illustrate the proposed control chart in this paper. In this example the items are evaluated using some classification criterion or even a GO/NO GO gauge for the samples of size n_a. In a sequence, a sample of size n_b is extracted, which items are measured and calculated its sample variance.

For that purpose, let us consider a machining process of a shaft, which an important quality characteristic is its length. Thus, every h inspection interval alternating samples of sizes n_a and n_b will be collected for monitoring of process variance. In this example, the $np_x - S^2$ control chart is used to monitor the variance of the length of the manufactured shafts using the sample sizes $n_a = 7$ and $n_b = 3$ (to yield $ASS = 5.004$). This configuration is selected in order to get a better performance than the S^2 chart with $n_{S^2} = 4$ (for type I error $\alpha = 0.0027$) that yields $ARL_1 = 117.832$ vs $ARL_1 = 116.938$, respectively for S^2 and $np_x - S^2$ charts, both for a shift $\delta = 1.10$. Some additional assumptions are: the length of shafts values are independent and following

a normal distribution. When the process is in control, the parameters are $\mu_0 = 150.005\ mm$ and $\sigma_0 = 2.39\ mm$. The standardized discriminant limit used for sample of size n_a is $z_a = 2.445$. The control limits of the $np_x - S^2$ control chart are $UCL_{n_a} = 1$ and $UCL_{n_b} = 6.729$.

In Table 2 the evaluation procedure for 10 samples extracted from shafts production line is presented. As mentioned before, the inspection procedure consists in applying alternating inspections by attribute and variable, using sequentially samples of size n_a and $n - b$. In attribute inspection, the items are classified as approved (A) or not approved (N), using a gauge, for example, according to the criterion above mentioned. Thus, after the inspection, we have Y_a, the number of items not approved in sample of size n_a is observed. If $Y_a > UCL_{na}$, the process is judged as out of control, otherwise, the process is judged in control. In variable inspection, the items are measured and the sample variance $S^2_{n_b}$ is obtained, if its value exceeds the UCL_{n_b}, is said that the process is out of control, otherwise, the process is judged in control. Note that at the sample $\#5$, the first and fourth units are classified as not approved, providing $Y_a = 2 > UCL_{n_a}$=1. This result indicates that the process was judged as out of control signaling the need to search for special causes and performing some adjustments. As stated , the inspection procedure restarts (by attribute) collecting the sample $\#6$, classifying $n_a = 7$.

Table 2: Samples Evaluation and Decision by the improved $np_x - S^2$ mixed control chart

Sample	Evaluation							n_a	Y_a	n_b	$S^2_{n_b}$	Type of inspection	Decision
1	A	A	A	A	A	A	A	7	0			attribute	In-control
2	151.0	146.65	147.25							3	5.557	variable	In-control
3	A	A	A	A	A	N	A	7	1			attribute	In-control
4	151.10	146.60	147.20							3	5.970	variable	In-control
5	N	A	A	N	A	A	A	7	2			attribute	Out-of-control
6	A	A	A	A	A	A	A	7	0			attribute	In-control
7	148.35	148.75	147.80							3	0.227	variable	In-control
8	A	A	A	A	A	N	A	7	1			attribute	In-control
9	147.20	150.950	146.750							3	5.317	variable	In-control
10	N	A	A	A	A	A	A	7	1			attribute	In-control

5. Conclusions

In this paper, a new control chart, the $np_x - S^2$ mixed control chart is proposed to monitor a process variance. It can be a good alternative to the use of traditional S^2 control chart. Their parameters are selected to satisfy a target value of $ARL_0 \approx 370.4$ and minimize ARL_1. When compared with the most commonly used competitor, the traditional S^2 control chart. The $np_x - S^2$ mixed control chart presents good results in all shifts of size in the variance ($\delta = 1.10$ to 2.00 by 0.1), that is, it provides an economically feasible alternative taking account the inspection strategy adopted and smaller ARL_1 values reached (the results show that is possible an average reduction about 10% in $AR1$ for large shift size in process variance) with just a small increase over the average sample size. When compared to the $np^{\sigma^2}_{x(I)}$ control chart provide better performance in small sample sizes, despite of the higher average inspection cost in this comparison.

It is relevant to mention that at the variable inspection few measurements are made on the sampled units, taking account that most of items are classified approved or not approved, using some criterion or a gauge, for example. Thus, searched the optimized parameters, the $np_x - S^2$ control chart proposed in this paper is a good option for the S^2 chart, as it presents better performance in terms of ARL_1 than S^2 chart.

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