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Fuzzy set based multiattribute decision making, its computing implementation, and applications

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ABSTRACT

This paper reflects results of research related to the analysis of models of multiattribute decision making in a fuzzy environment (within the so-called $\langle X, R \rangle$ models). These results can be used independently or within the framework of a general scheme of multicriteria decision making under conditions of uncertainty. The considered techniques for fuzzy preference modeling are directed at evaluating, comparing, choosing, prioritizing, and/or ordering alternatives. These techniques have served for developing a Multiattribute Decision Making System (MDMS2). It has been implemented in the C# programming language, utilizing the framework .NET. MDMS2 permits one to present preferences of decision makers in one of five preference formats. These formats as well as quantitative estimates are reduced to non-reciprocal fuzzy preference relations to provide homogeneous preference information for procedures of decision making. The paper results are of a general character and were used to analyzed problems of power engineering.

KEYWORDS. fuzzy preference modeling; preference formats; $\langle X, R \rangle$ models.

Eixo Temático Principal: ADM – Apoio à Decisão Multicritério; IC – Inteligência Computacional; EN&PG – PO na Área de Energia, Petróleo e Gás.

1. Introduction

Some researches in the field of operational research and systems analysis agree that, from the general point of view, the uncertainty of goals (related to a multicriteria character of many decision problems) is the most difficult to overcome because "we simply do not know what we want" [Ekel et al. 2019]. In reality, this type of uncertainty cannot be effectively captured only on the basis of applying formal models, since sometimes the unique information sources are the individuals who make decisions.

It is possible to identify two major classes of situations [Ekel et al. 1998, Ekel 2002] requiring the use of a multicriteria approach.

The first class is associated with problems whose solution consequences cannot be estimated with the use of a single criterion: these problems are associated with the analysis of models which include economic as well as physical indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimations are difficult or impossible.

The second class is related to problems that may be solved on the basis of a single criterion or several criteria. However, if the uncertainty of information does not permit one to obtain unique solutions, it is possible to reduce these problems to multicriteria decision making by applying additional criteria, including those of a qualitative character, whose utilization is based on knowledge, experience, and intuition of involved experts.

Taking this into account, it is necessary to distinguish two types of criteria: objectives and attributes. Thus, multicriteria decision making problems can be classified into two wide classes [Pedrycz et al. 2011, Ekel et al. 2020]:

- multiobjective decision making;
- multiattribute decision making.

Generally, multiobjective decision making is known as the continuous type of multicriteria decision making and its main characteristics are that the Decision Maker (DM) needs to achieve multiple objectives while these objectives are non-commensurable and conflict with each other. A multiobjective decision models include a vector of decision variables (which can be continuous and/or discrete), objective functions that describe the objectives, and constraints. The DM attempts to maximize or minimize the objective functions.

At the same time, multiattribute decision making is related to making preference decisions (that is, comparison, choice, prioritization, and/or ordering) over the available alternatives that are characterized by multiple, usually conflicting, attributes. The main peculiarity of multiattribute problems is that there are usually a limited number of predetermined alternatives, which are associated with a level of achieving the attributes. Based on the attributes, the decision is to be made.

According to this classification, two classes of models can be constructed: $\langle X, F \rangle$ models (as multiobjective models) and $\langle X, R \rangle$ models (as multiattribute models) [Pedrycz et al. 2011, Ekel et al. 2020]. The present work is devoted to analyzing $\langle X, R \rangle$ models. However, it is necessary to indicate that the analysis of $\langle X, F \rangle$ and $\langle X, R \rangle$ models serves as parts of a general scheme for multicriteria decision making under conditions of uncertainty [Pereira Jr. et al. 2015, Ekel et al. 2020]. This scheme is associated with a generalization of the classic approach to considering the uncertainty of information [Belyaev 1977] to multicriteria problems, based on analyzing special aggregations of characteristic estimates applied within the choice criteria [Belyaev 1977]. Its important feature is to use available quantitative information to the highest degree to reduce decision uncertainty regions. If the problem resolving capacity related to quantitative information processing does not allow one to obtain unique solutions, the general scheme assumes the use of qualitative information (within $\langle X, R \rangle$ models) based on knowledge, experience, and intuition of experts.

