

Transformation functions in Multi-Criteria Decision Analysis: a framework

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ABSTRACT

There is a consolidated understanding in the Multi-Criteria Decision Analysis literature about experts' cognitive stress when assessing many alternatives. This understanding relates the cognitive stress of experts to the intransitive assessments. This problem is relevant because intransitive assessments result in inconsistent matrices and unreliable information for decision-making. This research aims to develop a methodology to obtain transitive assessments even when the number of alternatives to be assessed is high. This methodology consists of simplifying the process of evaluating alternatives and generalizing its use to a broader range of decision-making problems. The theoretical and operational framework of the research includes the literature on group decision making, preference formats, transformation functions, and multi-criteria indicators. The contribution of this research is to provide reliable information for decision-making, in particular, a better representation of the costs of doing business in the twenty largest economies in the world.

Keywords. Preference formats. Transformation functions for converting preference formats. Composite Indicators.

Paper topics: ADM – Apoio à Decisão Multicritério; AD&GP – PO na Administração e Gestão da Produção.

1. Introduction

There is a consolidated understanding in the Multi-Criteria Decision Analysis literature about experts' cognitive stress when assessing many alternatives. This understanding relates the cognitive stress of experts to the intransitivity of assessments and is reflected in the works of Herrera-Viedma et al. [2004], Ishizaka and Lusti [2004], Benítez et al. [2011;2012], and Bortot et al. [2022]. This problem is relevant because intransitive assessments result in inconsistency matrices and unreliable information for decision-making.

This research aims to develop a methodology to obtain transitivity assessments even when the number of alternatives to be assessed is high. This methodology consists of simplifying the process of evaluating alternatives and generalizing its use to a broader range of decision-making problems. This methodology is based on the Group Decision Analysis literature theoretical framework, specifically the preference formats, including the transformation functions for converting preference formats (Section 2). The operational framework for applying the methodology is based on the multi-criteria indicators [El Gibari et al., 2019], specifically the Cost of Doing Business Index [Bernardes et al., 2021; Ekel et al., 2022].

The study results offer two contributions to the literature. The first contribution is associated with the development of the methodology and consists of providing reliable information for decision-making. The second contribution is associated with applying the methodology to the specific case and consists of the best representation of the costs of doing business in the twenty largest economies in the world.

2. Preference formats

Although there is an expressive number of researches on preference formats, the works carried out by Chiclana et al. [1998;2001], Pedrycz et al. [2011], Ramalho et al. [2019], and Ekel et al. [2019] gather sufficient information on the main theoretical, methodological and practical aspects of the topic, supporting the following discussion.

Experts express opinions about decision-making problems in different ways. The different ways of expressing opinions are known in the specialized literature as the format of preferences. The most commonly used preference formats are Ordering of Alternatives (**OA**), Utility Values (**UV**), Fuzzy Estimation (**FE**), Multiplicative Relations (**MR**), and Fuzzy Relations which can be Additive Reciprocal (**RR**) and Non-Reciprocal (**RN**). The decision on which format to use is associated with the: level of information uncertainty, expert domain on the topic, and nature of information (e.g., qualitative or quantitative).

There is no preferred format that is free from limitations. However, the operationalization of the **OA** format is considered simpler than the other formats. Decision-makers find it more challenging to assess alternatives quantitatively than to rank them. First, decision-makers have more confidence in the ratings of alternatives than in their respective values. Second, decision-makers often find it difficult to express how much one alternative is better than the other. Third, the assessment effectiveness is impaired due to differences between the experts' numerical interpretation of linguistic terms and the numerical representation of the model. Fourth, the quantitative assessment of many alternatives exerts cognitive stress on decision-makers, resulting in intransitive assessments. Researchers suggest that the number of alternatives should be less than ten so that assessments are carried out optimally [Saisana and Tarantola, 2002; Greco et al., 2019].

These characteristics suggest that assessing alternatives by the **OA** format is the simplest and fastest way to rank alternatives. However, many multi-criteria decision-making problems are interested in the scores of alternatives. Furthermore, the **OA** format is problematic when it is necessary to aggregate orderings performed by different decision-makers in a collective ordering, as the literature considers this operation very controversial [Bustince et al., 2020].

