

**NETWORK NEW MULTIPLE CRITERIA DATA ENVELOPMENT ANALYSIS
MODEL: AN APPLICATION ON EDUCATIONAL CASE**

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ABSTRACT

DEA seeks to optimize efficiency analysis processes and to identify which *Decision Making Units* - DMUs are efficient. When the number of DMUs does not meet the Golden Rule, traditional DEA models presents poor discrimination power and unrealistic weight distribution. Problems that used the Network DEA model do not consider the Golden Rule in their applications, generating an efficiency analysis with low discrimination power among DMUs. This paper proposes a new *Network DEA* model, called *Network New Multiple Criteria Data Envelopment Analysis* - NNMCD EA, based on relational NDEA models. To test the NNMCD EA model a real instance linked to an evaluation problem of academic departments of a public university was used. The results indicated that the new NNMCD EA model provides better discrimination of DMUs compared to the relational NDEA model used and, statistical testing in conjunction with university expert participation validated the logical structure of the NNMCD EA model.

KEYWORDS. Network Data Envelopment Analysis, Network Multiple Criteria Data Envelopment Analysis, Academic Departments.

Paper topics: Análise Envoltória de Dados.

1. Introduction

DEA is a quantitative method that employs linear optimization to optimize efficiency analysis processes, whose objective is to identify which *Decision Making Units* - DMUs are efficient [Silva et al., 2020]. It is known that, under certain circumstances, when the number of DMUs does not meet the Golden Rule [Banker et al. 1989], that is, when this number is not large enough compared to the total number of inputs (*Inputs*) and outputs (*Outputs*), traditional DEA models usually produce solutions that identify many DMUs as efficient [Silva et al. 2017]. This poor discrimination power and unrealistic weight distribution presented by DEA models remain a major challenge, leading to the development of models to improve this performance, such as *Multiple Criteria Data Envelopment Analysis* - MCDEA [Silva et al. 2021].

Since its creation by DEA has been used to analyze efficiency in problems related to the educational sector. It is noticeable in Figure 1 that the work by Sinuany-Stern et al. [1994] is the most-cited with over 180 citations, averaging 6.46 citations per year. This paper presented a case study in which academic departments at Ben-Gurion University were evaluated using DEA - CCR. The study by Kao and Chung [2008] was the second most cited research with over 170 citations, an average of 12.21 citations per year. Also, with respect to Figure 1, the research conducted by Yang et al. [2012] was the third most cited, with 90 more citations, averaging 9.5 citations per year.

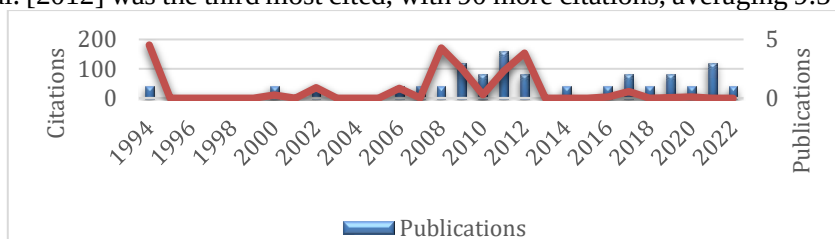


Figure 1- Bibliometric analysis of publications on "Data Envelopment Analysis" and "Academic Departments"

Alam et al. [2022] developed an investment model that was used to evaluate the efficiency of departments in an educational institution and to identify *benchmarks* for the least efficient departments. Chen and Chang [2021] applied DEA and the concept of guarantee region to evaluate the relative efficiency (including aggregate, technical, and scale efficiencies) of academic departments at National Chung Cheng University in Taiwan.

Figure 2 shows the results of a bibliometric analysis carried out by combining the keywords "Multiple Criteria Data Envelopment Analysis" or "Multi-Criteria Data Envelopment Analysis" or "MCDEA Model". It is noticed that over 50% of the publications were made as of 2017, which shows the contemporaneity of the theme addressed here.

