Time Series Models Combination for Forecasting Quarterly GDP Components by the Expenditure Side

Kleyton Vieira Sales da Costa
Departamento de Economia, Universidade Federal Rural do Rio de Janeiro
Km 07, BR-465, Seropédica - RJ, 23890-000
kleyton.vsc@gmail.com

Felipe Leite Coelho da Silva
Departamento de Matemática, Universidade Federal Rural do Rio de Janeiro
Km 07, BR-465, Seropédica - RJ, 23890-000
felipeleiterural@gmail.com

Josiane da Silva Cordeiro
Departamento de Matemática, Universidade Federal Rural do Rio de aneiro
Km 07, BR-465, Seropédica - RJ, 23890-000
josicordeiro@gmail.com

RESUMO

Este estudo tem como objetivo analisar o desempenho preditivo das abordagens univariada e de combinação de previsões para os componentes do PIB. Para os resultados empíricos, considera-se os dados trimestrais da economia brasileira entre 1996 e 2020. Na abordagem univariada foram utilizados o modelo sazonal autorregressivo integrado de médias móveis, o método de Holt-Winters, o modelo linear dinâmico e o modelo de redes neurais autorregressivas. O algoritmo usado nas combinações de previsão foi um preditor de média ponderada polinomial com taxas de aprendizado múltiplas (ML-Poly). Através da métrica de erro percentual absoluto médio, as combinações dos modelos propostos apresentaram melhores resultados de ajuste do que os modelos individuais. E para os resultados de previsão, não se observou uma superioridade das combinações aplicadas aos dados em análise.

PALAVRAS CHAVE. Previsão, Séries Temporais, Produto Interno Bruto

Tópicos (EST&MP, OA)

ABSTRACT

This study aims to analyze the predictive performance of the univariate and forecasting combination approaches for GDP components. For the empirical results, were used quarterly data from the Brazilian economy between 1996 and 2020. The univariate evaluation used the model: a seasonal autoregressive integrated moving average, the Holt-Winters method, dynamic linear model, and neural network autoregression. The algorithm used in the forecasting combinations was a polynomial weighted average predictor with multiple learning rates (ML-Poly). Through the mean absolute percentage error, the combinations of the proposed models presented better fitted results than the individual models. And for the forecasting results, there was no superiority of the combinations applied to the data under analysis.

KEYWORDS. Forecasting, Time Series, Gross Domestic Product

Paper topics (EST&MP, OA)

1. Introduction

The components of GDP on the demand side (household consumption, government spending, investments, imports, and exports) are variables that have a very significant forecasting interest, taking into account their impact on production, on foreign trade, in decisions investment. Thus, these variables are also important in the qualitative analysis of a given economy. Through the System of National Accounts (SNA), a family is a group of individuals who share the same household in addition to food and housing expenses. Thus, *household consumption* is a relevant component that is directly related to disposable income. The public administration consists of units that have the purpose of providing non-market services (provided at economically insignificant prices) that will be used for the needs of the community or to redistribute income. For the execution of these services, other sectors of the economy provide resources for the public administration through the mandatory payment of taxes, fees, and social contributions that are which are transformed into *government spending*. The investment consists of gross fixed capital formation and inventory variation. Gross fixed capital formation estimates the variation in the productive capacity of an economy of current investments/divestments in fixed assets, that is, goods used in the production process and have not been consumed or transformed over more than one year. And the changes in inventories represent the difference in the value of incoming and outgoing goods in stock during the period considered. Exports take into account new or used goods that leave the national territory and go to the rest of the world. Exports have the characteristic of being valued FOB (free on board), including only the commercialization costs to the port. On the other hand, imports represent all new or used goods that enter the national territory from the rest of the world, valued at CIF (cost, insurance, and freight) prices, including freight and insurance costs.

In this way, the present study aims to analyze a suitable time series model to describe and forecast Brazilian GDP components, also investigating the fit of these models to dynamics between periods of economic growth and recession. For this purpose, we compared the univariate and forecasting combination approaches. In the univariate approach, the integrated autoregressive seasonal model of moving averages, the Holt-Winters method, the dynamic linear model, and the model of autoregressive neural networks were used. The algorithm used in the forecasting combinations was a polynomial weighted average predictor with multiple learning rates (ML-Poly).