These two limitations substantially reduce the range of problems that the **OA** format can solve. For example, it prevents obtaining the group's decision on the weights of the criteria that make up a multi-criteria indicator. The methodology developed in this research is particularly interested in solving this problem. In other words, assign weights to criteria assessed by the **OA** format by a group of experts. For this purpose, the research employs the Group Decision Analysis literature, specifically the transformation functions.

2.1. Transformation functions for converting preference formats

The possibility of employing different preference formats offers psychological comfort to the decision-maker in assessing alternatives. Furthermore, allowing experts to express preferences in various formats is a way of considering the particular characteristics of each expert. However, assessing alternatives in different formats means, in practice, the production of heterogeneous information.

Several studies in the group decision-making field offer the means for treating heterogeneous information. In particular, the studies by Chen et al. [2015], Li et al. [2016], Zhang et al. [2019], Figueiredo et al. [2021], Wu and Liao [2021], and Libório et al. [2022] show that it is possible to homogenize assessments in different formats through transformation functions.

In short, transformation functions were initially conceived to homogenize assessments performed in preferred formats. However, nothing prevents these transformation functions from being used to convert assessments from the **OA** format to formats. This flexibility makes it possible to convert alternatives ordered by importance into scores and solve the two problems considered in this research. First, it allows for defining scores or weights for the criteria of a multi-criteria indicator. Second, it allows for aggregating individual opinions into a collective opinion.

The conversion of alternatives ordered by importance to weights can be performed by the transformation functions presented by Stillwell et al. [1981], Chiclana et al. [1998], Ekel et al. [2011], and Ramalho et al. [2019, p.258] and transcribed below

$$\mathbf{OA} \rightarrow \mathbf{UV}(X_k) = \frac{1/O_k}{\sum_{k=1}^n (1/O_k)} \quad (1)$$

where O_k is the order of the k -th alternative.

$$\mathbf{OA} \rightarrow \mathbf{RM}(X_k X_l) = m^{\left(\frac{O_l - O_k}{n-1}\right)}, n > 1 \quad (2)$$

where O_l is the order of the l -th alternative, $m = 9$, and corresponds to the upper limit of the scale proposed by Saaty [1977, p.246] for the construction of the multiplicative relationship matrix $\mathbf{RM}(X_k X_l)$.

$$\mathbf{OA} \rightarrow \mathbf{RN}(X_k X_l) = \begin{cases} \frac{1}{2} + \frac{O_l - O_k}{2(n-1)} & \text{if } O_k > O_l \\ 1 & \text{if } O_k \leq O_l \end{cases} \quad (3)$$

$$\mathbf{OA} \rightarrow \mathbf{RR}(X_k X_l) = \frac{1}{2} \left(1 + \frac{O_l - O_k}{n-1} \right) \quad (4)$$

The conversion of assessments in **OA** format to the formats **RM**, **RN** e **RR** through the transformation functions (2), (3) e (4) is especially valuable, as they guarantee the consistency of the matrices $\mathbf{RM}(X_k X_l)$, $\mathbf{RN}(X_k X_l)$ e $\mathbf{RR}(X_k X_l)$.

3. Materials and methods

The methodology that allows obtaining transitive assessments of many alternatives was developed and applied from the accomplishment of the nine steps below.

Problem definition: is assigning weights to the criteria that make up a multi-criteria indicator based on the opinion of a group of experts. The weighting of criteria (henceforth sub-indicators) is considered a critical step for constructing a multi-criteria indicator (henceforth composite indicator). The criticality of this step in the construction of composite indicators is shown in numerous studies, among which it is possible to mention Dialga and Le Giang [2017], Becker et al. [2017], and Otoi et al. [2021]. Specifically, the methodology presented in this research is based on assigning weights to the seventeen sub-indicators of the Cost of Doing Business Index or CDBI¹ [Bernardes et al., 2021; Ekel et al., 2022].

Selection of experts: was defined considering two criteria. First, have more than five years of professional experience in the business field. Second, have a minimum academic background of a bachelor's degree in business. Ten experts met these criteria and were invited to the sub-indicator assessment stage.

