A Mixed Integer Model for the Integrated Lot-Sizing and Raw Material Procurement Problem with Quantity Discounts and Age-Based Holding Costs

Caio Paziani Tomazella
Universidade de São Paulo - Instituto de Ciências Matemáticas e de Computação
Av. Trabalhador São-carlense, 400, 13560-970, São Carlos-SP, Brasil
caio.tomazella@usp.br

Maristela Oliveira Santos
Universidade de São Paulo - Instituto de Ciências Matemáticas e de Computação
Av. Trabalhador São-carlense, 400, 13560-970, São Carlos-SP, Brasil
mari@icmc.usp.br

Douglas Alem
University of Edinburgh Business School
29 Buccleuch Place, EH8 9JS, Edinburgh, United Kingdom
douglas.alem@ed.ac.uk

RESUMO

Este artigo apresenta uma abordagem integrada para lidar com dois problemas da gestão da cadeia de suprimentos, a compra de matéria-prima e dimensionamento de lotes. Esta abordagem tem como motivação o fato de que, conforme mostrado pela literatura recente, um planejamento integrado resulta em um processo de tomada de decisão mais eficiente, reduzindo custos operacionais. Vários fatores presentes na gestão da cadeia de suprimentos são incluídos nesta modelagem: múltiplos fornecedores oferecendo matérias-primas e descontos para compras em grandes quantidades, custo de estoque baseado no preço e idade do material e limitações de orçamento. Os resultados obtidos mostram que a abordagem integrada, a qual trata o problema considerando todas as restrições simultaneamente, resulta em soluções melhores que a sequencial, a qual resolve o problema de dimensionamento de lotes seguido do problema de compra. Ao final do artigo, são apresentadas propostas de pesquisas futuras.

PALAVRAS CHAVE. Dimensionamento de lotes. Integração. Compra de matéria-prima.

ABSTRACT

This article presents an integrated approach for addressing two problems from the supply chain management area, raw material procurement and lot sizing. This approach is motivated by the fact that, as shown by recent literature, an integrated planning leads to more efficient decision-making, reducing operational costs as a whole. Several factors present in supply chain cases are included in this problem: multiple suppliers offering material and discounts for bulk purchases, an unit holding cost calculation which considers material price and age and budget limitations. The results show that the integrated approach, which solves the problem considering all constraints simultaneously, results in better solutions than the sequential approach, which solves first the lot sizing than the procurement problem. Propositions for future research are made in order to improve the integrated approach.

KEYWORDS. Lot sizing. Integration. Raw material procurement.

1. Introduction

The integration of different Supply Chain Management (SCM) problems has, in the past years, received attention due to its efficiency in minimizing operational costs and delivery lead-times and the improving technological capacity for computers to solve more complex problems. This trend is illustrated by the reviews from Moons et al. [2017] and Melega et al. [2018], which show that integrated approaches allow for a more efficient planning than decoupled methods. Several authors have studied the problem of integrating decisions in raw material procurement and lot sizing problems [Kim et al., 2002; Crama et al., 2004; Pathumnakul et al., 2011; Cunha et al., 2015; Zouadi et al., 2018; Cunha et al., 2018]. This integration not only minimizes overall costs but also provides a better control of inventory levels in multiple period scenarios.

In lot sizing problems, several constraints and costs are taken into consideration. The production time is limited by machine availability, and setup operations between jobs account for machine occupation with non-value added activities. In problems with multiple periods and production stages, there are also holding costs for intermediate and finished products. Literature reviews that address this problem in different approaches are found in the articles from Jans e Degraeve [2008], Brahimi et al. [2017] and Melega et al. [2018]. Some applications of sizing in the industry including: pulp and paper mill [Santos e Almada-Lobo, 2012], glass [Almada-Lobo et al., 2008], chemical [Cunha et al., 2018], spinning [Camargo et al., 2014], brewing [Baldo et al., 2014] and beverages [Pagliarussi et al., 2016]. It is also important to observe that some articles in the literature also call ”lot sizing”problems in which a company order products from outside sources to fulfill a distribution demand, which is not the case of this article. Here, the lot sizing problem relates to production planning decisions.

The raw material procurement problem involves deciding when and how much to buy in order to provide enough material for the production process, also taking into consideration factors such as fixed ordering and transportation costs, holding costs, minimum order sizes and delivery lead-times. Wider technological reach allows for the SCM decision makers to buy from different suppliers with different characteristics, which turns the procurement problems into a more complex process. Aissaoui et al. [2007] provided an extensive review on supplier selection and procurement problems and their applications.