Regarding the contributions that use classic models, the analysis constructed by Abonazel e Abd-Elftah [2019] for Egypt's annual GDP between the years 1965 and 2016, with forecasting of 2017 to 2026, presented results that indicates to the country's GDP growth during the period under analysis; Wabomba et al. [2016] estimated Kenya's GDP between 2013 and in 2017. The result obtained was significant growth in the Kenyan economy in the period; Agrawal [2018] modeled the series of India's real GDP growth rate from 1996 to 2017. In the analyzed data, the ARIMA model did not show any more significant results than other models. The author also used the Holt-Winters model and linear trend, both showing similar results each other, and da Silva et al. [2020] found significant results using ARIMAX and SARIMAX models (take into account exogenous variables) for the forecast of Brazilian annual and quarterly real GDP for the year 2019.

For the Bayesian approach and the class of state-space models, Piccoli [2015] analyzed four dynamic linear models to identify the one with the best forecasting capacity for nominal GDP in the United States. Best results were obtained using a multivariate model SUTSE (Seemingly Unrelated Time Series Equations) that considered as variables the nominal GDP, the industry production index, the consumer price index (inflation), and the quarterly interest rate for US Treasury bills; Rees et al. [2015] built new measures for Australia's GDP growth, using state-space methods. The results found have a high correlation with the figures published officially for GDP growth.

However, the measures are less volatile, easier to predict, and achieved good results in nowcasting; Issler e Notini [2016] estimate Brazilian real monthly GDP with state-space representation and also find good results in forecasting when compared with Central Bank Economic Activity Index (IBC-Br)[1]; Migon et al. [1993] developed a study about the performance of Bayesian Dynamic Models applied to a set of Brazilian macroeconomics time series (industrial productivity index, the balance of trade, components of GDP and others) between the period 1970 to 1990. The comparison was made between the dynamic models and classical structured models and obtained results indicate that the Bayesian approach was similar to the classical approach. Another applied study was developed by Baurle et al. [2020], with the aim of forecasting GDP in the euro area and Switzerland with a Bayesian vector autoregressive structure (BVAR) and a factor model structure. He found evidence that the factor model structure performs satisfactorily.

Considering forecast combinations, Newbold e Granger [1974] analyzes the predictive performance of univariate time series models and the combination of models applied to economic series. In their study, Box & Jenkins models show significant results. However, the combination of the Box & Jenkins models, the Holt-Winters method, and the stepwise autoregressive model has substantial performance gains. Kapetanios et al. [2008] obtains results that point to good results through the combination of models applied to English macroeconomic variables when compared with the individual forecast results. Fang [2003] uses quarterly consumer spending data in the UK in an analysis that seeks to demonstrate that encompassing tests are useful tools for understanding the most significant results of forecast combinations. The investigated models were ARIMA, DHSY, and VAR.

This work is organized as follows: Section 2 describes the methodology. Section 3 presents the empirical results and discussion, and, finally, Section 4 provides the main conclusions and some possibilities for future research.

2. Methodology

To follow we outline the models that were investigated to fitted and forecast Gross Domestic Product. Care was also taken that the references used in the definition of models and metrics also correspond to studies and authors with wide use and quality proven by the academic community.

This study, are evaluated two approaches concerning time series forecasting models. Initially, were investigated the univariate models: the Holt-Winters method (HW), seasonal autoregressive integrated moving average (SARIMA), dynamic linear model (DLM), and neural networks autoregression (NNAR). And, finally, were proposed the combinations of the predictions of these models using a polynomial weighted average predictor with multiple learning rates (ML-Poly). The performance of the models was compared using the statistical metric MAPE (Mean Absolute Percentage Error), following the contributions of Hyndman e Koehler [2006], Armstrong [2001].

2.1. Univariate Approach

2.1.1. Holt-Winters Method

The HW was proposed by Holt [1957] and Winters [1960], using exponentially weighted moving averages to update those needed for seasonal adjustment of the mean (trend) and seasonality.