Assessment of sub-indicators: the experts assessed the sub-indicators using the **OA** format. The **OA** format is expressed through a vector $O = [o(x_1), o(x_2), o(x_k) \dots, o(x_n)]$. Where $o(x_k)$ is a swap function that indicates the position of the sub-indicator x_k among the integer values $\{1, 2, 3, \dots, n\}$. Among these integer values, 1 is the sub-indicator that most impacts the costs of doing business, and n is the sub-indicator that has the least impact.

Conversion of the assessments: the assessments in **OA** format were converted into the **UV** format through the transformation function (1). The result of this conversion is a set of n utility values $U = \{u(x_1), u(x_2), \dots, u(x_k), \dots, u(x_n)\}$, where $u(x_k) \in [0,1]$ and represents the weight assigned to the sub-indicator x_k . The transformation function (1) was chosen because it allows obtaining the weights directly, while the other functions require the calculation of the eigenvalues and eigenvectors of the matrices.

Definition of collective weights: was performed by aggregating the weights of the sub-indicators of each expert in weights that represent the collective opinion. This operation is performed by the arithmetic mean of the individual weights for each sub-indicator.

Obtaining the consensus degree: was carried out in two stages. First, the degree of individual consensus was obtained through the following expression.

$$C^I = \sum_{l=1}^m \left(1 - \frac{|O^G(X_k) - O^{E_i}(X_k)|}{n-1} \right) \quad (5)$$

where $O^G(X_k)$ corresponds to the position of the k -th sub-indicator of the collective opinion, $O^{E_i}(X_k)$ corresponds to the k -th position of the sub-indicator of the expert opinion E_i .

Second, the individual consensus degree C^I is used to obtain the group's consensus degree through the following expression.

$$C^G = \frac{\sum_{k=1}^n C^I}{n} \quad (6)$$

Selection and data collection: the world's twenty largest economies were selected based on 2020 Gross Domestic Product (GDP) at current prices in the U.S. dollars [International Monetary Fund, 2020]. Then, the seventeen sub-indicators from each of the twenty selected

¹ The CDBI only considers the sub-indicators of cost of the World Bank's Ease of Doing Business Index (EDBI). The EDBI is an aggregate of sub-indicators that reflect the ease of doing business in 190 countries, being widely used to analyze the quality of the regulatory environment and its relationship with economic growth (Kuc-Czarnecka et al., 2020).

countries were collected from the World Bank DataBank: Doing Business [The World Bank Group, 2020].

Construction of the CDBI: was carried out from the aggregation of normalized and weighted sub-indicators, applying the following expression

$$CDBI = \sum_{k=1}^n \left(\frac{x_{kr} - \min(x_{kr})}{\max(x_{kr}) - \min(x_{kr})} \right) u_k \quad (7)$$

where x_{kr} is the value of the k -th sub-indicator of the r -th country, $\min(x_{kr})$ is the minimum value of the k -th sub-indicator among all countries, $\max(x_{kr})$ is the maximum value of the k -th sub-indicator among all countries and u_k é o weight assigned to the k -th sub-indicator.

Consistency analysis: verified by Pearson's [1901] correlation between CDBI and GDP per capita in US\$ [International Monetary Fund, 2020]. This correlation makes it possible to verify the CDBI's power to represent the business environment, as countries with higher costs of doing business have worse economic performance [Bernardes et al., 2021].

4. Results

As illustrated in Figure 1, the results of this research can be separated into five parts. The data resulting from the first four parts are presented in the annex in Tables 1, 2, and 3. The resulting data of the fifth part are presented in the annex in Table 4.