$\langle X, R \rangle$ models and their analysis

Assume we are given a set X of alternatives coming from the decision uncertainty region and/or predetermined alternatives, which are to be examined by q criteria of a quantitative and/or qualitative nature. The problem of decision making may be presented as a pair $\langle X, R \rangle$ where $R = \{R_1, R_2, \dots, R_p, \dots, R_q\}$ is a vector of fuzzy preference relations [Fodor and Roubens 1994, Ekel et al. 1998, Ekel 2002] which can be presented as

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)], p = 1, 2, \dots, q, X_k, X_l \in X, \quad (1)$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of the p -th fuzzy preference relation.

In (1), R_p (also called a nonstrict fuzzy preference relation or fuzzy weak preference relation in literature) is defined as a fuzzy set of all pairs of the Cartesian product $X \times X$, such that the membership function $\mu_{R_p}(X_k, X_l)$ represents the degree to which X_k weakly dominates X_l , i.e., the degree to which X_k is not worse than X_l for the p -th criterion. In a somewhat loose sense, $\mu_{R_p}(X_k, X_l)$ also represents the degree of truth of the statement “ X_k is preferred over X_l ”.

A natural and convincing approach to constructing fuzzy preference relations (in particular, nonreciprocal fuzzy preference relations [Pedrycz et al. 2011, Ekel et al. 2020]) R_p is presented in [Ekel et al. 1998, Ekel and Neto 2006]. To better understand this approach, let us consider $F(X_k)$ and $F(X_l)$ as fuzzy sets reflecting evaluations of the attribute F for alternatives X_k and X_l , respectively. Then, the quantity $\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$ is the degree of preference $\mu[F(X_k)] \geq \mu[F(X_l)]$, while $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$ is the degree of preference $\mu[F(X_l)] \geq \mu[F(X_k)]$. Then, the membership functions of the generalized preference relations $\eta\{\mu[F(X_k)], \mu[F(X_l)]\}$ and $\eta\{\mu[F(X_l)], \mu[F(X_k)]\}$ take [Ekel et al. 1998, Ekel and Neto 2006] the following forms:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_k)], \mu[F(X_l)], \mu_R[F(X_k), F(X_l)]\}, \quad (2)$$

$$\eta\{\mu[F(X_l)], \mu[F(X_k)]\} = \sup_{F(X_k), F(X_l) \in F} \min\{\mu[F(X_l)], \mu[F(X_k)], \mu_R[F(X_l), F(X_k)]\}, \quad (3)$$

where $\mu_R[F(X_k), F(X_l)]$ and $\mu_R[F(X_l), F(X_k)]$ are the membership functions of the corresponding fuzzy preference relations which, respectively, reflect the essence of the preferences of X_k over X_l and of X_l over X_k and (for instance, “more attractive”, “more flexible”, etc.).

When F can be measured on a numerical scale and the essence of preference behind relation R is coherent with the natural order ($\leq$) along the axis of measured values of F , then (2) and (3), respectively, are reduced [Ekel et al. 1998, Ekel and Neto 2006] to the following expressions:

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_k) \leq F(X_l)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\}, \quad (4)$$

$$\eta\{\mu[F(X_k)], \mu[F(X_l)]\} = \sup_{\substack{F(X_k), F(X_l) \in F \\ F(X_l) \leq F(X_k)}} \min\{\mu[F(X_k)], \mu[F(X_l)]\}. \quad (5)$$

If F has a maximization character, the correlations (4) and (5) have to be written for $F(X_k) \geq F(X_l)$ and $F(X_l) \geq F(X_k)$, respectively.

The correlations (4) and (5) agree with some well-known fuzzy number ranking indices [Fodor and Roubens 1994, Ekel et al. 2020]. Examples of their utilization are given in [Ekel et al. 1998, Ekel et al. 2016]. The use of (4) and (5) is well founded. Taking this into account, it is necessary to indicate that in practice there are cases where the fuzzy quantities $F(X_k)$ and $F(X_l)$ have trapezoidal membership functions [Pedrycz et al. 2011] that are located in such a manner that it is not possible to distinguish X_k and X_l [Ekel and Neto 2006, Pedrycz et al. 2011]. For instance, we can say that alternatives X_1 and X_2 shown in Figure 1 are indistinguishable since

$$\eta\{\mu[F(X_1)], \mu[F(X_2)]\} = \eta\{\mu[F(X_2)], \mu[F(X_1)]\} = \alpha \quad (6)$$

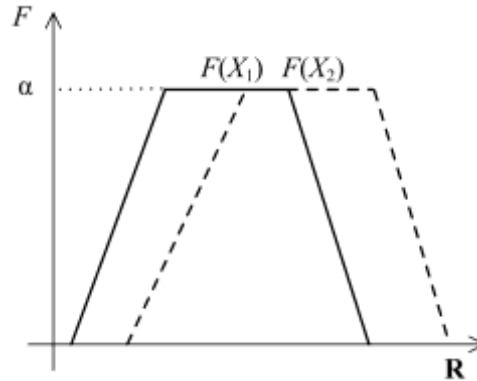


Fig. 1. Comparison of alternatives with trapezoidal membership functions

Let us return to construction matrices R_p .