In order to increase their competitiveness, suppliers offer lower prices for larger quantities of material bought in the form of discounts policies [Tempelmeier, 2002; Goossens et al., 2007]. Aissaoui et al. [2007] listed and classified these policies used in procurement problems with multiple suppliers. The most commonly used policies are Total, Incremental and Bundle. Total discounts are applied to all products when a condition is met, which can be based on the quantity of purchased products or the business volume of the order, that is, the total value of the products. Incremental discounts are given similarly to Total, but are only valid for products that are beyond a given discount threshold. Bundle discounts are applied when a number of different products are purchased from the same supplier.

Regarding the integrated lot sizing and procurement problem, Crama et al. [2004] discussed the importance of discount policies in both operational and strategical levels. In operational decisions, discount policies can lead the planning department to consolidate several purchases into one in order to get the most advantageous prices. In the strategical level, the results of the procurement model can be used for simulations and negotiations.

Discount policies also have an effect on the production decisions. Cunha et al. [2018] showed that, in a problem that integrates procurement and production with flexible recipes, the production is planned so that the raw material usage that takes more advantage of the discount policies. Significant cost reductions can be observed when the results between the integrated and subsequent approaches are compared.

The problem addressed in this article also provides the calculation of the holding costs of a product or material kept in stock based on its price and age. Xie et al. [2013] highlighted the importance of including the product purchase value in its holding costs since, among all factors of inventory keeping (eg. operational costs, losses and insurance), the capital cost of the products is the most significant one. As for the importance of considering product age, Tekin et al. [2001] showed that it benefits systems with perishable products. Regarding models for this type of inventory keeping, Coelho e Laporte [2014] and Costa et al. [2014] used integer variables to keep track of inventory age in problems with item perishability.

In this article, the raw material holding costs are initially calculated as a fraction of the purchasing costs, which is a more accurate model to value the total inventory when there are significant price variations, such as with seasonal products. Regarding the inventory age, the problem considers an increase over its initial holding cost for each period the item is kept in storage, in order to avoid unbalanced purchasing decisions, such as purchasing a large quantity of a cheap material in the first period to cover its demand during the entire planning horizon.

Overall, an integrated planning of raw material procurement and production planning creates more opportunities for minimizing costs in both parts of the supply chain. The integration allows the procurement planning to take advantage of suppliers discount policies and to minimize order and delivery costs, also considering the impact of large batches acquisitions in inventory. From the production point of view, more efficient planning is made optimizing setup operations, inventory levels and guaranteeing material availability. Lastly, the inventory pricing helps in avoiding cases such as holding items for long periods and keeping expensive material in storage.

The objective of this article is to present a Mixed Integer Problem (MIP) formulation for the lot sizing and raw material procurement problem with multiple suppliers offering total quantity discounts and a holding cost function based on the price and age of the product, and evaluate the efficiency of solving both decisions with an integrated approach. The proposed MIP is based on Cunha et al. [2015], which addressed the integrated model with a single supplier with no discounts and price based raw material holding costs. Additional constraints, based on Goossens et al. [2007], are presented to model the supplier selection and discount options. The variables and constraints used to model the inventory pricing policy are adapted from the article of Coelho e Laporte [2014].

This article is divided as follows: Section 2 describes the addressed problem in details and presents a Mixed Integer Problem (MIP) formulation; Section 3 details the computational experiments specifications, used instances and results; Section 4 contains the final remarks regarding the results.

2. Problem Description and Mathematical Model

In this problem, a facility produces J unique products over a planning horizon that is divided into T periods. There is an external deterministic demand for all products that needs to be fulfilled and no backlogging is allowed. The bill of materials of each product consists of raw material that needs to be purchased and from other products, forming a multi-level product structure and creating internal demand for the products in its lower level.

The production process is done by M machines with limited capacity, and each product is designated to a single machine. Since multiple products are products by each machine, setup operations are needed to be done whenever a change occurs. These operations do not depend on the production sequence, and setups can be carried over through multiple periods without any additional cost. In case the machine capacity is not enough to fulfill the total product demand, it can be extended by working overtime, although it is penalized by a significant increase in costs.

F raw materials are needed for the production, which are purchased from S available suppliers. The purchasing decisions are more flexible since each material is offered by multiple suppliers, and each supplier also sells multiple products. In order to increase competitiveness, suppliers offer price reductions in form of Total Quantity Discounts (TQD), in which discounts are applied in the base price of all units of material purchased if the ordered quantity falls within a given interval. The base price of a material is equal for all suppliers, with the difference laying on the discounted price and interval ranges given by each supplier that sells that material. Every supplier offers D discount intervals for each product that it sells. Since fixed costs are charged by the suppliers every time a purchase is made, there is an advantage in ordering more than one material and in large quantities.

The total volume spent in raw material purchases is bounded by a budget, which is set beforehand and is limited by the availability of resources from the manufacturer. With budget constraints, it becomes necessary to plan ahead for periods with higher material consumption by purchasing beforehand and making larger orders that achieve better discount intervals. In order to ensure feasibility, this set budget can be exceeded subject to a penalty cost. Here we set this penalty as a factor of 1, which means that any value spent over the budget is computed a second time in the objective function.