Table 1. Order of importance of sub-indicators in the cost of doing business.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
Cp1	2	3	1	5	13	5	1	10	1	8
Ec1	4	13	16	16	16	8	12	16	12	5
Ec2	8	14	6	4	1	9	10	13	3	6
Ec3	10	16	17	15	17	7	13	15	2	7
Ec4	11	15	15	13	2	6	11	14	7	3
Ge1	5	17	5	17	12	16	4	17	11	9
Pt1	1	2	2	3	3	2	5	3	6	4
Pt2	13	10	10	2	4	3	6	2	5	10
Pt3	3	1	9	1	5	1	2	1	4	11
Ri1	12	11	14	14	14	17	3	12	8	12
Rp1	9	12	13	6	15	4	7	11	9	13
St1	6	5	4	9	6	14	8	4	16	2
St2	7	4	3	10	7	15	9	5	17	1
Tb1	14	9	12	11	9	12	17	8	10	14
Tb2	15	7	11	12	11	13	16	9	13	15
Tb3	16	8	8	7	8	10	15	6	14	16
Tb4	17	6	7	8	10	11	14	7	15	17

1. Weights obtained after applying the transformation function (1)

2. Collective weights resulting from the arithmetic mean of individual weights

Table 2. Weights of sub-indicators according to individual and group opinions.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Group
Cp1	0.15	0.10	0.29	0.06	0.02	0.06	0.29	0.03	0.29	0.04	0.13
Ec1	0.07	0.02	0.02	0.02	0.02	0.04	0.02	0.02	0.02	0.06	0.03
Ec2	0.04	0.02	0.05	0.07	0.29	0.03	0.03	0.02	0.10	0.05	0.07
Ec3	0.03	0.02	0.02	0.02	0.02	0.04	0.02	0.02	0.15	0.04	0.04
Ec4	0.03	0.02	0.02	0.02	0.15	0.05	0.03	0.02	0.04	0.10	0.05
Ge1	0.06	0.02	0.06	0.02	0.02	0.02	0.07	0.02	0.03	0.03	0.03
Pt1	0.29	0.15	0.15	0.10	0.15	0.06	0.10	0.05	0.07	0.16	0.16
Pt2	0.02	0.03	0.03	0.15	0.10	0.05	0.15	0.06	0.03	0.03	0.03
Pt3	0.10	0.29	0.03	0.29	0.06	0.29	0.15	0.29	0.07	0.03	0.03
Ri1	0.02	0.03	0.02	0.02	0.02	0.02	0.10	0.02	0.04	0.02	0.03
Rp1	0.03	0.02	0.02	0.05	0.02	0.07	0.04	0.03	0.03	0.02	0.03
St1	0.05	0.06	0.07	0.03	0.05	0.02	0.04	0.07	0.02	0.15	0.06
St2	0.04	0.07	0.10	0.03	0.04	0.02	0.03	0.06	0.02	0.29	0.07
Tb1	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.04	0.03	0.02	0.03
Tb2	0.02	0.04	0.03	0.02	0.03	0.02	0.02	0.03	0.02	0.02	0.03
Tb3	0.02	0.04	0.04	0.04	0.04	0.03	0.02	0.05	0.02	0.02	0.03
Tb4	0.02	0.05	0.04	0.04	0.03	0.03	0.02	0.04	0.02	0.02	0.03



5. Construction of the Cost of Doing Business Index

Table 3. Consensus degree of experts and the group.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Group
Cp1	1.00	0.94	0.94	0.81	0.31	0.81	0.94	0.50	0.94	0.63	0.78
Ec1	0.44	1.00	0.81	0.81	0.81	0.69	0.94	0.81	0.94	0.50	0.78
Ec2	0.81	0.44	0.94	0.94	0.75	0.75	0.69	0.50	0.88	0.94	0.76
Ec3	0.94	0.56	0.50	0.63	0.50	0.88	0.75	0.63	0.56	0.88	0.68
Ec4	0.81	0.56	0.56	0.69	0.63	0.88	0.81	0.63	0.94	0.69	0.72
Ge1	0.63	0.63	0.63	0.63	0.94	0.69	0.56	0.63	1.00	0.88	0.72
Pt1	0.88	0.94	0.94	1.00	1.00	0.94	0.88	1.00	0.81	0.94	0.83
Pt2	0.56	0.75	0.75	0.75	0.81	1.00	0.75	0.94	0.75	0.83	0.75
Pt3	0.88	1.00	0.50	1.00	0.75	1.00	0.94	1.00	0.81	0.38	0.83
Ri1	1.00	0.94	0.88	0.88	0.88	0.69	0.44	1.00	0.75	1.00	0.84
Rp1	0.94	0.88	0.81	0.75	0.69	0.63	0.81	0.94	0.94	0.81	0.82
St1	0.94	0.88	0.81	0.88	0.94	0.56	0.94	0.81	0.44	0.69	0.79
St2	0.81	1.00	0.94	0.63	0.81	0.31	0.69	0.94	0.19	0.81	0.71
Tb1	0.88	0.56	0.75	0.69	0.56	0.75	0.94	0.50	0.63	0.88	0.71
Tb2	0.88	0.38	0.63	0.69	0.63	0.75	0.94	0.50	0.75	0.88	0.70
Tb3	0.88	0.63	0.63	0.56	0.63	0.75	0.94	0.50	1.00	0.88	0.74
Tb4	0.88	0.44	0.50	0.56	0.69	0.75	0.94	0.50	1.00	0.88	0.71