The method has two variations with four equations: one forecast equation and three smoothing equations. Hyndman e Athanasopoulos [2018] describes that in the additive method, the seasonal component is defined in absolute terms on the scale of the observed series. In the level equation, the series is seasonally adjusted by subtracting the seasonal component. Within each year, the seasonal component sums up to approximately zero. With the multiplicative method, the

[1]Monthly indicator of national economic activity published by Central Bank of Brazil.

seasonal component is defined in percentage terms, and the series is seasonally adjusted by dividing through by the seasonal component. Within each year the seasonal component will sum up to approximately m.

The additive method equations are described as following,

$$\begin{aligned}
&\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t+h-m(k+1)},\\
&\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1}),\\
&b_t = \beta(\ell_t - \ell_{t-1}) + (1-\beta)b_{t-1},\\
&s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m},
\end{aligned} \tag{1}$$

where $\hat{y}_{t+h|t}$ is the forecast equation. The ℓ_t, b_t and s_t are respectively level, trend and seasonality equations, with corresponding smoothing parameters α, β and γ. The parameter m denotes the frequency of seasonality, and for quarterly data $m = 4$. Finally, k is the integer part of $(\frac{h-1}{m})$ which ensures that the estimates of the seasonal indices used for forecasting come from the final year of the sample.

For the multiplicative method the same equations ℓ_t, b_t and s_t are defined. But the change in structure occurs because instead of sum the equations in $\hat{y}_{t+h|t}$ an operation is performed to multiply the sum of the level and trend equations by the seasonality equation.

2.1.2. SARIMA

Box & Jenkins models determine the proper stochastic process to represent a given time series by passing white noise through a linear filter [Box et al., 2015] The model used in this study was SARIMA, seeking to incorporate the seasonality component that is present in the data under analysis.

The SARIMA of order $(p, q, d) \times (P, Q, D)_s$ is defined by,

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\alpha_t, \tag{2}$$

where $\theta(B)$ is the moving average operator of *q* order, $\phi(B)$ is the autoregressive operator of *p* order, $\Phi(B^s)$ is the seasonal autoregressive operator of *P* order, $\Theta(B^s)$ is the seasonal moving average operator of *Q* order, ∇^d is the simple difference operator, ∇_s^D is the seasonal difference operator and α_t is the noise.

2.1.3. Dynamic Linear Model

DLMs are an important class of state-space models. Broadly used in the last decades, they have a high degree of efficiency for the analysis and forecast of time series, providing flexibility and applicability through an elegant and robust probabilistic apparatus.

The estimation and inference challenges are solved by recursive algorithms, which follow the Bayesian approach, calculating conditional distributions of quantities of interest given the observed information. Considering a series affected by time, through dynamic and random deformations, they associate seasonal or regressive components.

In this work were used contributions from West e Harrison [1997], Laine [2019], Petris et al. [2009] and Petris [2010]. For each time t, the general univariate DLM is defined by an observational equation,

$$Y_t = F_t\theta_t + v_t, \qquad v_t \sim N_m(0, V_t), \tag{3}$$

a system equation

$$\theta_t = G_t\theta_{t-1} + w_t, \qquad w_t \sim N_p(0, W_t) \tag{4}$$

and initial information given by

$$(\theta_0|D_0) \sim N(m_0, C_0), \tag{5}$$

where F_t e G_t are known matrices; v_t and w_t are two sequences of independent noises, with average zero and known covariance matrices V_t and W_t respectively. D_t is the current information set; m_0 and C_0 contains relevant information about the future, according usual statistical sense, given D_t, (m_t, C_t) is sufficient for $(Y_{t+1}, \theta_{t+1}, \ldots, Y_{t+k}, \theta_{t+k})$.

To take into account growth and seasonality, it is defined $\theta_t = (\mu_t, \beta_t, \gamma_t, \gamma_{t-1}, \gamma_{t-2})$, where μ_t is the current level, β_t is the slope of the trend, $\gamma_t,\ \gamma_{t-1}$ and γ_{t-2} are the seasonal components.

For the study, it was assumed the observational variance $V_t = \sigma^2$, and the covariance matrix of the system W_t is a diagonal matrix introduced by $W_t = diag(\sigma^2_\mu, \sigma^2_\beta, \sigma^2_\gamma, 0, 0)$. These unknown variances were also estimated using Bayesian inference. Thus, to complete the specification of the model, it was assumed independent inverse gamma priors distributions with means $a, a_{\theta_1}, a_{\theta_2}, a_{\theta_3}$ and variances $b, b_{\theta_1}, b_{\theta_2}, b_{\theta_3}$, respectively, fixed in known values.

