

**Greedy Randomized Construction with Vertex Degree for the Minimum
Independent Dominating Set Problem**

André Eduardo Alessi

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
andreedalessi@gmail.com

Lucas Caldeira de Oliveira

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
Caldeira.live@gmail.com

Bruno Duarte

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
brunoduarte@alunos.utfpr.edu.br

Marco Antonio de Castro Barbosa

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
marco.cb@gmail.com

Dalcimar Casanova

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
dalcimar@gmail.com

Ives Rene Venturini Pola

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
ivesrene@gmail.com

Fernanda Paula Barbosa Pola

Universidade Tecnológica Federal do Paraná
Via do Conhecimento, Km 01 – Pato Branco – Paraná
fernandapola@utfpr.edu.br

ABSTRACT

The Minimum Independent Dominating Set (MIDS) problem is a classic graph theory problem very often Applied to solve wireless networks issues, as well as wireless sensor networks and also aiding similarity sets extraction. As the exact approach solution to solve the MIDS problem leads to exponential execution time complexity, some works tackled it by using metaheuristics. However, they use a customized path cost approach in the solution. In this works, we will show that using a classic information – the vertex degree – allied with a Greedy Randomized Construction metaheuristic, can lead to better results than other works on the DIMACS and BHOSLIB benchmarks in most of the instances. We developed a new algorithm called GRC+VD to tackle the MIDS problem and we performed experiments to demonstrate the enhanced performance achieved by our new algorithm when compared with other approaches.



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1. Introduction

Given a graph $G = (V, E)$, a *dominating* set D of G is a set of vertices such that every vertex not in D is adjacent to a vertex in D . A set is *independent* if there are no adjacent pair of vertices. An *Independent Dominating Set* (IDS) of G is a set that is both dominating and independent [Goddard and Henning, 2013]. The *Minimum Independent Dominating Set* (MIDS) problem aims at identifying the IDS with minimum cardinality in a given graph [Irving, 1991].

The MIDS problem has some applications. Some studies show that clustering algorithms of wireless networks can adopt the MIDS scheme [Chen et al., 2004; Basagni, 1999; Erciuyes et al., 2007; Chen et al., 2002; Lin and Gerla, 1997]. Moreover, the MIDS problem has applications in wireless sensor networks [Santos et al., 2009] as well as wireless sensors with actors networks [Akyildiz and Kasimoglu, 2004]. Another application of the MIDS problem is the extraction of similarity sets, a concept of sets to seamlessly handle similarity in database management systems [Pola et al., 2015; Alessi et al., 2021].

The MIDS problem has been proven to be *NP*-hard [Irving, 1991], meaning that unless $P = NP$, there are no polynomial time algorithms to solve this problem within a ratio $|V|^{1-\theta}$, for any $\theta > 0$ [Halldórsson, 1993]. Some exact algorithms have been proposed to solve the MIDS problem [Moon and Moser, 1965; Liu and Song, 2006; Bourgeois et al., 2013].

However, due to its *NP*-Hardness, metaheuristics have been used to tackle this kind of problems. Metaheuristics are solutions methods that have an interaction between local improvement procedures and higher-level strategies to create a process capable of escaping from local optima and performing a robust search of solution space [Gendreau et al., 2010]. They have been used to solve combinatorial problems, like the Capacitated Vehicle Routing Problem [Oliveira et al., 2020], the Traveling Salesman Problem with Draft Limit [Morais et al., 2017; Duarte et al., 2020], to name a few.

Metaheuristics cannot guarantee an optimal solution, but they can find high quality solutions in polynomial time. A few metaheuristics have been proposed to solve the MIDS problem over the years: GRASP+PC [Wang et al., 2017] uses a path cost heuristic based on a greedy randomized adaptative search procedure (GRASP) framework. MEMETIC [Wang et al., 2018] is a hybrid search method which takes advantage of a local search framework with a population-based search. Recently, [Wang et al., 2020] proposed a new local search algorithm with a tabu method and two different strategies to adapt its local search phase. This last method is considered by its authors as the state-of-the-art approach for handling the MIDS problem.