3. Individual consensus degree after applying equation (5)

4. Group consensus degree after applying equation (6)

Figure 1. The results of ordering sub-indicators by importance, converting them to weights, and obtaining the individual and group consensus degree.

First part: the average, maximum, and minimum weight of the sub-indicators are 0.06, 0.29, and 0.02, respectively. The cost to get connected to the electrical grid is the sub-indicator most associated with the minimum weight, and the total tax rate is the sub-indicator most associated with the maximum weight. The minimum weight was assigned to the cost to get connected to the electrical grid three times, and the maximum weight was assigned to the total tax rate four times.

Second part: the sub-indicators with the most significant impact on the costs of doing business are the cost of construction permits, labor tax and contributions, and total tax rate. The weights of these three sub-indicators are 0.13, 0.12, and 0.16, respectively. The combination of these sub-indicators has a weight of 0.41 in the countries' CDBI scores.

Third part: the highest individual consensus degree was 0.83, corresponding to the assessments of specialists 1 and 7. Specialist 8 presented more divergent opinions with the group. Expert 8's consensus degree with the group was 0.71.

Fourth part: the consensus degree of the expert group was 0.76. The sub-indicator with the lowest consensus degree was 0.68, corresponding to court fees. The highest consensus degree on sub-indicators corresponds to labor tax and contributions, with a consensus degree of 0.93.

Fifth part: the three countries with the highest business costs are Mexico, Italy, and Brazil. Switzerland, Canada, and the United Kingdom have the lowest business costs. CDBI scores and GDP per capita in US\$ are strongly and negatively correlated. The correlation coefficient between these two indicators was -0.71. This correlation suggests that countries with better socioeconomic conditions have lower business costs. Finally, Figure 2 shows the scores and ranking of the twenty largest economies in the world concerning the costs of doing business.

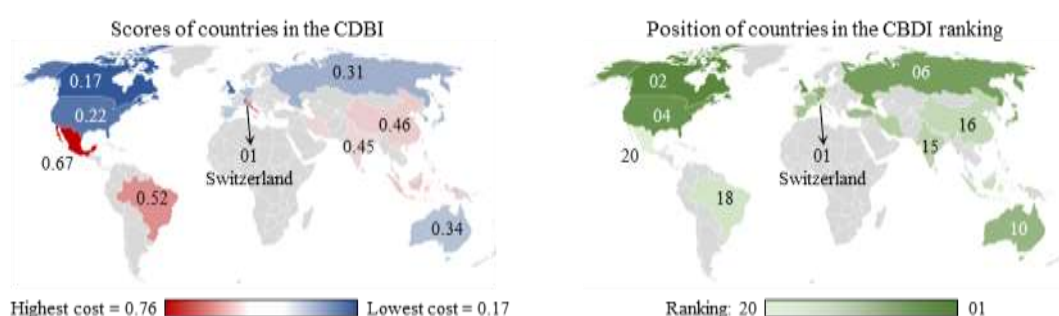


Figure 2. Representation of the costs of doing business of the twenty largest economies in the world.