Therefore, by using the unobserved states as latent variables, a Gibbs sampler can be run on the basis of the following full conditional densities:

$$\begin{aligned}
\sigma^2 &\sim IG\left(\tfrac{a^2}{b} + \tfrac{n}{2}, \tfrac{a}{b} + \tfrac{1}{2}SS_y\right), \\
\sigma^2_\mu &\sim IG\left(\tfrac{a^2_{\theta_1}}{b_{\theta_1}} + \tfrac{n}{2}, \tfrac{a_{\theta_1}}{b_{\theta_1}} + \tfrac{1}{2}SS_{\theta_1}\right), \\
\sigma^2_\beta &\sim IG\left(\tfrac{a^2_{\theta_2}}{b_{\theta_2}} + \tfrac{n}{2}, \tfrac{a_{\theta_2}}{b_{\theta_2}} + \tfrac{1}{2}SS_{\theta_2}\right), \\
\sigma^2_\gamma &\sim IG\left(\tfrac{a^2_{\theta,3}}{b_{\theta_3}} + \tfrac{n}{2}, \tfrac{a_{\theta_3}}{b_{\theta_3}} + \tfrac{1}{2}SS_{\theta_3}\right),
\end{aligned} \tag{6}$$

with $SS_y = \sum_{t=1}^{n} (y_t - F_t\theta_t)^2$ and $SS_{\theta_i} = \sum_{t=1}^{T} (\theta_{t,i} - (G_t\theta_{t-1})_i)^2$, for $i = 1,\ 2,\ 3$. The full conditional density of the states is a normal distribution and it is covered in the used dlm package [Petris, 2010].

2.1.4. NNAR

The artificial neural networks model seeks to model the relationship between a set of input signals and an output signal. We can describe a feedforward neural network through a hidden layer and a layer of lagged inputs, being a useful approach for forecasting univariate time series. When lagged values of the time series are uses as inputs to a feedforward neural network, this process is called neural network autoregression or NNAR model [Hyndman e Athanasopoulos, 2018]. We can consider the relationship between the output and the inputs of neural network autoregression as

$$y_t = w_0 + \sum_{j=1}^{h} w_i \cdot g\left(w_{0,j} + \sum_{i=1}^{n} w_{i,j} \cdot y_{t-j}\right) + \varepsilon \tag{7}$$

where y_t is the output, $(y_{t-1}, \ldots, y_{tp})$ are the inputs, the model parameters (weights) are $w_{ij} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, h)$, and $w_j (j = 1, 2, \ldots, h)$.

The usually activation function used is a sigmoid function, given by

$$sig(x) = \frac{1}{1 + e^{-x}} \tag{8}$$

The neural network autoregression model used in this study considers seasonality. That is, the input layer is given by $(y_{t-1}, y_{t-2}, \ldots, y_{tp}, y_{tm}, y_{t-2m}, y_{t-Pm})$ with a hidden layer with k nodes, NNAR$(p, P, k)_m$.

2.2. Combination Approach

2.2.1. ML-Poly

The combination of models for the prediction of time series is an approach that seeks to improve the predictive capacity when working with the composition of results of the models. The pionner article by Bates e Granger [1969] presents evidence that through the combination of the forecasts it is possible to obtain a mean square error less than that of the original predictions. In this work, was used a polynomially weighted average aggregation rule with multiple learning rates (ML-Poly) for each model, with the learning rates calibrated by theoretical values [Cesa-Bianchi e Lugosi, 2006]. Algorithm 1 presents an implementation for this combination technique.