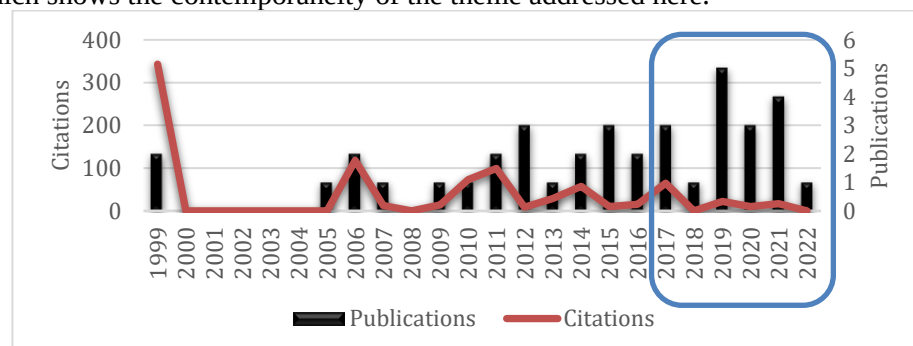


Figure 2 - Bibliometric analysis of publications on *Data Envelopment Analysis* and *Academic Departments*.

Figure 3 shows the result of a bibliometric analysis carried out by combining the keywords "*Data Envelopment Analysis*" and "*Network Models*". Färe and Grosskopf's [2000] research is the most cited work on NDEA. The research of [Castelli et al. 2010] is the second most cited. However, more than 50% of the papers were published in the last five years.

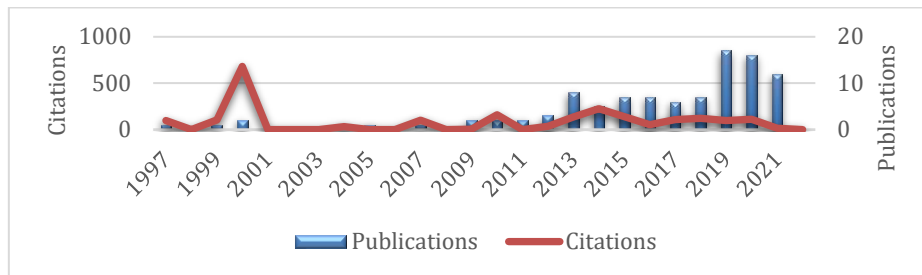


Figure 3 - Bibliometric analysis of publications on *Data Envelopment Analysis* and *Network Models*.

Table 1 summarises a bibliometric analysis of the volume of publications by combinations of several keywords. One can see that there is a significant volume of publications when the analysis is done individually (Combinations 1 to 6). However, when combining "*Data Envelopment Analysis*" and "*Academic Departments*" (Combination 7) only 29 publications were obtained. Finally, all other combinations from 8 to 12 did not return any publications, thus evidencing a research gap that this article aimed to explore.

This paper proposes a new *Network DEA* model, called *Network New Multiple Criteria Data Envelopment Analysis - NNMCD EA*, based on Kao and Hwang [2008] relational models and on the New- MCDEA model proposed by Silva et al. [2020].

Table 1-Bibliometric Analysis of the Publications

Search	Keywords	Scopus Publication (a)	WoS Publications (b)	GAP $\left(\frac{a}{b} - 1\right)$
1	"Data Envelopment Analysis"	21,234	17,627	20.46%
2	"Academic Departments"	2,622	1,130	132.04%
3	"Multiple Criteria Data Envelopment Analysis" or "Multi-Criteria Data Envelopment Analysis" or "MCDEA Model".	38	28	35.71%
4	"Network Data Envelopment Analysis"	452	392	15.31%
5	"Network Data Envelopment Analysis or "NDEA"	1,235	978	26.28%
6	"Data Envelopment Analysis" and "Network Models".	101	22	359.09%
7	"Data Envelopment Analysis and Academic Departments"	29	26	11.54%
8	"Multiple Criteria Data Envelopment Analysis" and "Academic Departments"	0	0	-
9	"Multi-Criteria Data Envelopment Analysis" and "Academic Departments"	0	0	-
10	"MCDEA Model and Academic Departments"	0	0	-
11	"Network Data Envelopment Analysis and Academic Departments"	0	0	-
12	"NDEA" and "Academic Departments"	0	0	-

The idea of this model is to be used in network structures which do not meet the Golden Rule. To test the NNMCD EA model, a real instance linked to an evaluation problem of academic departments of a public university was used. The results show that the new NNMCD EA model provides better discrimination of DMUs in all tested instances and, via statistical tests combined with the analysis of the experts from the studied public university, validated the logical structure of the NNMCD EA model. Figure 4 presents the methodological flowchart of the research, with seven steps defined for applying the NNMCD EA.

The overall objective of this research was to develop a NNMCD EA model to evaluate the efficiency of academic departments at a public university. The specific objectives were: Model and optimize the NNMCD EA model using the Simplex algorithm by applying programming in

VBA-Excel; Apply the NNMCDDEA model to evaluate the efficiency of academic departments at a public university; Compare the efficiencies generated by Kao and Hwang [2008] *Network Data Envelopment Analysis* (NDEA) model to the efficiencies generated by the NNMCDDEA model; Validate the results generated by the NNMCDDEA model optimization.