In this paper, we propose a new heuristic approach to deal with the MIDS problem, using *Greedy Randomized Construction* and considering the vertex degree instead of the path cost commonly employed in other works. We use the same benchmarks used by drMIDS method in order to produce comparison results.

2. Basic concepts

In the next subsections we introduce the necessary basic concepts to understand this work.

2.1 Minimum Independent Dominating Set Problem Formulation

We shall consider a graph $G = (V, E)$ as finite, undirected, with no multiple edges, and with no loops. Also, consider $N(v)$ as the neighborhood of v and $N[v]$ the closed neighborhood of v , that is, the neighborhood of v and v itself. Then, we can define [Allan and Laskar, 1978]:

- **Definition 1:** A set $D \subseteq V$ is a *dominating* set of G if for all $v \in V - D$, $N(v) \cap D \neq \emptyset$.
- **Definition 2:** A set $I \subseteq V$ is an *independent* set of G if for all $u, v \in I$, $N(u) \cap \{v\} = \emptyset$.

- **Definition 3:** A set $IDS \subseteq V$ is an *independent dominating set* of G if IDS is both an independent and dominating set.
- **Definition 4:** The minimum independent dominating set problem aims to identify the independent dominating set with minimum cardinality in G [Irving, 1991].

2.2 Greedy Randomized Construction

The Greedy Randomized Construction (GRC) is a step of the GRASP metaheuristic [Resende and Ribeiro, 2010]. By its own, it is comparable to the semi-greedy heuristic [Hart and Shogan, 1987], which is a multi-start approach. Algorithm 1 illustrates the procedure with pseudo-code. The set of candidate elements is formed by all elements that can be incorporated into the partial solution being built, while allowing the construction of a feasible solution. The selection of the next element depends on the evaluation of all candidate elements according to a greedy evaluation function. This evaluation leads to the creation of a Restricted Candidate List (RCL) formed by the best elements. The element to be incorporated into the partial solution is randomly selected from those on the RCL. Once the selected element is incorporated into the partial solution, the candidate list is updated, and the greedy evaluation function is reevaluated. These steps are repeated if there is at least one candidate element in C , the set of candidate elements.

Algorithm 1: Greedy Randomized Construction Algorithm [Resende e Ribeiro, 2010]

Data: α
Result: *Solution*
Solution $\leftarrow \emptyset$;
 Initialize the set of candidate elements C ;
 Evaluate the incremental costs $c(e)$ for all $e \in C$;
while $C \neq \emptyset$ **do**
 $c_{min} \leftarrow \min\{c(e) | e \in C\}$;
 $c_{max} \leftarrow \max\{c(e) | e \in C\}$;
 $RCL \leftarrow \{e \in C | c(e) \leq c_{min} + \alpha(c_{max} - c_{min})\}$;
 Select an element s from the RCL at random;
 Solution $\leftarrow$ *Solution* $\cup \{s\}$;
 Update the candidate set C ;
 Reevaluate the incremental cost $c(e)$ for all $e \in C$;
end

We denote by $c(e)$ the greedy evaluation function associated with the incorporation of the element $e \in C$ into the solution under construction. At any iteration, let c_{min} and c_{max} be the smallest and the largest incremental cost, respectively. The RCL is associated with a threshold parameter $\alpha \in [0,1]$. When $\alpha = 0$, we have a purely greedy algorithm, while when $\alpha = 1$, we have a random construction.

3. GRC with vertex degree for the MIDS problem

For the MIDS problem, we consider the incremental cost $c(e)$ as the vertex degree. This differs to other published works [Wang et al., 2017, 2018, 2020], where authors used a customized path cost as the incremental cost. The set of candidate elements C is initialized as the vertex set V , as initially any vertex can be chosen. The solution is initially empty. Then, the degree of each vertex is evaluated. The following steps are repeated while there is a candidate element left in C . The maximum vertex degree c_{max} is evaluated. Then, the restriction candidate list is set by selecting elements in C with a vertex degree less or equal to the parameter α times c_{max} . A random element s is selected from the RCL and is inserted into the solution set. Then, the closed

neighborhood of s is removed from the graph G . The candidate set C is updated with the remaining vertices and the vertex degree is reevaluated for all remaining elements. These steps are illustrated in Algorithm 2 and are denoted as the Greedy Construction phase.