The availability of fuzzy or linguistic estimates of alternatives $F_p(X_k)$, $p=1, 2, \dots, q$, $X_k \in X$ with the membership functions $\mu[F_p(X_k)]$, $p=1, 2, \dots, q$, $X_k \in X$ permits one, using (3) and (4), to construct R_p , $p=1, 2, \dots, q$ applying [Ekel et al. 1998, Ekel and Neto 2006] the following relationships:

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_k) \leq F_p(X_l)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\}, \quad (7)$$

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_l) \leq F_p(X_k)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\}. \quad (8)$$

Another approach to constructing matrices R_p on the basis of preference functions is given in [Parreiras and Ekel 2013]. The most recent our results on constructing fuzzy preference relations are presented in [Kokshenev et al. 2014]. If estimates $F_p(X_k)$, $p=1, 2, \dots, q$, $X_k \in X$ are defined on a unit value scale, these results permit one to obtain $\mu_{R_p}(X_k, X_l)$, $p=1, 2, \dots, q$, $X_k, X_l \in X$ as follows:

$$\mu_{R_p}(X_k, X_l) = 1 - \delta_p(X_l, X_k), \quad (9)$$

where $\delta_p(X_l, X_k)$ corresponds to the volume of all positive differences between the worst outcomes of $F_p(X_l)$ and the best outcomes of $F_p(X_k)$.

The results of [Kokshenev et al. 2014] are suitable for dealing with the both fuzzy and crisp estimates on the same domain while preserving the preference measures on interval scale.

Fuzzy preference relations are not a unique form of preference representation. For instance, the authors of [Zhang et al. 2004] indicate eight formats which can be used to establish preferences among analyzed alternatives. Among these formats, one can distinguish nonreciprocal fuzzy preference relations discussed above, additive reciprocal fuzzy preference relations, ordering of alternatives, utility values, fuzzy estimates, and multiplicative preference relations [Ramalho et al. 2019, Ekel et al. 2020]), which cover all practical situations of preference elicitation. Besides, in many cases, deterministic information should also be taken into account. Although the possibility to use diverse preference formats provides psychological comfort for experts, it creates a need to convert all utilized formats to a unique format which can be processed and analyzed. Considering the advantages and rationality of applying nonreciprocal fuzzy preference relations for this goal, the results of [Herrera-Viedma et al. 2002, Ramalho et al. 2019, Ekel et al. 2020] permit one to utilize so-called transformation functions to convert different preference formats to nonreciprocal fuzzy preference relations, providing homogeneous preference information for decision making procedures.

Let us consider the situation of setting up a single fuzzy nonstrict preference relation R . It can be processed [Orlovsky 1983] to construct a fuzzy strict preference relation as

$$R^S = R \setminus R^{-1} \quad (10)$$

where R^{-1} is the inverse relation.

The membership function corresponding to (10) is:

$$\mu_R^S(X_k, X_l) = \max \{ \mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0 \}. \quad (11)$$

It serves as the basis for the choice procedure introduced in [Orlovsky 1983]. Its properties as well as questions of its axiomatic characterization are discussed, for instance, in [Banerjee 1993].

The utilization of (11) permits one to construct a set of nondominated alternatives with the membership function

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X} [1 - \mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_R^S(X_l, X_k), \quad (12)$$

which allows the evaluation of the level of nondominance of each alternative X_k . Considering that it is natural to choose alternatives providing the highest level of nondominance, one can choose alternatives X^{ND} as

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu_R^{ND}(X_k)\}. \quad (13)$$

The expressions (11)-(13) are applicable to solve choice problems as well as other problems, related to the evaluation, comparison, prioritization, and/or ordering of alternatives with a single criterion. These expressions may also be used when R is a vector of fuzzy preference relations, under different approaches to the multiattribute analysis.