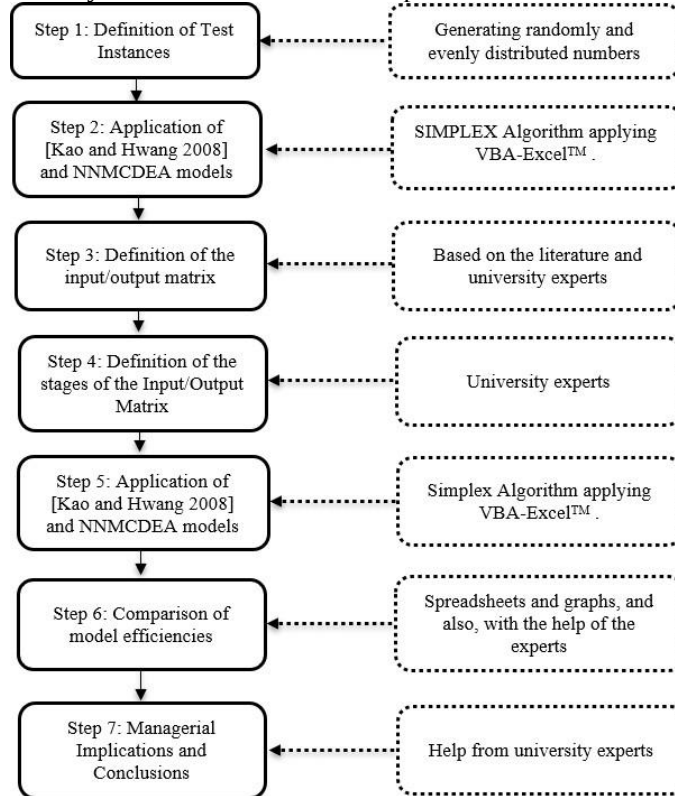


Figure 4 – Methodological Flowchart

2. Network Data Envelopment Analysis - NDEA

DEA is a quantitative method that uses linear optimization to optimize efficiency analysis processes, whose objective is to identify which *Decision Making Units* - DMUs are efficient. From a statistical point of view, DEA is a non-parametric method of analysis, being well accepted by companies that apply it intending to analyze the relative efficiency for decision-making. This method can be applied both to the public and private sectors. The expressions (1) - (4) refer to the original DEA model, which is an *input-oriented* multiplier model with constant returns to scale (*Constant Return to Scale* - CRS):

$$\max \theta = \sum_{r=1}^s u_r y_{r0} / \sum_{i=1}^m v_i x_{i0} \quad (1)$$

subject to:

$$\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \leq 1, \quad j = 1, 2, \dots, n \quad (2)$$

$$u_r \geq 0, \quad r = 1, 2, \dots, s \quad (3)$$

$$v_i \geq 0, \quad i = 1, 2, \dots, m \quad (4)$$

where, for each DMU_j, $j \in \{1, \dots, n\}$, y_{rj} is the value of output $r \in \{1, \dots, s\}$; x_{ij} is the value of the input $i \in \{1, \dots, m\}$; u_r is the weight associated with the output r e v_i is the weight associated

with the input i ; y_{r0} e x_{i0} are the values for the output $r \in \{1, \dots, s\}$ and for the input $i \in \{1, \dots, m\}$ of the DMU under analysis (DMU_0), whose relative efficiency is represented by θ .

The initial *Network Data Envelopment Analysis* - NDEA was developed by Färe and Grosskopf [2000], who proposed opening the so-called "black box". Early works using NDEA sought the solution of a DEA model for each stage independently, disregarding the continuity of relationships between stages [Soares de Mello et al., 2006]. A two-stage network structure (see Figure 5) has been widely studied as a basic structure for DEA networks [Cook et al., 2010]. Specifically, in Figure 5, the two subprocesses are connected in the sense that the same set of weights is applied to the intermediate products, regardless of their treatment as inputs (in the second stage) or outputs (in the first stage) [Lim and Kim, 2022].