Holding costs of both products and raw materials increase over the time they are kept in storage. The age of an intermediate product or material does not affect other parameters such as units consumed for production or machine processing time. There is no perishability and, for modelling purposes, a maximum age of T periods was set in order to limit the number of variables and parameters. Raw materials can be consumed in the same period they are ordered, while intermediate products require a provisioning lead-time, which means that they can only be used for production in the periods that follow the one in which they were finished. Since this model is designed to be applied in rolling horizon cases with demand and production cycles, the final inventory is set to be within a range of the initial values, in order to avoid shortages when the following cycle starts.

The indexes, subsets, parameters and variables used in this model are listed below.

Indexes

$f \in \{1, 2, \dots, F\}$	Raw material
$s \in \{1, 2, \dots, S\}$	Supplier
$d \in \{1, 2, \dots, D\}$	Discount interval
$j, i \in \{1, 2, \dots, J\}$	Product
$m \in \{1, 2, \dots, M\}$	Machine
$t \in \{0, 1, \dots, T\}$	Period
$k \in \{0, 1, \dots, T\}$	Product age

Subsets

$\mathbb{R}(f)$	Products that consume raw material f in their production
$\mathbb{P}(j)$	Products that consume product j in their production
$\mathbb{K}(m)$	Products that use machine m in their production

Parameters

$\overline{a}_{f,j}$	Quantity of material f consumed for one unit of product j
$a_{j,i}$	Quantity of product j consumed for one unit of product i
b_j	Production time of a single unit of product j
s_j	Machine setup time for the production of product j
$u_{f,s,d,t}$	Upper bound of interval d given by supplier s for material f in t
$dem_{j,t}$	Demand for product j in period t
$CAP_{m,t}$	Production capacity of machine m in period t
B_t	Purchasing budget in period t
$\overline{I}_f^{k,0}$	Inventory of material f with age k at the start of the planning period
$I_j^{k,0}$	Inventory of product j with age k at the start of the planning period
I_α	Range factor for the final inventory values
Δt_j	Production lead-time of product j
D_j	Total internal and external demand of product j
cs_j	Machine setup cost for the production of product j
co_m	Overtime cost of machine m for a time unit
$\overline{h}_{f,t}^k$	Holding cost for an unit of material f with age k in t
$h_{j,t}^k$	Holding cost for an unit of product j with age k in t
$\overline{p}_{f,s,d,t}$	Price of an unit of material f from supplier s in interval d in t
o_s	Fixed order cost of supplier s

Variables

$\overline{I}_{f,t}^k$	Inventory of material f with age k at the end of t
$I_{j,t}^k$	Inventory of product j with age k at the end of t
$\overline{W}_{f,t}^k$	Units of material f with age k consumed in t
$W_{j,t}^k$	Units of product j with age k consumed in t
$Q_{f,s,d,t}$	Units of material f purchased from s in interval d in t
$X_{j,t}$	Units of product j produced in t
$\overline{O}_t$	Purchased volume that exceeds the budget in t
$O_{m,t}$	Overtime of machine m in t
$Y_{j,t}$	Setup configuration for the production of product j in t
$U_{j,t}$	Setup carry-over for the production of product j from $t-1$ to t
$V_{m,t}$	Auxiliary variable for machine m in t
$Z_{f,s,d,t}$	1 if material f is purchased from supplier s in interval d in t, 0 otherwise
$Z'_{s,t}$	1 if there is a purchase from supplier s in t, 0 otherwise

2.1. Objective Function

The total cost function to be minimized is divided in three parts for simplification purposes. Equation (1) is the sum of the raw material procurement variable and fixed costs and penalties for exceeding the budget. Equation (2) is the sum of the holding costs for both raw materials and manufactured products. Equation (3) is the sum of the setup operations and overtime capacity usage costs. The final cost function is giving by adding these three terms, as shown in Equation (4). These three sums are added in the main function, Equation (3).

$$PC = \sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{d=1}^{D}\sum_{t=1}^{T}(\overline{p}_{f,s,d,t} \cdot Q_{f,s,d,t}) + \sum_{s=1}^{S}\sum_{t=1}^{T}(o_s \cdot Z'_{s,t}) + \sum_{t=1}^{T}\overline{O}_t \tag{1}$$

$$IC = \sum_{f=1}^{F}\sum_{k=0}^{T}\sum_{t=1}^{T}(\overline{h}^k_{f,t} \cdot \overline{I}^k_{f,t}) + \sum_{j=1}^{J}\sum_{k=0}^{T}\sum_{t=1}^{T}(h^k_{j,t} \cdot I^k_{j,t}) \tag{2}$$

$$OC = \sum_{j=1}^{J}\sum_{t=1}^{T}(cs_j \cdot (Y_{j,t} - U_{j,t})) + \sum_{m=1}^{M}\sum_{t=1}^{T}(co_m \cdot O_{m,t}) \tag{3}$$

$$\min z = PC + IC + OC \tag{4}$$

2.2. Raw Material Procurement Constraints

Constraints (5) set the purchased quantity in its proper discount interval. Constraints (6) guarantee that at most one discount interval is chosen per product. Constraints (7) limit the monthly procurement volume to a budget, with a penalty for overspending.