In particular, when R is a vector of fuzzy preference relations, the expressions (11)-(13) can serve as a basis for the **first technique** for multiattribute decision making in a fuzzy environment if we take $R = \bigcap_{p=1}^q R_p$, i.e.,

$$\mu_R(X_k, X_l) = \min_{1 \leq p \leq q} \mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X. \quad (14)$$

When using the intersection (14), the set X^{ND} fulfils the role of a Pareto set [Orlovsky 1983]. Its contraction is possible on the basis of differentiating the importance of $R_p, p=1, 2, \dots, q$ with the application of the following convolution (of the aggregation of monobjective fuzzy preference relations) as follows:

$$\mu_T(X_k, X_l) = \sum_{p=1}^q \lambda_p \mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X, \quad (15)$$

where $\lambda_p \geq 0, p=1, 2, \dots, q$ are weights (importance factors) for the corresponding criteria normalized as

$$\sum_{p=1}^q \lambda_p = 1. \quad (16)$$

The construction of $\mu_T(X_k, X_l), X_k, X_l \in X$ allows one to obtain the membership function $\mu_T^{ND}(X_k)$ of the set of nondominated alternatives according to an expression similar to (12). The intersection of $\mu_R^{ND}(X_k)$ and $\mu_T^{ND}(X_k)$ defined as

$$\mu^{ND}(X_k) = \min \{ \mu_R^{ND}(X_k), \mu_T^{ND}(X_k) \}, \quad X_k \in X \quad (17)$$

provides us with

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu^{ND}(X_k)\}. \quad (18)$$

The expressions (12) and (13) can also serve as the basis for constructing the **second technique**, which is of a lexicographic character. It is associated with step-by-step introduction of criteria for comparing alternatives. The technique permits one to construct a sequence $X^1, X^2, \dots, X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq \dots \supseteq X^q$ with the use of the following expressions:

$$\mu_{R_p}^{ND}(X_k) = \inf_{X_l \in X^{p-1}} [1 - \mu_{R_p}^S(X_l, X_k)] = 1 - \sup_{X_l \in X^{p-1}} \mu_{R_p}^S(X_l, X_k), p=1, 2, \dots, q, \quad (19)$$

$$X^p = \{X_k^{ND,p} | X_k^{ND,p} \in X^{p-1}, \mu_{R_p}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{p-1}} \mu_{R_p}^{ND}(X_k)\}. \quad (20)$$

It should be noted that if R_p is transitive [Ekel 2002, Ekel et al. 1998], we can bypass the pairwise comparison of alternatives at the p -th step. In this situation, the comparison can be done on a serial basis (the direct use of (7) and (8)) with memorizing the best alternatives.

Finally, the **third technique** implemented within MDMS2 is associated with the following considerations.

The use of (12) represented in the form of

$$\mu_{R_p}^{ND}(X_k) = 1 - \sup_{X_l \in X} \mu_{R_p}^S(X_l, X_k), p=1, 2, \dots, q \quad (21)$$

permits one to construct the membership functions of the set of nondominated alternatives for each fuzzy preference relation.

The membership functions $\mu_{R_p}^{ND}(X_k), p=1, 2, \dots, q$ play a role identical to membership functions replacing objective functions $F_p(X), p=1, 2, \dots, q$ in solving multiobjective problems [Pereira Jr et al. 2015] on the basis of modifying the Bellman-Zadeh approach to decision making in a fuzzy environment [Bellman and Zadeh 1970]. Therefore, it is possible to construct

$$\mu^{ND}(X_k) = \min_{1 \leq p \leq q} \mu_{R_p}^{ND}(X_k) \quad (22)$$

to obtain X^{ND} .

If necessary to differentiate the importance of different preference relations, it is possible to transform (22) to

$$\mu^{ND}(X_k) = \min_{1 \leq p \leq q} [\mu_{R_p}^{ND}(X_k)]^{\lambda_p}. \quad (23)$$

The utilization of (23) does not require the normalization of $\lambda_p, p=1, \dots, q$ in the way similar to (16).

The results presented above have been used to solve the problem of supporting the analysis and prioritization of new electric transmission line projects, in the prospection stage, aimed at greater efficiency and effectiveness of the advanced preparation of concessionaires and investors for participating in the auctions [Ekel et al. 2019]. These results have also been applied to considering qualitative information in solving spatial problems with their particular application to the determination of generating sites for renewable energy systems [Silva et al. 2020].

Computing Implementation

The results described above have been implemented within the framework of the Multicriteria Decision Making System MDMS2. It has been developed in the C# programming language and is executed in the graphical environment of the Microsoft Windows Operating System. In particular, the window MDMS2 (see Figure 2) is divided in tabs. The use of tab "Descrição do problema" permits one to select the technique to be used to solve the problem, to indicate the numbers of alternatives and criteria, and to show the solution results.