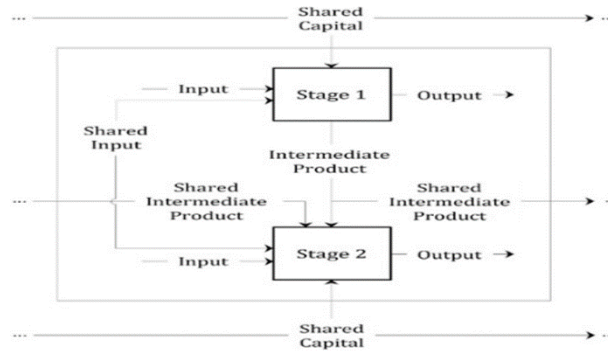


Figure 5- A two-stage network structure. Source: [Lim and Kim 2022]

Kao and Hwang [2008] presented the multiplicative relational model for the NDEA formulation, based on the DEA-CCR model, and which is given in (5) - (11):

$$\max \theta = \sum_{r=1}^s u_r y_{r0} \quad (5)$$

Subject to:

$$\sum_{i=1}^m v_i x_{i0} = 1 \quad (6)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^e w_d Z_{dj} \leq 0, j = 1, 2, \dots, n \quad (7)$$

$$\sum_{d=1}^e w_d Z_{dj} - \sum_{i=1}^m v_i x_{i0} \leq 0, j = 1, 2, \dots, n \quad (8)$$

$$u_r \geq 0, r = 1, 2, \dots, s \quad (9)$$

$$v_i \geq 0, i = 1, 2, \dots, m \quad (10)$$

$$w_d \geq 0, d = 1, 2, \dots, e \quad (11)$$

where d is the index associated with the intermediate products (*inputs* and *outputs*) or endogenous $d \in \{1, \dots, e\}$; Z_{dj} is the value of the intermediate *input* or *output* $d \in \{1, \dots, e\}$ for the j th DMU $j \in \{1, \dots, n\}$. The other indexes, parameters and variables are analogous to the model (1) - (4).

It is known that the optimal weights in the model (5) - (11) may not be unique, as well as the decomposition of the overall efficiency θ^* of the efficiencies of each stage θ_1 e θ_2 may not be unique either. In any case, Kao and Hwang [2008] proposed to calculate the individual efficiencies by expressions (12) and (13) and to calculate the overall efficiency θ^* by (14):

$$\theta_1 = \frac{\sum_{d=1}^e w_d Z_{d0}}{\sum_{i=1}^m v_i x_{i0}} \quad (12)$$

$$\theta_2 = \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{d=1}^e w_d Z_{d0}} \quad (13)$$

$$\theta^* = \theta_1 \times \theta_2 \quad (14)$$

Assuming the gap established by the Golden Rule, the MCDEA - *Multiple Criteria Data Envelopment Analysis* model (15) - (23) was presented to increase the discrimination power in cases where this rule is not met [Li and Reeves, 1999]:

$$\min d_0 \text{ or } \max \sum_{r=1}^s u_r y_{r0} \quad (15)$$

$$\min M \quad (16)$$

$$\min \sum_{j=1}^n d_j \quad (17)$$

subject to:

$$\sum_{i=1}^m v_i x_{i0} = 1 \quad (18)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j \leq 0, j = 1, 2, \dots, n \quad (19)$$

$$M - d_j \geq 0, j = 1, 2, \dots, n \quad (20)$$

$$u_r \geq 0, r = 1, 2, \dots, s \quad (21)$$

$$v_i \geq 0, i = 1, 2, \dots, m \quad (22)$$

$$d_j \geq 0, j = 1, 2, \dots, n \quad (23)$$

Where d_j measures the inefficiency for j -th DMU and $M = \max \{d_j\}$, that is, the variable M is the maximum inefficiency value, and d_0 indicates the inefficiency for the DMU under analysis.

Other models based on the MCDEA model have been presented in the literature, for example, the *Goal Programming* DEA (GPDEA) model proposed by Bal et al. [2010], in which the concepts of Goal Programming were added to increase the discrimination of DMUs and the dispersion of the weights. However, Ghasemi et al. [2014] criticized the formulation of the latter model and presented the *Bi-Objective-MCDEA* (BiO-MCDEA) model. This model was also criticized by Hatami-Marbini et al. [2017], who warned about the importance of calculating the parameter ε to find the optimal value. Rubens et al. [2017] resumed the use of *Goal Programming*, correcting the problems identified in the Bal et al. [2010] model and proposed the *Weighted Goal Programming* MCDEA (WGP-MCDEA) model. In this line of research, Silva et al. [2020] proposed the New- MCDEA-CCR model, given by (24) - (32), and proved its superiority compared to the other MCDEA models:

$$\min \sum_{j=1}^n d_j^- + M + \theta + d_0^+ \quad (24)$$

subject to:

$$\sum_{i=1}^m v_i x_{i0} = 1 \quad (25)$$

$$\sum_{r=1}^s u_r y_{r0} - d_0^+ \leq \theta \quad (26)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j^- - d_0^+ = 0, \quad j = 1, 2, \dots, n \quad (27)$$

$$M - d_j^- \geq 0, \quad j = 1, 2, \dots, n \quad (28)$$

$$u_r \geq 0, \quad r = 1, 2, \dots, s \quad (29)$$

$$v_i \geq 0, \quad i = 1, 2, \dots, m \quad (30)$$

$$d_j^- \geq 0, j = 1, 2, \dots, n \quad (31)$$

$$\theta \geq 0 \quad (32)$$

where θ is the measure of the inefficiency of the DMU under analysis, d_0^+ is the positive deviation associated with the aspiration level θ . The other indices, parameters and variables are analogous to model (15) - (23).

Based on the model proposed by Kao and Hwang [2008] it was possible to extend the same idea to the New- MCDEA and, in this way, this paper proposes the *Network New Multiple Criteria Data Envelopment Analysis* - NNMCD EA model, given by (33) - (43):

$$\text{Min} \left(\sum_{j=1}^n d_j^- + d_{2j}^- \right) + M + \theta + d_0^+ \quad (33)$$

subject to:

$$\sum_{i=1}^m v_i x_{i0} = 1 \quad (34)$$

$$\sum_{r=1}^s u_r y_{r0} - d_0^+ \leq \theta \quad (35)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^e w_d Z_{dj} + d_j^- - d_0^+ = 0, \quad j = 1, 2, \dots, n \quad (36)$$

$$\sum_{d=1}^e w_d Z_{dj} - \sum_{i=1}^m v_i x_{ij} + d_{2j}^- - d_0^+ = 0, \quad j = 1, 2, \dots, n \quad (37)$$

$$M - d_j^- \geq 0, \quad j = 1, 2, \dots, n \quad (38)$$

$$M - d_{2j}^- \geq 0, \quad j = 1, 2, \dots, n \quad (39)$$

$$u_r \geq 0, \quad r = 1, 2, \dots, s \quad (40)$$

$$v_i \geq 0, \quad i = 1, 2, \dots, m \quad (41)$$

$$d_j^-, d_{2j}^- \geq 0, j = 1, 2, \dots, n \quad (42)$$

$$w_d \geq 0, \quad d = 1, 2, \dots, e \quad (43)$$

Where d_{2j}^- e d_j^- are the inefficiencies for the first and second stages, respectively. The other indexes, parameters and variables are analogous to the models already mentioned. The efficiency of each stage can be obtained by (44) and (45), while the global efficiency is obtained by (46).

$$\theta_1 = 1 - d_j^- \quad (44)$$

$$\theta_2 = 1 - d_{2j}^- \quad (45)$$

$$\theta^* = \theta_1 \times \theta_2 \quad (46)$$

3. Modeling and problem description

The real problem addressed in this article consists in analyzing the efficiency of ten academic departments belonging to a Brazilian university. Each of these ten departments is endowed with a certain budget for its activities and has its faculty responsible for the department's teaching, research, and extension activities. We considered applying the *Network Data Envelopment Analysis* model, rather than a classical DEA model to the problem presented, because we believe that, in the process of transforming inputs into outputs, academic departments can be associated with a networked production structure with sub-processes, so that an intermediate product of a sub-process becomes the input of another subsequent process in the network.

Steps 3 - 4 of Figure 4 were used to implement the *Network New Multiple Criteria Data Envelopment Analysis* (NNMCDEA) model, given in (33) - (43), and they consisted in the definition of variables and data collection for the representation of the problem proposed in Figure 6. It was decided that the intermediary products should belong to the Teaching indicators (first stage), leaving the Research and Extension indicators as products of the second stage.

As presented in Figure 6, the academic departments efficiency analysis problem has two exogenous inputs (x_1 - budget and x_2 - number of faculty members), three intermediate products (z_1 - Undergraduate workload, z_2 - Graduate workload and z_3 - average grade of the courses) and seven exogenous products (y_1 - number of publications with Qualis, y_2 - total number of scientific articles published, y_3 - number of research fellows, y_4 - number of projects lasting more than a year, y_5 - resources obtained in projects lasting more than a year, y_6 - number of projects lasting less than a year, y_7 - resources obtained in projects lasting less than a year). In Table 2 are the variables of the problem with the network structure of Figure 6. In Table 3 are the actual data for the parameters of exogenous inputs, intermediate products, and exogenous outputs from the efficiency analysis problem of a university using the NNMCDEA model.