$$u_{f,s,d-1,t} \cdot Z_{f,s,d,t} \leq Q_{f,s,d,t} \leq u_{f,s,d,t} \cdot Z_{f,s,d,t} \qquad \forall f,s,d,t \geq 1 \tag{5}$$

$$\sum_{d=1}^{D} Z_{f,s,d,t} \leq Z'_{s,t} \qquad \forall f,s,t \geq 1 \tag{6}$$

$$\sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{d=1}^{D}(\overline{p}_{f,s,d,t} \cdot Q_{f,s,d,t}) + \sum_{s=1}^{S}(o_s \cdot Z'_{s,t}) \leq B_t + \overline{O}_t \qquad \forall t \geq 1 \tag{7}$$

2.3. Raw Material Inventory Flow Constraints

Constraints (8) set the values for the initial inventory, while constraints (9) are responsible for keeping track of the unused inventory age from one period to its following one. Constraints (10) define the material quantity that is purchased and not consumed in a period as brand new inventory. Constraints (11) describe the entire inventory flow, considering materials with all ages. Constraints (12) set that the amount of consumed raw material considering all ages is equal to the amount required by the production processes, given that the material age does not have an influence in the product. Constraints (20) guarantee a minimum inventory of raw material to start the following planning horizon.

$$\overline{I}_f^{k,0} = \overline{I}_{f,0}^k \qquad \forall f, k \geq 1 \qquad (8)$$

$$\overline{I}_{f,t-1}^{k-1} - \overline{W}_{f,t}^k = \overline{I}_{f,t}^k \qquad \forall f, t \geq 1, k \geq 1 \qquad (9)$$

$$\sum_{s=1}^{S}\sum_{d=1}^{D} Q_{f,s,d,t} - \overline{W}_{f,t}^0 = \overline{I}_{f,t}^0 \qquad \forall f, t \geq 1 \qquad (10)$$

$$\sum_{k=0}^{T} \overline{I}_{f,t-1}^k + \sum_{s=1}^{S}\sum_{d=1}^{D} Q_{f,s,d,t} - \sum_{k=0}^{T} \overline{W}_{f,t}^k = \sum_{k=0}^{T} \overline{I}_{f,t}^k \qquad \forall f, t \geq 1 \qquad (11)$$

$$\sum_{k=0}^{T} \overline{W}_{f,t}^k = \sum_{j \in \mathbb{R}(f)} (\overline{a}_{f,j} \cdot X_{j,t}) \qquad \forall f, t \geq 1 \qquad (12)$$

$$I_\alpha \cdot \sum_{k=0}^{T} \overline{I}_f^{k,0} \leq \sum_{k=0}^{T} \overline{I}_{f,T}^k \qquad \forall f \qquad (13)$$

2.4. Product Inventory Flow Constraints

These constraints are similar to those used for the raw material inventory flow. Constraints (14) and (15) are the equivalent of (8) and (9), respectively. Also, in Constraints (14), it is also necessary to subtract from the initial inventory the units that are consumed by another products already in the first period due to the provisioning lead-time. Constraints (16) set zero-age inventory as the number of units produced minus the amount of these units that consumed in that period. Constraints (17) guarantee the inventory flow. Constraints (18) and (19) equal the total number of consumed units in a period to the demand plus the amount used in order to manufacture other units in case of an intermediate product. Finally, Constraints (20) set a minimum final inventory value for the products.

$$I_j^{k,0} - X_{j,0}^k = I_{j,0}^k \qquad \forall j, k \geq 1 \qquad (14)$$

$$I_{j,t-1}^{k-1} - W_{j,t}^k = I_{j,t}^k \qquad \forall j, t \geq 1, k \geq 1 \qquad (15)$$

$$X_{j,t} - W_{j,t}^0 = I_{j,t}^0 \qquad \forall j, t \geq 1 \qquad (16)$$

$$\sum_{k=0}^{T-1} I_{j,t-1}^k + X_{j,t} - \sum_{k=0}^{T} W_{j,t}^k = \sum_{k=0}^{T} I_{j,t}^k \qquad \forall j, t \geq 1 \qquad (17)$$

$$\sum_{k=0}^{T} X_{j,0}^k = \sum_{i \in \mathbb{P}(j)} a_{j,i} \cdot Q_{i,\Delta t_j} \qquad \forall j \qquad (18)$$

$$\sum_{k=0}^{T} W_{j,t}^k = dem_{j,t} + \sum_{i \in \mathbb{P}(j)} (a_{j,i} \cdot Q_{i,t+\Delta t_j}) \qquad \forall j, t \geq 1 \qquad (19)$$

$$I_\alpha \cdot \sum_{k=0}^{T} I_j^{k,0} \leq \sum_{k=0}^{T} I_{j,T}^k \qquad \forall j \qquad (20)$$

2.5. Production Constraints

Constraints (21) limit the machine usage to its maximum capacity, and any necessary extra usage is counted as overtime work. Constraints (22) guarantee that the production occurs only if the needed setup operations are done. These constraints also bound the value of the $X_{j,t}$ variable to the total demand of product j from period t onwards.