Algorithm 1: The polynomially weighted average forecaster with multiple learning rates (ML-Poly) [Gaillard e Goude, 2014]

Initialization: $\boldsymbol{p}_1 = (1/K, \ldots, 1/K)$ and $\boldsymbol{R}_0 = (0, \ldots, 0)$;
For each instance: $t = 0, 1, \ldots, T$;
0. pick the learning rates;
$$\eta_{k,t-1} = 1/\left(1 + \sum_{s=1}^{t-1} \left(\ell_s\left(\widehat{y}_s\right) - \ell_s\left(x_{k,s}\right)\right)^2\right);$$
1. form the mixture $\hat{p}_t$ defined component-wise by;
$$\widehat{p}_{k,t} = \eta_{k,t-1}\left(R_{k,t-1}\right)_+ / \boldsymbol{\eta}_{t-1} \cdot \left(\boldsymbol{R}_{t-1}\right)_+;$$
where x_+ denotes the vector of non-negative parts of the components of x;
2. output prediction $\widehat{y}_t = \widehat{\boldsymbol{p}}_t \cdot \boldsymbol{x}_t$;
3. for each expert k update the regret;
$$R_{k,t} = R_{k,t-1} + \ell_t\left(\widehat{y}_t\right) - \ell_t\left(x_{k,t}\right)$$

The combinations of the proposed models in this study are C^1 (HW-SARIMA-MLD), C^2 (HW-SARIMA-NNAR-MLD), C^3 (SARIMA-MLD), C^4 (HW-SARIMA-NNAR), and C^5 (HW-SARIMA).

3. Empirical Results

This section presents the results obtained using the Holt-Winters method, SARIMA model, dynamic linear model and NNAR model to fit the interest data. For each model, it was plotted the observed and predicted values, and also the 95% confidence interval for the predicted values. Graphics are effective tools to understand the behavior of the series and whether the models generate reasonable fit and predictions in relation to the observed data.

Data used for the analysis are quarterly and comprise the first quarter of 1996 until the third quarter of 2020, in Brazilian Real (BRL) at 1995 prices. Statistical analyzes, as well as graphic representations, were built using open-source software R Core Team [2020].

3.1. Description of Data

The United Nations [2010] says that GDP derives from the concept of value added. Therefore, GDP is the sum of gross value added of all resident producer units plus that part of taxes on products, fewer subsidies on products. GDP is also equal to the sum of finalizes of goods and services measured at purchasers' prices, less the value of imports goods and services. And GDP is too equal to the sum of primary incomes distributed by resident producer units. The time series constructed in this work was built from the perspective of expenditure. It is calculated by the sum of private consumption, government spending, investment (gross fixed capital formation plus changes in inventories), and net exports (exports minus imports). Thus, the five variables used in this study are presented in Figure 1 and defined as follows: **Pri** - Brazilian quarterly private consumption; **Pub** - Brazilian quarterly government spending; **Inv** - Brazilian quarterly investment (gross fixed capital formation plus changes in inventories); **Exp** - Brazilian quarterly exports; and **Imp** - Brazilian quarterly imports.

The Table 1 show the variables statistical description. Our models consider a sample with 100 observations for each variable split in a training set of 92 observations and a testing set of 8 observations.