Once the inputs and outputs, as well as their corresponding values, were defined, the second stage of the NNMCDEA model implementation started with the implementation of the model proposed in this work, expressed by (33) - (43), in the *Microsoft Excel software* through the SIMPLEX algorithm by applying VBA programming.

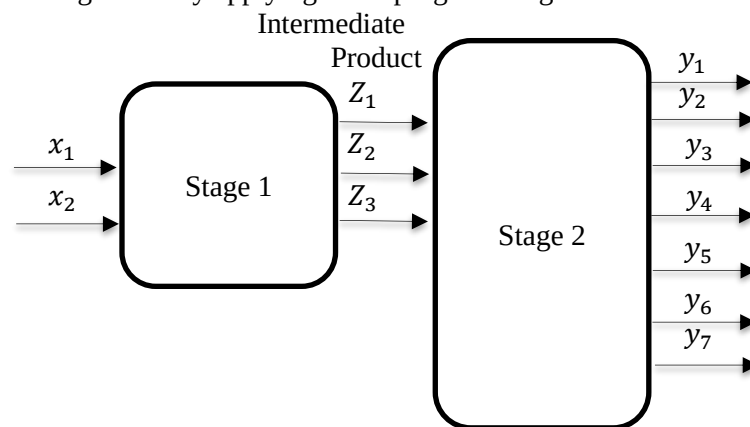


Figure 6 - Representation of the Efficiency Analysis Problem of a University using the NNMCDEA model
Table 2 - Exogenous Inputs, Intermediate Products and Exogenous Products of the problem of efficiency analysis of a University through the NNMCDEA model

Variables	Description	Unit
x_1 Budget	Budget allocated to each department to fund the execution of its teaching, research and extension activities.	thousand [\$]
x_2 Number of Teachers	Total number of faculty members assigned to a given academic department.	units
z_1 Workload in Graduate Studies	Sum of the workload taught by the teachers of that academic unit in post-graduation <i>stricto sensu</i> subjects.	hours
z_2 Workload in Graduation	Sum of the workload taught by the teachers of that academic unit in undergraduate courses.	hours
z_3 Average grade of the courses	Average grade of the undergraduate and graduate courses under the responsibility of an academic department.	Dimensionless (from 0 to 5)
y_1 Number of publications with Qualis	Total number of publications by department faculty members that have Qualis ratings (A1 to B5).	units
y_2 Total number of scientific articles	Total number of publications of the department's faculty members with and without Qualis classification.	units

y_3	Number of scholarship teachers	Number of faculty members with research grants recognized by a funding agency.	units
y_4	Number of projects lasting more than one year	Total number of extension projects conducted by faculty in the department ending more than one year.	units
y_5	Resources obtained in projects that are more than one year old	Total number of financial resources handled by projects lasting more than one year.	thousand [\$]
y_6	Number of projects with a duration of less than one year	Total number of extension projects conducted by department faculty with a completion of less than one year.	units
y_7	Resources obtained in projects that are less than one year old	Total number of financial resources handled by projects with a duration of less than one year.	thousand [\$]

Table 3 - Values for Exogenous Inputs, Intermediate Products and Exogenous Products

DMU	x_1 (thousand \$)	x_2 (un)	z_1 (h)	z_2 (h)	z_3 (adi)	y_1 (un)	y_2 (un)	y_3 (un)	y_4 (un)	y_5 (thousand \$)	y_6 (un)	y_7 (thousand \$)
A	256,1	62	302	17645	4.2	44	70	0	5	562,0	10	55,5
B	179,4	50	216	11635	4.4	11	13	0	6	2630,1	7	116,9
C	255,8	61	992	15422	3.9	42	68	1	2	5600,0	8	24,0
D	331,4	71	2741	18530	4.3	67	86	16	28	27933,1	24	293,3
E	211,7	54	2633	12924	4.8	65	104	7	29	3527,9	53	329,4
F	201,4	50	1253	10915	4.4	19	20	6	22	11482,6	18	187,9
G	314,6	68	2891	18512	4.1	62	90	7	4	32,4	25	134,1
H	265,0	58	1294	18293	4.0	30	47	0	6	2245,3	19	97,1
I	344,6	68	2865	17304	4.5	116	160	4	24	4874,6	41	148,3
J	160,0	36	1600	8134	4.4	27	31	4	24	11991,0	7	74,6

Next, Steps 5 and 6 of the flowchart in Figure 4 were performed. Note that the proposed NNMCDDEA model is appropriate because it allows measuring the performances of the internal structures of a DMU, integrating them into a corresponding unit of its global efficiency, even when the Golden Rule is not met, which would prevent the use of NDEA models available in the literature. Using the NDEA and NNMCDDEA models for the efficiency analysis problem of a university, with the data from Table 3, it was possible to calculate the efficiencies of the first and second stages, as well as the global efficiency of the process, for each model, as shown in Table 4. The NNMCDDEA model provided greater discrimination in all stages, as shown in the GAP column.