Fig. 2. MDMS2 window and "Descrição do problema" tab.

After the number of criteria is set, tabs are created: each criterion taken into account corresponds to a separate tab. Each "Critério N" tab is designed to choose the preference format (nonreciprocal fuzzy preference relations, ordering of alternatives, utility values, fuzzy estimates or multiplicative preference relations) for the criteria and also to load information on each alternative.

Choice of an Alternative of an Energy Source

As an illustrative example, below we consider the solution of the problem of choosing an alternative of an energy source to be installed in an isolated system. A mining site is located in an isolated location remote from the interconnected power system. The mine is to have a total demand of 20 MW of assured energy.

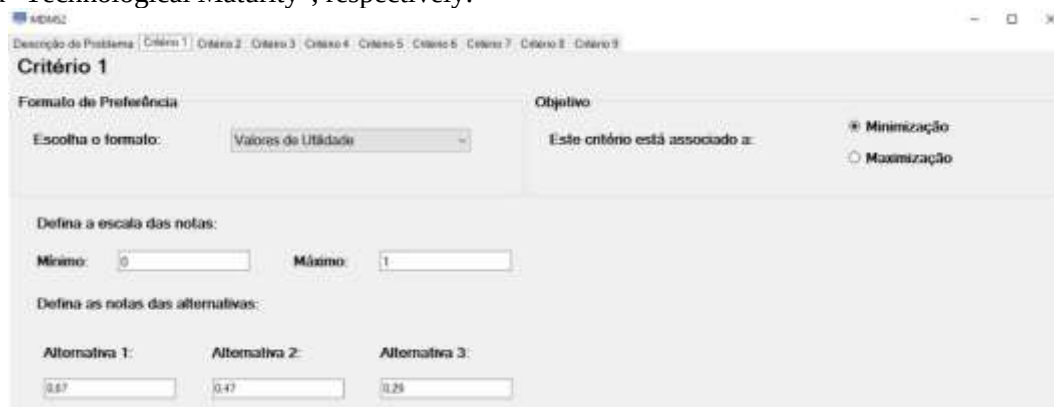
To meet this demand, a study is to be carried out on the choice of one of the following energy sources: diesel generation source (Alternative 1), wind energy source (Alternative 2), and solar energy source (Alternative 3). To analyze these alternatives, the following criteria of a quantitative character are to be taken into account: 1. Capacity factor, 2. Levelized Cost of Energy, 3. Deployment Time, 4. Space Requirement, 5. Plant Lifetime, and 6. Greenhouse Gases Emissions. Besides the following criteria of a quantitative character are to be considered: 1. Environmental Risk, 2. Corporate Image Risk, and 3. Technological Maturity. The estimates of the alternatives from the point of view of the indicated above criteria are given in Table 1.

Table 1. Assessing alternatives according to the criteria under consideration.

Attributes		Diesel Generation	Wind Generation	Solar Generation
Installed Power	kW	23.000,00	42.600,00	69.000,00
Capacity Factor	%	87%	47%	29%
Assured Energy	kW	20.010,00	20.022,00	20.010,00
Levelized Cost of Energy	\$/kW	37,00	36,93	0,08
Deployment Time	Months	24,00	30,00	22,00
Space Requirement	m2/kW	4,00	43,00	23,00
Lifespan	Years	15,00	30,00	25,00
Greenhouse Gas Emissions	tCO2/MWh	0,76	-	-
Environmental Risk		High	Low	High
Corporate Image Risk		High	Low	Low
Technological Maturity		High	High	Medium

Among the considered criteria, the Levelized Cost of Energy, Deployment Time, Space Requirement, Greenhouse Gases Emission, Environmental Risk, and Corporate Image Risk are related to minimization. On the other hand, the criteria of the Capacity Factor, Plant Lifetime, and Technological Maturity are related to maximization. Thus, the analysis is associated with 9 criteria and 3 alternatives. This information is to be included in the "Descrição do Problema" (Figure 2).

Then, for each criterion, the corresponding preference format should be chosen. For the criteria of a quantitative character, numerical scales (as the utility values format) are utilized. At the same time, for the criteria of a qualitative character, the fuzzy estimates are used. As an example, Figures 3-5 provide information on the criteria "Capacity Factor", "Levelized Cost of Energy", and "Technological Maturity", respectively.