5. Conclusions

The research shows that the proposed methodology brings promising results but that it is necessary to deepen the analysis on at least two points. First, invite experts to evaluate the sub-indicators using the **RM** format to deepen the analysis on the advantages of applying the methodology and answer questions such as: What is the consistency ratio of the matrices? How much time is required of each expert who evaluates the sub-indicators by the formats **OA** and **RM**. What is the difference between the consensus degree obtained from assessments in **RM** and **OA** format? Second, to analyze the uncertainty associated with variations in the input parameters to obtain a more stable composite indicator. This uncertainty analysis should answer questions such as: What is the average change in the position of countries in the ranking after excluding the experts who most diverge from the group? How much do country positions vary when sub-indicators are normalized by z-scores or aggregated by geometric or harmonic mean. How much do the weights of the sub-indicators vary when the evaluations in **OA** format are converted by the other transformation functions? What is the impact of varying these weights on the position of countries in the ranking? Those are questions the authors are considering for present and future research.

Annex

This annex presents the four tables resulting from assessing alternatives, conversion of assessment formats, consensus degree computation, and the composite indicator construction and validation. Table 1 shows the importance order of the seventeen sub-indicators according to each of the ten experts consulted.

Table 1. Order of importance of sub-indicators in the cost of doing business.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
Cp1	2	3	1	5	13	5	1	10	1	8
Ec1	4	13	16	16	16	8	12	16	12	5
Ec2	8	14	6	4	1	9	10	13	3	6
Ec3	10	16	17	15	17	7	13	15	2	7
Ec4	11	15	15	13	2	6	11	14	7	3
Ge1	5	17	5	17	12	16	4	17	11	9
Pt1	1	2	2	3	3	2	5	3	6	4
Pt2	13	10	10	2	4	3	6	2	5	10
Pt3	3	1	9	1	5	1	2	1	4	11
Ri1	12	11	14	14	14	17	3	12	8	12
Rp1	9	12	13	6	15	4	7	11	9	13
St1	6	5	4	9	6	14	8	4	16	2
St2	7	4	3	10	7	15	9	5	17	1
Tb1	14	9	12	11	9	12	17	8	10	14
Tb2	15	7	11	12	11	13	16	9	13	15
Tb3	16	8	8	7	8	10	15	6	14	16
Tb4	17	6	7	8	10	11	14	7	15	17

Note: Cp1. Cost with construction permits; Ec1. Attorney fees; Ec2. Cost of Enforcing contracts; Ec3. Court fees; Ec4. Enforcement fees; Ge1. Cost to get connected to the electrical grid; Pt1. Labor tax and contributions; Pt2. Profit tax; Pt3. Total tax rate; Ri1. Cost with resolving insolvency; Rp1. Cost with registering property; St1. Costs with starting a business (Men's income); St2. Costs with starting a business (Women's income); Tb1. Cost to export: Border compliance; Tb2. Cost to export: Documentary compliance; Tb3. Cost to import: Border compliance; Tb4. Cost to import: Documentary compliance.

The results of converting the sub-indicators ordered by importance into weights and the aggregation of individual weights into collective weights are presented in Table 2.

Table 2. Weights of sub-indicators according to individual and group opinions.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Group
Cp1	0.15	0.10	0.29	0.06	0.02	0.06	0.29	0.03	0.29	0.04	0.13
Ec1	0.07	0.02	0.02	0.02	0.02	0.04	0.02	0.02	0.02	0.06	0.03
Ec2	0.04	0.02	0.05	0.07	0.29	0.03	0.03	0.02	0.10	0.05	0.07
Ec3	0.03	0.02	0.02	0.02	0.02	0.04	0.02	0.02	0.15	0.04	0.04
Ec4	0.03	0.02	0.02	0.02	0.15	0.05	0.03	0.02	0.04	0.10	0.05
Ge1	0.06	0.02	0.06	0.02	0.02	0.02	0.07	0.02	0.03	0.03	0.03
Pt1	0.29	0.15	0.15	0.10	0.10	0.15	0.06	0.10	0.05	0.07	0.12
Pt2	0.02	0.03	0.03	0.15	0.07	0.10	0.05	0.15	0.06	0.03	0.07
Pt3	0.10	0.29	0.03	0.29	0.06	0.29	0.15	0.29	0.07	0.03	0.16
Ri1	0.02	0.03	0.02	0.02	0.02	0.02	0.10	0.02	0.04	0.02	0.03
Rp1	0.03	0.02	0.02	0.05	0.02	0.07	0.04	0.03	0.03	0.02	0.03
St1	0.05	0.06	0.07	0.03	0.05	0.02	0.04	0.07	0.02	0.15	0.06