The setup carry-over is modelled using Constraints (23) to (28): Constraints (23) limit the carry-over to one product setup per machine; Constraints (24) and (25) allow for the same setup to be carried over to subsequent periods; Constraints (26) and (27) eliminate the need of setup operations being performed again when the setup is carried over from the previous period, and Constraints (28) set that no setup is carried over to the start of the first period.

$$\sum_{j \in \mathbb{K}(m)} ((b_j \cdot X_{j,t}) + s_j \cdot (Y_{j,t} - U_{j,t})) \leq CAP_{m,t} + O_{m,t} \qquad \forall m, t \geq 1 \tag{21}$$

$$X_{j,t} \leq D_j \cdot Y_{j,t} \qquad \forall j, t \geq 1 \tag{22}$$

$$\sum_{j \in \mathbb{K}(m)} U_{j,t} \leq 1 \qquad \forall m, t \geq 1 \tag{23}$$

$$U_{j,t} \leq Y_{j,t-1} \qquad \forall j, t \geq 1 \tag{24}$$

$$U_{j,t} \leq Y_{j,t} \qquad \forall j, t \geq 1 \tag{25}$$

$$U_{j,t} + U_{j,t+1} \leq 1 + V_{m,t} \qquad \forall m, j \in \mathbb{K}(m), t \geq 1 \tag{26}$$

$$Y_{j,t} - U_{j,t} + V_{m,t} \leq 1 \qquad \forall m, j \in \mathbb{K}(m), t \geq 1 \tag{27}$$

$$U_{j,1} = 0 \qquad \forall j \tag{28}$$

2.6. Variable Domains

These last constraints sets are used to define the domain of the variables in the model.

$$\overline{I}^k_{f,t}, I^k_{j,t}, \overline{W}^k_{f,t}, W^k_{j,t}, Q_{f,s,d,t}, X_{j,t}, \overline{O}_t, O_{m,t} \geq 0 \quad \forall f, j, s, d, m, k, t \tag{29}$$

$$Y_{j,t}, U_{j,t}, V_{m,t}, Z_{f,s,d,t}, Z'_{s,t} \in \{0, 1\} \qquad \forall f, j, s, d, m, t \tag{30}$$

2.7. Solution Approaches

This article presents two approaches to solve the lot sizing and raw material procurement problem. The first one, Integrated Model, solves the problem in an integrated manner, considering all variables and constraints, and takes advantage of the effect a production decision has on the procurement problem and vice versa. The second approach, the Sequential Model, firstly solves the lot sizing problem and then, with a fixed production plan, solves the procurement problem.

- **Integrated Model**: solves both the lot sizing and procurement problems simultaneously, minimizing the cost function in Equation (4) subject to Constraints (5)-(30).

- **Sequential Model**: solves the lot sizing problem, subject to Constraints (14)-(30), then uses the production values ($X_{j,t}$) as inputs for the procurement problem, subject to Constraints (5)-(13),(29)-(30). In both problems the objective to be minimized is the sum of the respective costs of the problems in Equation (4).

3. Computational Experiments and Results

The models were coded in Python 3.6.9 and solved by Gurobi 7.5.2 using its default parameters, number of threads limited to 8 and a time limit of 3.600 seconds for each instance. The experimentation was performed in a cluster containing nodes with 2 Intel Xeon E5-2680v2 processors each, 10 cores, 2.8GHz ad 128GB DDR3 1866MHz RAM.

3.1. Experimentation Data

This experimentation used the lot-sizing problem instances from Tempelmeier e Buschkühl [2009], with data regarding raw material purchasing and holding costs, consumption and initial inventory from [Cunha et al., 2015]. In order to adapt these instances for this problem, additional data for discount intervals and budget was generated. Totally, 216 instances were used.

In more detail, the instances chosen were those of Class 2 from the database of Tempelmeier e Buschkühl [2009], which is named in their article as *New invented data*. These instances have the following parameters: number of products $J = 10$, number of machines $M = 3$, number of periods $T = 8$. Using the same nomenclature from the authors, the other characteristics for instances are: assembly and general product structure; non-cyclic process structure; three demand profiles; setup costs, setup times and machine capacity profiles 1, 2 and 1, respectively (as named by the authors). The instance data from Cunha et al. [2015] consist of two raw material profiles, with $F = 6, 12$, and three raw material price scenarios. Further information on these instances can be found in the original articles.