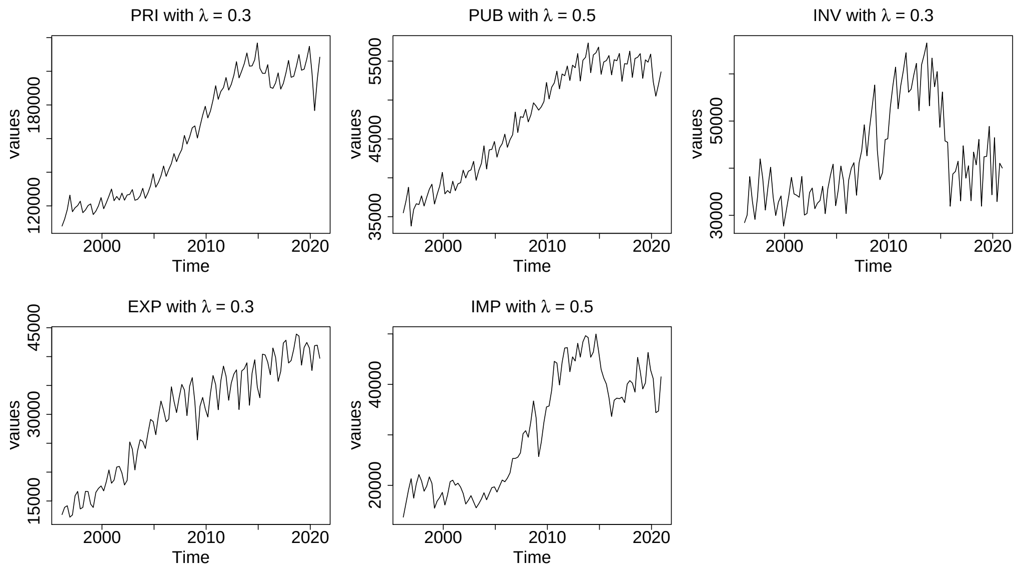

Figure 1: Brazilian quarterly Pri, Pub, Inv, Exp, and Imp variables with Box-Cox transformation (λ) in the period from 1Q1996 to 4Q2020, at 1995 prices.

Table 1: Summary statistics for Brazilian quarterly GDP components from 1Q1996 to 4Q2020.

Description	Pri	Pub	Inv	Exp	Imp
Sample size	100	100	100	100	100
Min	108013	33803	27728	12205	13697
1st Qu.	125886	40808	34076	20750	19685
Median	161407	48746	39999	31881	30525
Mean	161704	47396	42177	29812	30602
3rd Qu.	196597	53819	48474	37536	40841
Max	216829	57330	66554	43903	49971
Kurtosis	-1.670	-1.455	-0.573	-1.170	-1.559
Skewness	0.013	-0.256	0.7402	-0.399	0.139

3.2. Performance Comparison

In Table 2, it is possible to observe the MAPE result about the predictions of the training set of the models used in the study. It is possible to observe that all series had the best fit through model combinations. The combination C^4 obtained the best adjustment result for variables Pri and Pub. And the combination C^5 gets the best adjustment results for Inv, Exp, and Imp. Thus, the results indicate the combinations are superior to the predictions that consider the models individually - for data sets in analysis.

Table 2: MAPE comparison between Holt-Winters method, SARIMA, DLM, NNAR, and the combinations C^1, C^2, C^3, C^4, and C^5 in relation to the fitted of models to Brazilian quarterly GDP components data from 1Q1996 to 4Q2018, at 1995 prices. Note: *HW-Additive and **HW-Multiplicative.

Model	Pri ($\lambda = 0.3$)	Pub ($\lambda = 0.5$)	Inv ($\lambda = 0.3$)	Exp ($\lambda = 0.3$)	Imp ($\lambda = 0.5$)
HW	*0.307	*0.636	**1.953	**1.411	*2.460
SARIMA	0.328	0.682	1.817	1.346	2.446
DLM	3.003	3.255	2.977	4.099	4.127
NNAR	0.485	0.649	2.279	1.572	3.017
C^1	0.9062	0.789	1.792	1.928	2.355
C^2	0.9061	0.787	1.858	1.927	2.379
C^3	1.248	0.910	1.798	2.224	2.524
C^4	**0.270**	**0.531**	1.878	1.355	2.306
C^5	0.275	0.544	**1.789**	**1.327**	**2.272**

In Table 3, it is possible to observe the MAPE result about the predictions of the test set of the models used in the study. In this case, the specifications were not superior to the individual models for all variables. The results are heterogeneous, indicating that an individual model or a combination was not able to generate results for more than one variable. The combination C^4 generated the best forecast results for the variable Pri; the Holt-Winters additive method for a variable pub; the dynamic linear model for inv. variable; the autoregressive neural network model for the Exp variable; and the C^1 combination for the Imp variable.

Table 3: MAPE comparison between Holt-Winters method, SARIMA, DLM, NNAR, and the combinations C^1, C^2, C^3, C^4, and C^5 in relation to the forecast of models to Brazilian quarterly GDP components data from 1Q2019 to 4Q2020, at 1995 prices. Note: *HW-Additive and **HW-Multiplicative.