Algorithm 2: Greedy_Construction

Data: $\alpha, G = (V, E)$
Result: *Solution*
 $Solution \leftarrow \emptyset;$
 $C \leftarrow V;$
 Evaluate the vertex degree;
while $C \neq \emptyset$ **do**
 $c_{max} \leftarrow$ maximum vertex degree;
 $RCL \leftarrow \{e \in C \mid c(e) \leq \alpha c_{max}\};$
 Select an element s from RCL at random;
 $Solution \leftarrow Solution \cup \{s\};$
 Remove $N[s]$ from G ;
 Update the candidate set C ;
 Reevaluate the vertex degree $c(e)$ for all $e \in C$;
end

The algorithm proposed in this work is then called GRC+VD (Greedy Randomized Construction + Vertex Degree) and is illustrated in Algorithm 3.

Algorithm 3: GRC+VD algorithm

Data: α, N_Iter
Result: *Best_Solution*
 $Best_Solution \leftarrow Greedy_Construction(\alpha);$
for $i = 0; i \leq n_iter; i++$ **do**
 $Solution \leftarrow Greedy_Construction(\alpha);$
 if $|Solution| < |Best_Solution|$ **then**
 $Best_Solution \leftarrow Solution;$
 end
end

The best solution is initialized with a Greedy Construction. Then, for a N_Iter number of iterations, a solution is built with the Greedy Construction and, if its size is smaller than the best solution size, the best solution is updated with this candidate solution. These steps are like those of a GRASP metaheuristic, but without the local search phase. We did not implement a local search because any disturbance in the solution can easily make it unfeasible, thus a GRASP approach would heavily rely on the quality of the greedy construction phase. Because of that, we only perform the greedy construction, without the local search phase step. This, allied with the vertex degree usage, differs this work from the GRASP approach already present in the literature [Wang et al., 2017].

4. Computational experiments and results

The algorithm GRC+VD was implemented using the C++ programming language. The instances used in the experiment are the same used by drMIDS: the DIMACS [Johnson and Trick, 1996] and the BHOSLIB [Xu et al., 2007] benchmarks, both acquired at the Network Repository [Rossi and Ahmed, 2015]. The experiment was made on a machine with 64 bits Intel Core i5-

7200U 2.50 GHz, 8 GB RAM and Linux Ubuntu 20.10. The results can be seen in Tables 1, 2 and 3. A value of $\alpha = 0.8$ was used.

Table 1: Experiment results of drMIDS and GRC+VD on the DIMACS benchmark I.

Instance	Length	Edge density (%)	drMIDS		GRC+VD			
			card min	card avg	card min	card avg	t (s) min	t (s) avg
brock200.2	200	49.62	4	4	6	6.3	0.015	0.016
brock200.4	200	65.77	6	6	4	4.5	0.016	0.017
brock400.2	400	74.91	9	9	4	4.2	0.113	0.116
brock400.4	400	74.89	9	9	4	4.4	0.113	0.116
brock800.2	800	65.13	8	8	5	5.9	0.776	0.784
brock800.4	800	64.96	8	8	5	6	0.781	0.794
C1000.9	1000	90.1	25	25.48	3	3.4	1.743	1.762
C125.9	125	89.84	14	14	2	2.8	0.005	0.005
C2000.5	2000	50.01	7	7	8	9.8	10.354	10.489
C2000.9	2000	90.02	32	32.03	3	3.4	13.543	14.15
C250.9	250	89.9	17	17	2	3	0.033	0.034
C4000.5	4000	50.01	8	8	9	10.4	82.24	83.05
C500.9	500	90.04	21	21	3	3	0.238	0.241
c-fat200-1	200	7.7	13	13	14	14.7	0.014	0.015
c-fat200-2	200	16.25	6	6	6	7.1	0.014	0.014
c-fat200-5	200	42.57	3	3	3	3	0.015	0.015
c-fat500-1	500	3.57	27	27	31	31.8	0.182	0.199
c-fat500-2	500	7.32	14	14	15	16.1	0.161	0.165
c-fat500-5	500	18.58	6	6	6	6.4	0.163	0.165
DSJC1000.5	1000	50.01	6	6	7	8.2	1.344	1.37
DSJC500.5	500	50.19	5	5	6	7.2	0.187	0.192
gen200.p0.9.44	200	90	16	16	2	2.9	0.018	0.02
gen200.p0.9.55	200	90	16	16	2	2.6	0.018	0.018
gen400.p0.9.55	400	90	20	20	3	3.5	0.124	0.13
gen400.p0.9.65	400	90	20	20	2	3.2	0.123	0.125
gen400.P0.9.75	400	90	20	20	2	2.9	0.126	0.128
hamming10-4	1024	82.89	12	12	8	8	1.675	1.773
hamming6-2	64	90.47	12	12	2	2	0.001	0.001
hamming6-4	64	34.92	2	2	8	8	< 1ms	< 1ms
hamming8-2	256	96.86	32	32	2	2	0.036	0.037
hamming8-4	256	63.92	4	4	8	8	0.028	0.031