MDM2

Descrição do Problema | Critério 1 | Critério 2 | Critério 3 | Critério 4 | Critério 5 | Critério 6 | Critério 7 | Critério 8 | Critério 9

Critério 1

Formato de Preferência

Escolha o formato:

Objetivo

Este critério está associado a:

☒ Minimização

☐ Maximização

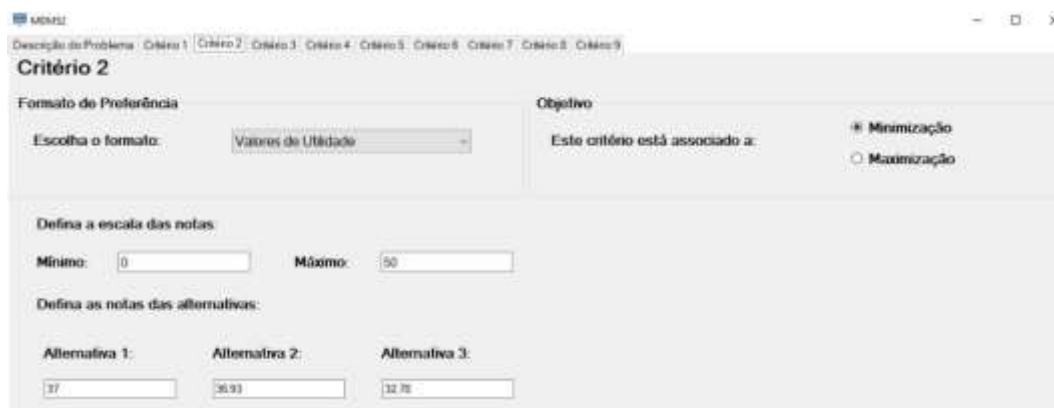
Defina a escala das notas:

Mínimo: Máximo:

Defina as notas das alternativas:

Alternativa 1: Alternativa 2: Alternativa 3:

Fig. 3. Estimates for the first criterion (Capacity Factor).



MDM2

Descrição do Problema | Critério 1 | Critério 2 | Critério 3 | Critério 4 | Critério 5 | Critério 6 | Critério 7 | Critério 8 | Critério 9

Critério 2

Formato de Preferência

Escolha o formato:

Objetivo

Este critério está associado a:

☒ Minimização

☐ Maximização

Defina a escala das notas:

Mínimo: Máximo:

Defina as notas das alternativas:

Alternativa 1: Alternativa 2: Alternativa 3:

Fig. 4. Estimates for the second criterion (Levelized Cost of Energy).