Table 4 - Results of first (θ_1) and second stage (θ_2) efficiencies and overall efficiency (θ^*) with the NDEA and NNMCDDEA models

DMU	θ_1 - NDEA	θ_2 - NDEA	θ^* - NDEA	θ_1 - NNMCDDEA	θ_2 - NNMCDDEA	θ^* - NNMCDDEA	GAP ($\frac{\text{NNMCDDEA}}{\text{NDEA}} - 1$)		
1	83.71%	56.48%	47.28%	82.19%	61.08%	50.20%	-1.82%	8.14%	6.18%
2	89.34%	50.48%	45.10%	72.38%	52.18%	37.77%	-18.99%	3.37%	-16.26%
3	81.45%	66.44%	54.12%	80.54%	58.86%	47.41%	-1.12%	-11.41%	-12.40%
4	95.02%	100.00%	95.02%	94.28%	100.00%	94.28%	-0.78%	0.00%	-0.78%
5	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	0.00%	0.00%	0.00%
6	90.70%	93.57%	84.87%	80.97%	70.54%	57.11%	-10.73%	-24.61%	-32.70%
7	100.00%	70.52%	70.52%	100.00%	44.35%	44.35%	0.00%	-37.11%	-37.11%
8	100.00%	39.91%	39.91%	100.00%	37.71%	37.71%	0.00%	-5.50%	-5.50%
9	96.27%	100.00%	96.27%	79.54%	100.00%	79.54%	-17.38%	0.00%	-17.38%
10	100.00%	100.00%	100.00%	89.72%	80.28%	72.02%	-10.28%	-19.72%	-27.98%

In Figure 7 is a Box-plot analysis, with the overall efficiency for the NDEA given by Box-Plot (1) and the overall efficiency for the NNMCDDEA given by Box-Plot (2). The interquartile range for the NDEA model (quartiles Q1 - Q3) was larger compared to the range generated for the NNMCDDEA, thus evidencing that discrimination by the NNMCDDEA model was greater than that of the NDEA model. When analyzing the interquartile range (Q3 - Q1), one notices that the efficiency values for the NNMCDDEA model were lower compared to the efficiency values

generated by the NDEA model. This result shows, again, that the NNMCDDEA model has practical advantages in its use in Network DEA problems that do not meet the Golden Rule.

Figure 8 depicts the result of the equal variance test for the real instance, one can see based on the p-values 0.083 and 0.166, that one can conclude that the variance of the weights (v_i, w_d, u_r) for the NDEA and NNMCDDEA models are statistically equal, at a significance level of $\alpha = 5\%$. In the next section, the impacts of the application of Stages 6 and 7 of the methodological flowchart in Figure 4 are presented to manage the real problem of the university studied, taking Figure 8 as a basis. This Figure 9 presents the comparative graph of first (θ_1) and second stage efficiencies (θ_2) and overall efficiency (θ^*), considering the NDEA and NNMCDDEA models for the ten academic departments analyzed.

5. Managerial Implications and Conclusions

It should be noted that for the NDEA model to be applied, the Golden Rule should be respected, that is, there should be at least 15 DMUs for stage 1 and 30 DMUs for stage 2. Adopting as minimum quantity the stage that indicates the largest number of DMUs, we have that, for the classic NDEAs models to be employed in this problem in question, at least 30 DMUs would be necessary to make this analysis feasible. The NNMCDDEA model has a higher discrimination power than the NDEA model, since the efficiencies, per stage and overall, were lower in the NNMCDDEA model.