Model	Pri ($\lambda = 0.3$)	Pub ($\lambda = 0.5$)	Inv ($\lambda = 0.3$)	Exp ($\lambda = 0.3$)	Imp ($\lambda = 0.5$)
HW	*1.301	***1.422**	**3.092	**3.275	*4.934
SARIMA	1.045	1.496	3.413	2.722	4.035
DLM	1.205	1.681	**3.028**	5.729	4.237
NNAR	1.124	1.756	3.351	**1.640**	5.493
C^1	1.052	1.462	3.121	3.037	**3.778**
C^2	0.953	1.480	3.118	1.909	3.903
C^3	0.965	1.505	3.036	3.039	3.798
C^4	**0.935**	1.475	3.404	1.933	3.891
C^5	1.107	1.459	3.114	2.952	3.844

Figure 2 presents the best forecasting models for each variable. It can be observed that the models have a significant ability to adjust to the behavior of the series, even for series that have challenging behaviors such as Inv and Imp. The forecast considered a horizon of 8 steps ahead, with a confidence interval of 95%.

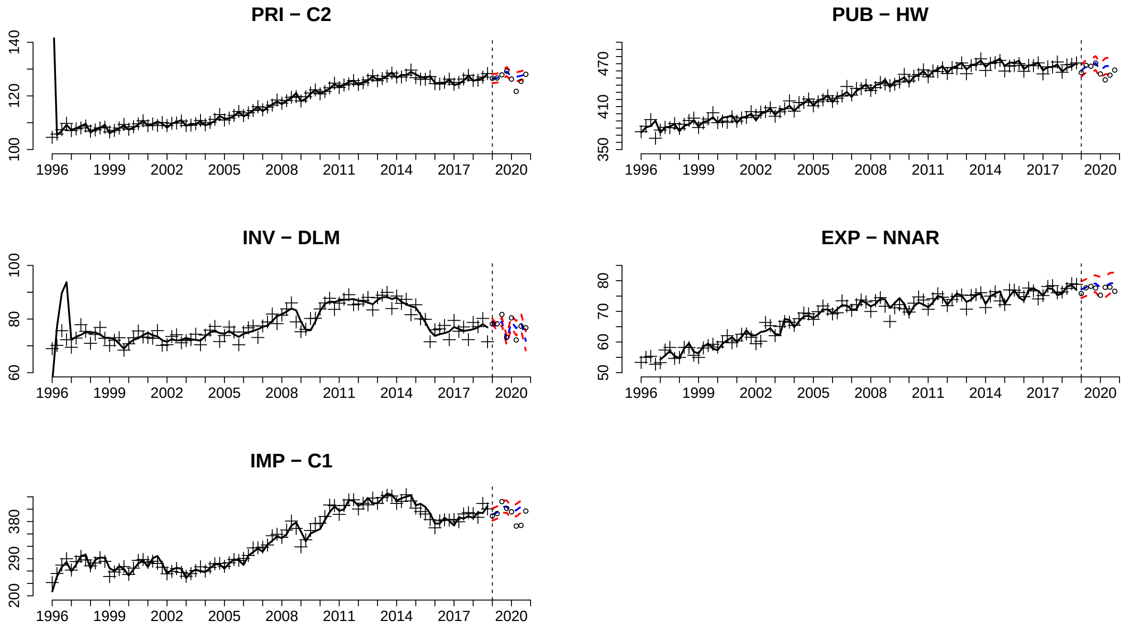

Figure 2: Models fitted (solid line) to the observed Brazilian quarterly GDP components ("+") in the period from 1996 to 2018, at 1995 prices. Forecast (blue line) for the horizon of 8 steps ahead with its interval of 95% confidence (red line), superimposed on the values observed in this period (circles).

4. Conclusion

The components of the GDP are variables that have relevant information regarding the economic activity of a country in a given period. They serve as an auxiliary instrument for decision-making by domestic and foreign investors, and also to identifying patterns of economic behavior useful for the construction of evidence-based public policies. Thus, this study proposed the comparison between univariate models and a combination of forecasts applied to the components of GDP on the demand side.

Our results indicate that the fit of the combinations was superior to that of the univariate models for all components of the GDP. However, the predictive capacity of the univariate models outperformed the combinations for the variables government spending, investment, and exports, showing that the combinations considered are not capable of reducing the mean absolute percentage error in all cases.

For future research, different forecast combinations - for example the weighted averages, linear forecast combinations, and variance-covariance method - can be used in comparison to the ML-Poly algorithm. Furthermore, time series used are at a lower hierarchical level, enabling the application of a bottom-up forecasting strategy.

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