Tables 1 and 2 illustrate the results of GRC+VD in comparison with the drMIDS approach [Wang et al., 2020] in the DIMACS benchmark. The instance name is in the first column, the graph length of that instance is in the second column, the percentual edge density is in the third column, the minimum set cardinality and average set cardinality achieved by the drMIDS approach are in the fourth and fifth columns, respectively. The minimum set cardinality and average set cardinality achieved by the GRC+VD approach are in the sixth and seventh columns, respectively. Finally, the minimum execution time and average execution time of the GRC+VD approach are in the last two columns.

The bold numbers in sixth and seventh columns mean that GRC+VD achieved an equal or lower set cardinality for that instance than drMIDS. GRC+VD achieved this in 65% of the instances for the minimum cardinality and 60% for the average cardinality.

Table 2: Experiment results of drMIDS and GRC+VD on the DIMACS benchmark II.

Instance	Length	Edge density (%)	drMIDS		GRC+ND			
			card min	card avg	card min	card avg	t (s) min	t (s) avg
johnson16-2-4	120	76.47	8	8	3	3	0.004	0.004
johnson32-2-4	496	87.87	16	16	3	3	0.229	0.239
johnson8-2-4	28	55.55	4	4	3	3	< 1ms	< 1ms
johnson8-4-4	70	76.81	7	7	5	5	0.001	0.001
keller4	171	64.91	5	5	4	4	0.01	0.011
keller5	776	75.15	9	9	4	7	0.761	0.8
keller6	3361	81.81	15	17.16	4	6.6	60	62.523
MANN_a27	378	99.01	27	27	2	2	0.11	0.113
MANN_a45	1035	99.62	45	45	2	2	2.039	2.047
MANN_a81	3321	99.88	81	81	2	2	64.722	65.002
MANN_A9	45	92.72	9	9	2	2	< 1ms	< 1ms
p_hat1500-1.clq	1500	25.34	12	12.71	19	21.2	3.947	4.022
p_hat1500-2.clq	1500	50.6	7	7.68	12	13.9	4.394	4.589
p_hat1500-3.clq	1500	75.36	3	3	5	5.4	5.286	5.533
p_hat300-3.clq	300	74.44	3	3	4	4.3	0.048	0.051
p_hat700-1.clq	700	24.93	11	11	14	17.7	0.42	0.443
p_hat700-2.clq	700	49.75	6	6	8	11.2	0.477	0.512
p_hat700-3.clq	700	74.8	3	3	5	5.2	0.544	0.582
san1000	100	50.15	4	4	23	26.7	1.402	1.464
san200_0.7_1	200	70	6	6	5	5.8	0.016	0.017
san200_0.7_2	200	70	15	15	5	6.2	0.016	0.017
san200_0.9_2	200	90	16	16	2	2.8	0.02	0.023
san200_0.9_3	200	90	15	15	2	3	0.017	0.018
san400_0.5_1	400	50	4	4	16	18.8	0.099	0.104
san400_0.7_1	400	70	7	7	5	5.9	0.112	0.113
san400_0.7_2	400	70	7	7	5	7	0.112	0.114
san400_0.7_3	400	70	7	7	6	7.2	0.107	0.112

Table 3 shows the results of GRC+VD in comparison with the drMIDS approach on the BHOSLIB benchmark. The column disposition is the same as Tables 1 and 2. In this case, GRC+VD performed better than drMIDS for all instances, both for the minimum and for the average cardinalities.