The suppliers and discount data generated for this article follow these assumptions: number of suppliers $S = 12$; each raw material is sold by three different suppliers; discounts are given following the Total Quantity Discount Policy [Goossens et al., 2007] for each individual raw material; the discounts are divided into $D = 5$ discount intervals for each raw material. Two scenarios were generated, with different average maximum discounts (20% and 30%). The fixed orders costs were randomly generated in an uniform distribution in the $[1000, 3000]$ range.

The purchasing budget calculation was done by estimating how much it is going to be spent during the planning horizon based on the product demand and raw material base prices. The budget per period is then obtained by dividing this total value by T and multiplying it by a factor that changes how flexible the purchasing process is. In this case, flexibility means how much the manufacturer can anticipate purchases to take advantage of discounts and stock material for periods with higher consumption, in which the budget will not allow for all the materials to be bought then. The budget scenarios are listed in Table 1, and there are 72 instances for each scenario.

Budget Scenario	Multiplier	Flexibility
1	1.5	Loose
2	1.25	Intermediate
3	1	Tight

Tabela 1: Characteristics of the budget scenarios.

Holding costs of products were directly taken from the instances of Tempelmeier e Buschkühl [2009], while the holding costs of raw materials were set as 20% of the purchasing price. The holding cost function over time is linear: for each period an item is kept in inventory, its holding cost is increased by 15% over its initial value.

Given the parameters, it is possible to calculate the size of the integrated model: with $F = 6$ there are 5484 constraints and 8200 variables (4280 of them binaries); and with $F = 12$, 9522 constraints and 14728 variables (7160 binaries).

3.2. General Results

From the 216 instances, the integrated model optimally solved 26 (12%) within the time limit, with an optimality gap of 3.12%. This gap is given by the solver and is the relative difference between the best solution found and the lower bound of the problem.

As for the sequential model, all instances were solved in an average of 48 seconds, taking 46 seconds for the lot sizing problem and 2 for the procurement. Even though this model is faster, in all cases the integrated model found better solutions. Since the sequential approach solves the lot sizing problem optimally, the production related costs from its solution are lower than those from the integrated approach. Therefore, the cost reduction obtained when using the integrated approach comes from the purchasing and raw material holding costs. The improvement gained with the solution of the integrated model (IM) over the sequential (SM) is calculated using the equation $Improvement(\%) = \frac{SM-IM}{SM}$.

The values of the optimality gap of the integrated model solutions and their improvement over the solutions of the sequential model are shown in Table 2.

	Budget 1		Budget 2		Budget 3	
	Average	Std. Dev.	Average	Std. Dev.	Average	Std. Dev.
Gap	2.62%	1.72%	2.96%	2.05%	3.78%	2.54%
Improvement	9.60%	3.19%	9.14%	3.30%	9.02%	3.52%

Tabela 2: Optimality gap and improvement over sequential solution obtained by solving the integrated model.

3.3. Detailed Results

Table 3 illustrates the cost reduction obtained by the integrated approach when compared to the sequential by showing the how much the purchasing volume is allocated in each discount interval. It can be observed that the integrated approach maximizes the amount of purchases in the interval with higher discounts, and this is done by adjusting the lot sizing decisions in a way that make bulk purchases viable, that is, without excessively increasing inventory costs. It is also possible to notice that with looser budgets, more volume is allocated into better discount intervals, whereas with tighter budget, the volume is distributed more evenly.

Discount Interval	Budget 1		Budget 2		Budget 3	
	Integrated	Sequential	Integrated	Sequential	Integrated	Sequential
1	17.94%	30.20%	18.83%	30.16%	20.71%	29.75%
2	16.77%	29.91%	19.18%	30.71%	21.02%	29.40%
3	18.90%	21.51%	17.39%	21.55%	19.87%	21.95%
4	21.70%	13.01%	20.60%	12.20%	16.78%	13.97%
5	24.69%	5.37%	23.99%	5.38%	21.63%	4.94%

Tabela 3: Percentage of purchasing volume per discount interval.

The graphs in Figure 1 show how the purchasing and raw material holding costs vary over time. The integrated approach focuses on purchasing more material in the earlier periods and reducing inventory to a minimum value for the latter. Both approaches purchase the materials for the final inventory in the last period in order to have no additional costs from inventory age. An effect of the proposed holding costs pricing policy proposed in this model is seen on the sequential model solutions, in which the holding costs are higher in the latter periods even though a lot less material is being purchased, which means that in these cases inventory is being kept for longer.