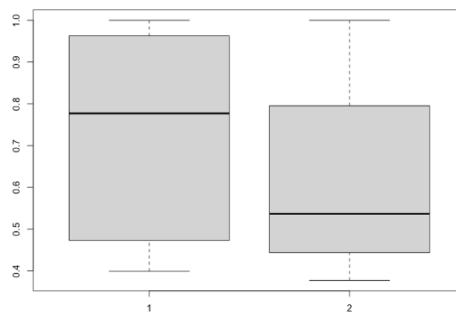


Figure 7 - Box-Plot for the Real Instance. Source: R-Language

The NDEA Model classified four DMUs as being efficient in the first stage (E, G, H, J), unlike the NNMCDDEA Model, with three efficient DMUs (E, G, H). In the second stage four DMUs (D, E, I, J) were considered efficient for the NDEA model, whereas for the NNMCDDEA model there were 3 efficient DMUS (D, E, I). Finally, regarding global efficiency, the NDEA model considered DMUs E and J to be efficient, unlike the NNMCDDEA model that considered only DMU E to be efficient. Regarding the first stage, focusing on the analysis of Teaching indicators, one clearly sees greater efficiency for departments with high workloads (undergraduate and graduate), associated with lower budgets. This fact is corroborated academically by the fact that departments G and H focus mainly on teaching activities at the university. Departments E and J, on the other hand, have fewer faculty members and are considerably dedicated to post-graduate activities, which also positively influences their indicators related to research and extension. However, it is noteworthy that the NNMCDDEA model did not consider DMU J as efficient, being considered inefficient in both stages.

95% Bonferroni Confidence Intervals for Standard Deviations

Sample	N	StDev	CI
REAL	130	0.0067064	(0.0048503, 0.0094354)
NNMCDDEA			
REAL NDEA	130	0.0132964	(0.0076996, 0.0233644)

Tests

Method	TestStatistic	P-Value
Multiple comparisons	3.01	0.083
Levene	1.93	0.166

Figure 8 - Test for equal variance for the real instance. Source: R-Language.

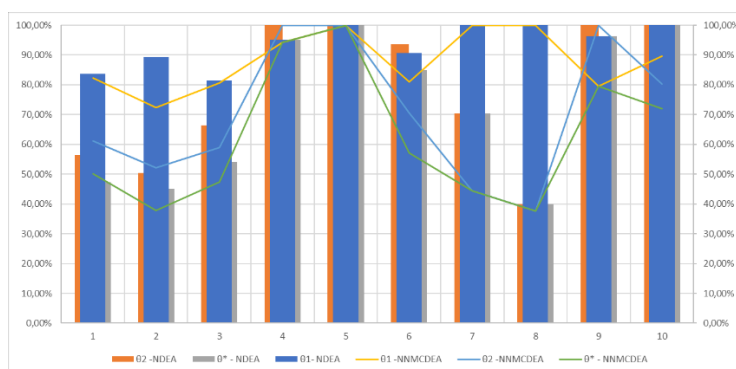


Figure 9 - Comparison plot of first (θ_1) and second stage (θ_2) efficiencies and overall efficiency (θ^*) with the NDEA and NNMCDDEA models

As for the second stage, which focused on the analysis of Research and Extension indicators, a change in the profile of the most efficient DMUs is observed for those that have a greater number of publications (with and without *Qualis*) and for those departments that manage to generate a greater number of extension projects and, thus, obtain a greater contribution of external resources. In this sense, for the NNMCDDEA model, the academic departments D, E and I stood out as being efficient. As expected, DMUs G and H were considered inefficient at this stage, since their research and extension indicators are considerably lower than their teaching indicators. One notices that, except for DMU E, departments D and I have the largest budgets and number of faculty members at the university. However, in the second stage, they are responsible for a large portion of the number of publications, number of fellows, and number of extension projects (number of projects and accumulated resources). On the other hand, as in the first stage, the NDEA model also considered DMU J as efficient, a fact not corroborated by the NNMCDDEA model. This department, as already mentioned, has a reduced number of faculty members and a low budget. However, it manages to have a considerable number of publications and projects.

Again, DMU E was considered efficient, being the only department considered to have 100% efficiency in both stages. Regarding this department, it is verified that it has an average number of teachers and budgets in relation to the numbers of other departments. However, it manages to present good results, both in terms of teaching and research and extension. This fact leads this DMU E to have an overall efficiency of 100%, by both models tested. The NDEA model also considers department J as being globally efficient, but this classification may be refuted by the University's management due to the low efficiency of this department, especially in the second stage, in terms of number of publications and projects lasting less than a year. Finally, it should be noted that, in summary, the model proposed in this article had greater discrimination power of the analyzed DMUs, being able to identify the most efficient departments in each of the considered stages, besides pointing out to the University Management points of improvement regarding the best use of its resources for each of its academic departments.

Acknowledgements: CNPq, CAPES, FAPESP, FAPEMIG.

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