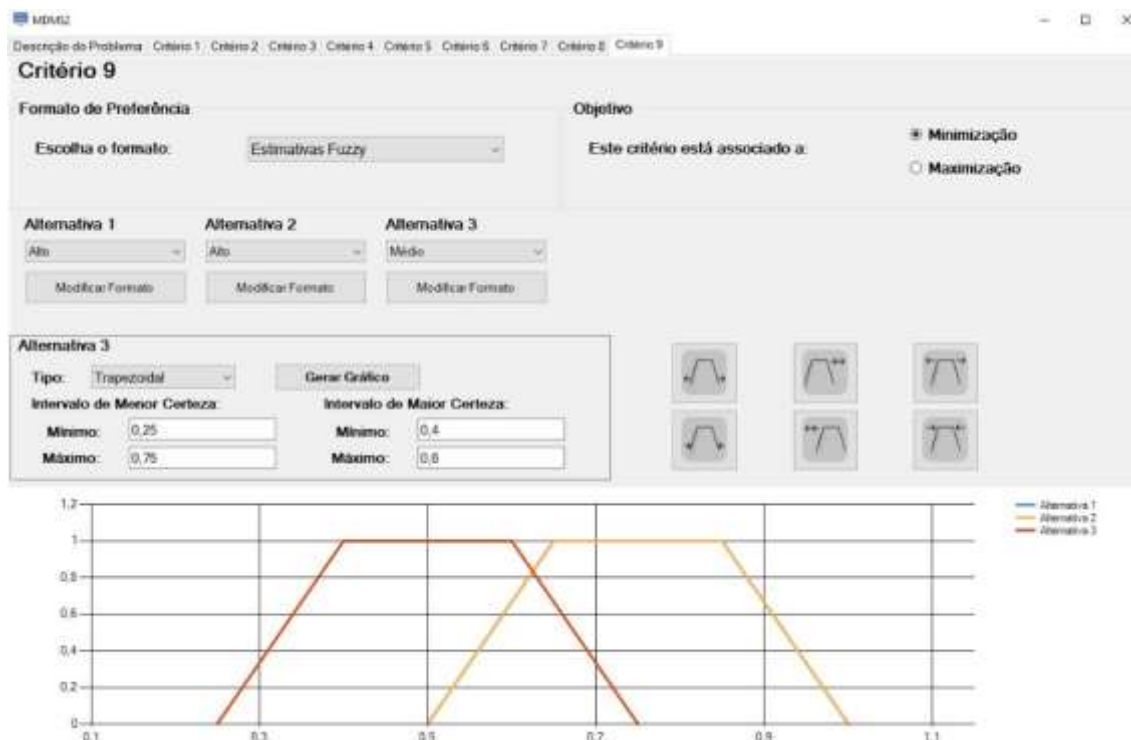


Fig. 5. Estimates for the ninth criterion (Technological Maturity).

The application of the results of [Ekel et al. 2020] to the estimates related to the criteria of a quantitative character permits one to construct the corresponding nonreciprocal fuzzy preference relations represented in Figure 6. At the same time, the use of the results of [Ekel et al. 2020] to the estimates related to the criteria of a qualitative character provides the nonreciprocal fuzzy preference relations represented in Figure 6 as well.

Resultado

Relação de Preferência do 1º Critério:

[1,000 1,000 1,000]
[0,600 1,000 1,000]
[0,420 0,820 1,000]

Relação de Preferência do 4º Critério:

[1,000 1,000 1,000]
[0,220 1,000 0,600]
[0,620 1,000 1,000]

Relação de Preferência do 7º Critério:

[1,000 0,000 0,000]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 2º Critério:

[1,000 0,999 0,916]
[1,000 1,000 0,917]
[1,000 1,000 1,000]

Relação de Preferência do 5º Critério:

[1,000 0,571 0,857]
[1,000 1,000 1,000]
[1,000 0,714 1,000]

Relação de Preferência do 8º Critério:

[1,000 0,000 0,000]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 3º Critério:

[1,000 1,000 0,958]
[1,000 1,000 0,958]
[1,000 1,000 1,000]

Relação de Preferência do 6º Critério:

[1,000 0,238 0,238]
[1,000 1,000 1,000]
[1,000 1,000 1,000]

Relação de Preferência do 9º Critério:

[1,000 1,000 0,833]
[1,000 1,000 0,833]
[1,000 1,000 1,000]

Fig. 6. Nonreciprocal fuzzy preference relations.

The results of applying the first technique to the nonreciprocal fuzzy preference relations represented in Figure 6 (execution of (14), (11)-(13), respectively) are given in Figure 7. These results generate the Alternative 3 as the problem solution.

Interseção entre as matrizes de preferência fuzzy:

$$\begin{bmatrix} 1,000 & 0,000 & 0,000 \\ 0,220 & 1,000 & 0,600 \\ 0,420 & 0,714 & 1,000 \end{bmatrix}$$

Relação de Preferência Fuzzy Estrita da Interseção:

$$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,220 & 0,000 & 0,000 \\ 0,420 & 0,114 & 0,000 \end{bmatrix}$$

Conjunto Não Dominado das Alternativas:

$$[0,580 \ 0,886 \ 1,000]$$

Alternativa Escolhida: 3

Fig. 7. The problem solution on the basis of the first technique.

The application of the second technique presupposes the ordering of the criteria. Let us consider the application of the second technique with the following ordering of the criteria: 6, 7, 8, 9, 2, 3, 4, 5, and 1.

The process of decision making is reflected by Figure 8. In particular, the processing of the criterion "Greenhouse Gases Emissions", applying (11), (19), and (20), permits us to cut the Alternative 1. The application of the criterion "Environmental Risk" to analyze the Alternatives 2 and 3 does not permit us to reduce the decision uncertainty region (this criterion has no resolution power to distinguish the analyzed alternatives). Then, go to the analysis of the Alternative 2 and 3, applying the criterion "Corporate Image Risk". The use of (11), (19), and (20) generates the Alternative 2 as the problem solution.

Relação de Preferência Fuzzy Estrita adicionando o 6º Critério:

$$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,762 & 0,000 & 0,000 \\ 0,762 & 0,000 & 0,000 \end{bmatrix}$$

Conjunto Não Dominado das Alternativas adicionando o 6º Critério:

$$[0,238 \ 1,000 \ 1,000]$$

Relação de Preferência Fuzzy Estrita adicionando o 7º Critério:

$$\begin{bmatrix} 0,000 & 0,000 \\ 0,000 & 0,000 \end{bmatrix}$$

Conjunto Não Dominado das Alternativas adicionando o 7º Critério:

$$[1,000 \ 1,000]$$

Relação de Preferência Fuzzy Estrita adicionando o 8º Critério:

$$\begin{bmatrix} 0,000 & 0,000 \\ 1,000 & 0,000 \end{bmatrix}$$

Conjunto Não Dominado das Alternativas adicionando o 8º Critério:

$$[0,000 \ 1,000]$$

Alternativa Escolhida: 2

Fig. 8. The problem solution on the basis of the second technique.