However, the purchasing strategy obtained by the integrated solution approach is achieved by an unbalanced production planning, as shown by the machine occupancy rates in Figure 2. The lot sizing instances used consider a non-cyclic process structure, in which the products produced in

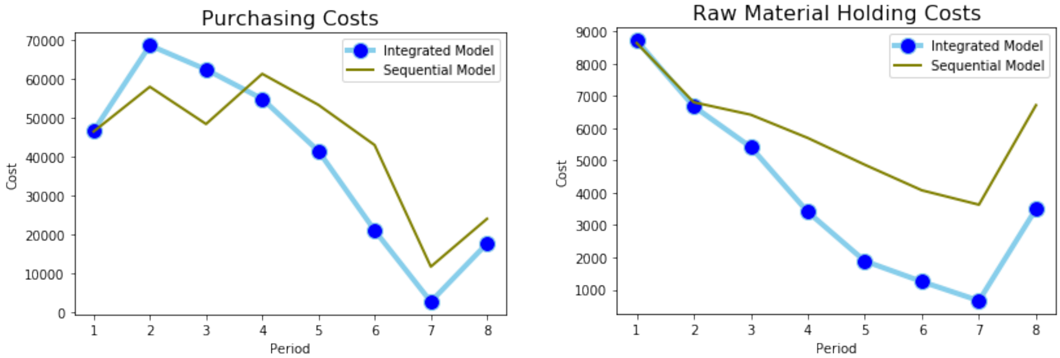

Figura 1: Purchasing and raw material inventory costs per period.

Machine 3 are consumed for those produced in Machine 2, and the same happens with Machines 2 and 1. Therefore, due to the provisioning lead-times, the machine occupancy in Machines 2 and especially 3, is reduced to close to zero in the last periods, only producing what is needed for the final inventory. A solution for this problem is to add constraints that set a lower bound for the machine occupancy, which approximate the model even more to a real life industrial case, in which machines need to operate at a minimum rate so the process is viable.

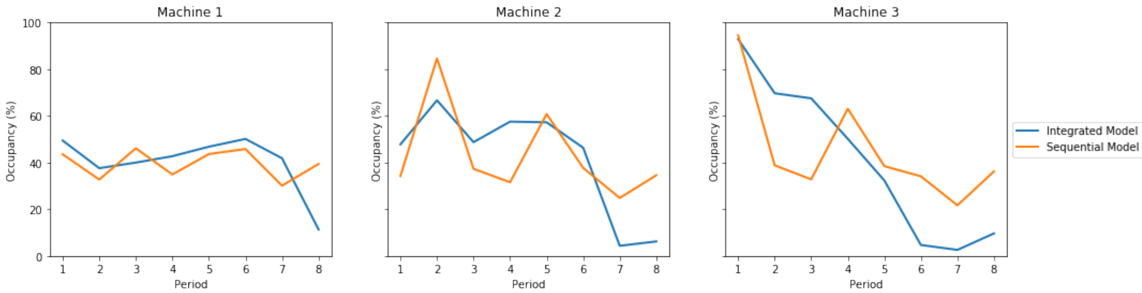

Figura 2: Machine occupancy per period.

4. Conclusion

This article presented an integrated approach to the lot sizing and raw material procurement problem with supplier selection, quantity discounts, age-based holding costs and budget limitations. The proposed model is an extension of the one presented by Cunha et al. [2015] that considers the additional constraints, and the computational experiments used problem instances of the databases from Tempelmeier e Buschkühl [2009] and Cunha et al. [2015], with additional data being generated when needed.

The experimentation results show that the integrated model obtained solutions with total costs that are on average 9.25% lower than those from the sequential model. This cost reduction is achieved by adjusting the lot sizing production plan in order to increase the raw material orders sizes, getting higher discount rates from suppliers. A trade-off exists between these bulk purchases and other limitations that appear in real life cases that are included in this model (budget constraints and aging inventory costs), and the integrated model is shown to be more flexible to optimize the total costs than the sequential, since in this approach the purchasing model is limited by fixed production decisions made when optimally solving the lot sizing model.

Suggestions for future researches address some limitations found while implementing the integrated model, such as the lower machine occupancy rates in the latter periods and more accurate inventory pricing functions. Another improvement is to model this problem using different time scales for procurement and production decisions, considering lead-times for raw material delivery, which are significantly longer than provisioning lead-times. Lastly, it is suggested to analyse the impact on the solutions when backlogging is allowed. In terms of methods, our literature review shows that no heuristics were yet proposed for this integrated problem. Since the results from this article show the limitations of solving the MIP formulation, we aim to develop heuristics and matheuristics for this problem in future works.

Acknowledgements

This research was funded by FAPESP (grant 2019/10824-7) and carried out using the computational resources of the Center for Mathematical Sciences Applied to Industry (CeMEAI) funded by FAPESP (grant 2013/07375-0).

Referências

Aissaoui, N., Haouari, M., e Hassini, E. (2007). Supplier selection and order lot sizing modeling: A review. *Computers & Operations Research*, 34(12):3516–3540.