St2	0.04	0.07	0.10	0.03	0.04	0.02	0.03	0.06	0.02	0.29	0.07
Tb1	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.04	0.03	0.02	0.03
Tb2	0.02	0.04	0.03	0.02	0.03	0.02	0.02	0.03	0.02	0.02	0.03
Tb3	0.02	0.04	0.04	0.04	0.04	0.03	0.02	0.05	0.02	0.02	0.03
Tb4	0.02	0.05	0.04	0.04	0.03	0.03	0.02	0.04	0.02	0.02	0.03

The individual and collective consensus degrees computed from the weights of the sub-indicators are presented in Table 3.

Table 3. Individual and collective consensus degree.

Sub-indicator	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Group
Cp1	1.00	0.94	0.94	0.81	0.31	0.81	0.94	0.50	0.94	0.63	0.78
Ec1	0.44	1.00	0.81	0.81	0.81	0.69	0.94	0.81	0.94	0.50	0.78
Ec2	0.81	0.44	0.94	0.94	0.75	0.75	0.69	0.50	0.88	0.94	0.76
Ec3	0.94	0.56	0.50	0.63	0.50	0.88	0.75	0.63	0.56	0.88	0.68
Ec4	0.81	0.56	0.56	0.69	0.63	0.88	0.81	0.63	0.94	0.69	0.72
Ge1	0.63	0.63	0.63	0.63	0.94	0.69	0.56	0.63	1.00	0.88	0.72
Pt1	0.88	0.94	0.94	1.00	1.00	0.94	0.88	1.00	0.81	0.94	0.93
Pt2	0.56	0.75	0.75	0.75	0.88	0.81	1.00	0.75	0.94	0.75	0.79
Pt3	0.88	1.00	0.50	1.00	0.75	1.00	0.94	1.00	0.81	0.38	0.83
Ri1	1.00	0.94	0.88	0.88	0.88	0.69	0.44	1.00	0.75	1.00	0.84
Rp1	0.94	0.88	0.81	0.75	0.69	0.63	0.81	0.94	0.94	0.81	0.82
St1	0.94	0.88	0.81	0.88	0.94	0.56	0.94	0.81	0.44	0.69	0.79
St2	0.81	1.00	0.94	0.63	0.81	0.31	0.69	0.94	0.19	0.81	0.71
Tb1	0.88	0.56	0.75	0.69	0.56	0.75	0.94	0.50	0.63	0.88	0.71
Tb2	0.88	0.38	0.63	0.69	0.63	0.75	0.94	0.50	0.75	0.88	0.70
Tb3	0.88	0.63	0.63	0.56	0.63	0.75	0.94	0.50	1.00	0.88	0.74
Tb4	0.88	0.44	0.50	0.56	0.69	0.75	0.94	0.50	1.00	0.88	0.71

Four pieces of information are presented in Table 4. The first two are the scores and the countries' positions in the CDBI ranking. The last two are associated with the indicator used to validate the CDBI. That is, the ranking and values of GDP per capita in US\$ of the twenty countries analyzed.

Table 4. CDBI and GDP per capita of the twenty largest economies in the world in 2020.

Country	CDBI Scores	CDBI Ranks	GDP per capita in US\$	GDP per capita Rank
Switzerland	0.17	1	87,351	1
Canada	0.17	2	43,307	6
United Kingdom	0.20	3	41,127	7
United States	0.22	4	63,078	2
Netherlands	0.29	5	52,456	4
Russian Federation	0.31	6	10,148	15
South Korea	0.32	7	31,638	11
Japan	0.33	8	40,049	9
Germany	0.33	9	46,216	5
Australia	0.34	10	52,848	3
Australia	0.35	11	8,610	16
Spain	0.35	12	27,039	12
France	0.38	13	40,162	8

Iran	0.45	14	11,149	13
India	0.45	15	1,935	20
China	0.46	16	10,525	14
Indonesia	0.46	17	3,923	19
Brazil	0.52	18	6,841	18
Italy	0.52	19	31,707	10
Mexico	0.67	20	8,507	17

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