Let's consider a null hypothesis H_0 that the populational mean of the distribution of the results of both methods is the same. We shall perform a statistical test on this. Using a Wilcoxon test [Wilcoxon, 1992], we can test H_0 against the alternative hypothesis that drMIDS scores are higher than those of GRC+VD, that is, that the method proposed in this work is better than drMIDS. Doing the test, with 1% of significancy, we can reject the null hypothesis.

5. Conclusions

The MIDS problem is a classic graph theory problem that has a series of applications in wireless networks, wireless sensor networks and similarity sets extraction. Due to its importance, finding a heuristic to solve the problem in polynomial time is being studied in the literature. In this work, we showed that the vertex degree information used in a Greedy Randomized Construction metaheuristic can lead to better results in most of the instances than the drMIDS approach, considered by its authors as the state-of-the-art approximate approach.

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Table 3: Experiment results of drMIDS and GRC+VD on the BHOSLIB benchmark.

Instance	Length	Edge Density (%)	drMIDS		GRC+ND			
			card min	card avg	card min	card avg	t min (s)	t avg (s)
frb100-40	4000	92.83	43	44.35	5	6.3	108.939	110.678
fbr30-15-1	450	82.35	11	11	4	4.5	0.161	0.169
fbr30-15-2	450	82.3	11	11	4	4.6	0.164	0.167
fbr30-15-3	450	82.37	11	11	3	4.9	0.165	0.169
fbr30-15-4	450	82.34	11	11	3	4.8	0.157	0.169
fbr30-15-5	450	82.38	11	11	4	4.6	0.162	0.168
fbr35-17-1	595	84.23	13	13	3	4.8	0.364	0.382
fbr35-17-2	595	84.24	13	13.03	4	4.8	0.36	0.374
fbr35-17-3	595	84.19	13	13	4	4.6	0.37	0.381
fbr35-17-4	595	84.24	13	13.29	4	5.1	0.366	0.384
fbr35-17-5	595	84.07	13	13.65	3	4.7	0.364	0.376
fbr40-19-1	760	85.67	15	15.39	4	4.6	0.761	0.786
fbr40-19-2	760	85.69	15	15.03	4	4.9	0.761	0.78
fbr40-19-3	760	85.75	15	15.03	3	5	0.753	0.772
fbr40-19-4	760	85.57	15	15	4	4.8	0.757	0.779
fbr40-19-5	760	85.57	15	15.19	4	4.7	0.763	0.786
fbr45-21-1	945	86.73	17	17.77	4	4.7	1.469	1.478
fbr45-21-2	945	86.85	17	17.87	4	4.9	1.407	1.447
fbr45-21-3	945	86.94	17	17.39	3	4.7	1.423	1.462
fbr45-21-4	945	86.87	17	17.55	4	5.2	1.431	1.455
fbr45-21-5	945	86.86	17	17.45	3	4.8	1.41	1.446
fbr50-23-1	1150	87.88	19	19.94	4	4.8	2.589	2.617
fbr50-23-2	1150	87.76	19	19.9	4	5.2	2.503	2.594
fbr50-23-4	1150	87.85	19	19.9	5	5.1	2.548	2.617
fbr50-23-5	1150	87.88	20	20.03	4	5	2.549	2.654
fbr53-24-4	1272	88.33	20	20.9	4	5.1	3.468	3.568
fbr53-24-5	1272	88.34	20	21.06	4	4.9	3.409	3.577
fbr59-26-1	1534	89.23	23	23.61	4	4.8	5.865	6.213
fbr59-26-2	1534	89.27	23	23.9	4	4.6	6.144	6.242
fbr59-26-3	1534	89.27	23	23.84	4	4.9	6.125	6.22
fbr59-26-4	1534	89.19	23	23.94	3	4.5	6.115	6.215
fbr59-26-5	1534	89.28	24	24.19	4	4.9	6.121	6.217

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