Almada-Lobo, B., Oliveira, J. F., e Carravilla, M. A. (2008). Production planning and scheduling in the glass container industry: A VNS approach. *International Journal of Production Economics*, 114(1):363–375.

Baldo, T. A., Santos, M. O., Almada-Lobo, B., e Morabito, R. (2014). An optimization approach for the lot sizing and scheduling problem in the brewery industry. *Computers and Industrial Engineering*, 72(1):58–71.

Brahimi, N., Absi, N., Dauzère-Pérès, S., e Nordli, A. (2017). Single-item dynamic lot-sizing problems: An updated survey. *European Journal of Operational Research*, 263(3):838–863.

Camargo, V. C., Toledo, F. M., e Almada-Lobo, B. (2014). HOPS - Hamming-oriented partition search for production planning in the spinning industry. *European Journal of Operational Research*, 234(1):266–277.

Coelho, L. C. e Laporte, G. (2014). Optimal joint replenishment, delivery and inventory management policies for perishable products. *Computers and Operations Research*, 47:42–52.

Costa, A. M., dos Santos, L. M. R., Alem, D. J., e Santos, R. H. (2014). Sustainable vegetable crop supply problem with perishable stocks. *Annals of Operations Research*, 219(1):265–283.

Crama, Y., Pascual, R. J., e Torres, A. (2004). Optimal procurement decisions in the presence of total quantity discounts and alternative product recipes. *European Journal of Operational Research*, 159(2 SPEC. ISS.):364–378.

Cunha, A. L., Santos, M. O., Morabito, R., e Barbosa-Póvoa, A. (2015). Um Modelo para o Problema de Dimensionamento de Lotes com Aquisição de Matérias-Primas. *XLVII SBPO - Simpósio Brasileiro de Pesquisa Operacional*, 1:145–154.

Cunha, A. L., Santos, M. O., Morabito, R., e Barbosa-Póvoa, A. (2018). An integrated approach for production lot sizing and raw material purchasing. *European Journal of Operational Research*, 269(3):923–938.

Goossens, D. R., Maas, A. J. T., Spieksma, F. C. R., e van de Klundert, J. J. (2007). Exact algorithms for procurement problems under a total quantity discount structure. *European Journal of Operational Research*, 178(2):603–626.

Jans, R. e Degraeve, Z. (2008). Modeling industrial lot sizing problems: A review. *International Journal of Production Research*, 46(6):1619–1643.

Kim, B., Leung, J. M., Park, K., Zhang, G., e Lee, S. (2002). Configuring a manufacturing firm's supply network with multiple suppliers. *IIE Transactions (Institute of Industrial Engineers)*, 34 (8):663–677.

Melega, G. M., de Araujo, S. A., e Jans, R. (2018). Classification and literature review of integrated lot-sizing and cutting stock problems. *European Journal of Operational Research*, 271(1):1–19.

Moons, S., Ramaekers, K., Caris, A., e Arda, Y. (2017). Integrating production scheduling and vehicle routing decisions at the operational decision level: A review and discussion. *Computers and Industrial Engineering*, 104:224–245.

Pagliarussi, M. S., Morabito, R., Santos, M. O., Pagliarussi, M. S., Morabito, R., e Santos, M. O. (2016). Optimizing the production scheduling of fruit juice beverages using mixed integer programming models. *Gestão & Produção*, 24(1):64–77.

Pathumnakul, S., Ittiphalin, M., Piewthongngam, K., e Rujikietkumjorn, S. (2011). Should feed mills go beyond traditional least cost formulation? *Computers and Electronics in Agriculture*, 75 (2):243–249.

Santos, M. O. e Almada-Lobo, B. (2012). Integrated pulp and paper mill planning and scheduling. *Computers and Industrial Engineering*, 63(1):1–12.

Tekin, E., Gürler, Ü., e Berk, E. (2001). Age-based vs. stock level control policies for a perishable inventory system. *European Journal of Operational Research*, 134(2):309–329.

Tempelmeier, H. (2002). A simple heuristic for dynamic order sizing and supplier selection with time-varying data. *Production and Operations Management*, 11(4):499–515.

Tempelmeier, H. e Buschkühl, L. (2009). A heuristic for the dynamic multi-level capacitated lotsizing problem with linked lotsizes for general product structures. *OR Spectrum*, 31(2):385–404.

Xie, Z., Park, C. S., e Zheng, L. (2013). Procurement models under purchase price uncertainty for Chinese oil refineries. *International Journal of Production Research*, 51(10):2952–2968.

Zouadi, T., Yalaoui, A., e Reghioui, M. (2018). Hybrid manufacturing/remanufacturing lot-sizing and supplier selection with returns, under carbon emission constraint. *International Journal of Production Research*, 56(3):1233–1248.