Finally, let us apply the third technique. The process of its utilization is reflected by Figure 9. The use of (11) and (21) permits us to construct the membership functions of the set of nondominated alternatives for each considered criterion. Their intersection in accordance with (22) provides the following solution: Alternative 3.

Thus, the first and third techniques indicate the Alternative 3 as the problem solution. At the same time, the second technique indicates the Alternative 2 as the problem solution. Taking this into account, it should be stressed that the fact of the possibility to obtain different solutions on the basis of different approaches is natural, and the choice of the approach is a prerogative of the decision maker.

Relação de Preferência Fuzzy Estrita do 1º Critério:	Conjunto Não Dominado das Alternativas do 5º Critério:
$\begin{bmatrix} 0,000 & 0,400 & 0,580 \\ 0,000 & 0,000 & 0,180 \\ 0,000 & 0,000 & 0,000 \end{bmatrix}$	$\begin{bmatrix} 0,571 & 1,000 & 0,714 \end{bmatrix}$
Conjunto Não Dominado das Alternativas do 1º Critério:	Relação de Preferência Fuzzy Estrita do 6º Critério:
$\begin{bmatrix} 1,000 & 0,600 & 0,420 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,762 & 0,000 & 0,000 \\ 0,762 & 0,000 & 0,000 \end{bmatrix}$
Relação de Preferência Fuzzy Estrita do 2º Critério:	Conjunto Não Dominado das Alternativas do 6º Critério:
$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,001 & 0,000 & 0,000 \\ 0,084 & 0,083 & 0,000 \end{bmatrix}$	$\begin{bmatrix} 0,238 & 1,000 & 1,000 \end{bmatrix}$
Conjunto Não Dominado das Alternativas do 2º Critério:	Relação de Preferência Fuzzy Estrita do 7º Critério:
$\begin{bmatrix} 0,916 & 0,917 & 1,000 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 1,000 & 0,000 & 0,000 \\ 1,000 & 0,000 & 0,000 \end{bmatrix}$
Relação de Preferência Fuzzy Estrita do 3º Critério:	Conjunto Não Dominado das Alternativas do 7º Critério:
$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,000 & 0,000 & 0,000 \\ 0,042 & 0,042 & 0,000 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 1,000 & 1,000 \end{bmatrix}$
Conjunto Não Dominado das Alternativas do 3º Critério:	Relação de Preferência Fuzzy Estrita do 8º Critério:
$\begin{bmatrix} 0,958 & 0,958 & 1,000 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 1,000 & 0,000 & 0,000 \\ 1,000 & 0,000 & 0,000 \end{bmatrix}$
Relação de Preferência Fuzzy Estrita do 4º Critério:	Conjunto Não Dominado das Alternativas do 8º Critério:
$\begin{bmatrix} 0,000 & 0,780 & 0,380 \\ 0,000 & 0,000 & 0,000 \\ 0,000 & 0,400 & 0,000 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 1,000 & 1,000 \end{bmatrix}$
Conjunto Não Dominado das Alternativas do 4º Critério:	Relação de Preferência Fuzzy Estrita do 9º Critério:
$\begin{bmatrix} 1,000 & 0,220 & 0,620 \end{bmatrix}$	$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,000 & 0,000 & 0,000 \\ 0,167 & 0,167 & 0,000 \end{bmatrix}$
Relação de Preferência Fuzzy Estrita do 5º Critério:	Conjunto Não Dominado das Alternativas do 9º Critério:
$\begin{bmatrix} 0,000 & 0,000 & 0,000 \\ 0,429 & 0,000 & 0,286 \\ 0,143 & 0,000 & 0,000 \end{bmatrix}$	$\begin{bmatrix} 0,833 & 0,833 & 1,000 \end{bmatrix}$
	Interseção dos Conjuntos Não Dominados das Alternativas:
	$\begin{bmatrix} 0,000 & 0,220 & 0,420 \end{bmatrix}$
	Alternativa Escolhida: 3

Fig. 9. The problem solution on the basis of the third technique.

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