

TOWARDS MULTI-CRITERIA CLUSTERING WITH STOCHASTIC PARAMETERS: APPLICATION TO THE HUMAN DEVELOPMENT INDEX PROBLEM

Javier Pereira
Universidad Tecnológica de Chile Inacap
Santiago, Chile
xjavierpereira7@gmail.com; jpereirar@inacap.cl

Pedro Contreras
The University of Warwick
Coventry, UK
pedro.contreras@acm.org

Danielle C. Morais
Universidade Federal de Pernambuco, CDSID
Recife, Brasil
dcmorais@cdsid.org.br

Pilar Arroyo-López
Tecnologico de Monterrey, Campus Toluca
Toluca, México
pilar.arroyo@itesm.mx

RESUMO

O Índice de Desenvolvimento Humano (IDH), que incentiva os formuladores de políticas a se concentrarem nas capacidades das pessoas para desenvolver um país, baseia-se em três índices: a expectativa de vida, os anos de educação e a Renda Nacional Bruta. No entanto, vários estudos criticam o processo de classificação subjacente ao cálculo do IDH, devido à sua natureza compensatória e à aparente imprecisão e incerteza das fontes de informação. Neste artigo, é apresentada uma abordagem de agrupamento de múltiplos critérios, integrando o algoritmo K -means, no qual são consideradas avaliações imprecisas e parâmetros incertos. Esta é uma abordagem de agrupamento de múltiplos critérios, não compensatória e estocástica, aplicada aos países do grupo incluídos no Relatório do IDH. Os resultados obtidos são comparados ao ranking determinístico do IDH de 2018, constatando que o tipo de incerteza modela as diferenças entre essas abordagens. Limitações são discutidas e pesquisas futuras são propostas.

PALAVRAS CHAVE. Agrupamento, K -means, sobreclassificação.

ABSTRACT

The Human Development Index (HDI), which encourages policy makers to focus on people capabilities to develop a country, is based on three indices: the life expectancy,

the years of education and the Gross National Income. However, several studies criticize the the ranking process underlying the HDI calculation because its compensatory nature and seeming precision and uncertainty of information sources. In this paper, a multi-criteria clustering approach, integrating the K -means algorithm, is presented in which imprecise evaluations and uncertain parameters are considered. This is a non-compensatory and stochastic multi-criteria clustering approach applied to group countries included in the HDI Report. Results obtained with this approach are compared to the deterministic 2018 HDI ranking, finding that the type of uncertainty shapes the differences between these approaches. Limitations are discussed and future research is proposed.

KEYWORDS. Clustering, K -means, outranking.

1. Introduction

Since 1990, the United Nations Development Programme (UNDP) releases the Human Development Index (HDI) to emphasize that people and their capabilities should be the ultimate criteria for assessing the development of a country, not only economic growth. The 2018 HDI ranking process evaluates 189 United Nations countries on the basis of three criteria: the life expectancy, the years of education and the Gross National Income (GNI). In addition, based on the HDI quartiles, four ordered groups of countries can be identified: very high human development, high human development, medium human development, and low human development. Despite the broad use of HDI in business, policy-making, research, development-political debates, allocation of development aid, international climate accord designs, and economics [Wolff et al., 2011], it has been object of three main criticisms [Noorbakhsh, 1998; Berenger e Chouchane, 2007; Klugman et al., 2011; Martinez, 2013]: (1) the compensatory effect; (2) the underlying concepts to define the dimensions; (3) the categorization process that uses the COP quartiles. Regarding the compensatory effect, it is easy to see that a country exhibiting a bad performance in one index, can compensate it with a good performance in another. However, this is not desirable since, for instance, bad health indices can be hidden by good GNI, which can exacerbate inequality in access to health systems.

Non-compensatory methods have been proposed as an alternative to the HDI calculation method. For instance, Natoli e Zuhair [2011] analyze three aggregation techniques designed for benchmarking and ranking of countries according to aggregated dimensions: additive methods, geometric aggregations, and non-compensatory methods. They focus on the Condorcet approach as a non-compensatory aggregation technique for progress measures. Lozano e Gutiérrez [2009] propose to use the minimum of the component indexes in HDI instead of the arithmetic average. Mazziotta e Pareto [2015] compare two non-compensatory composite indices for measuring multidimensional phenomena and monitoring their changes over time: a non-linear composite index and a two-parameter function, which is an intermediate case between a compensatory and a full non-compensatory index. The authors found that the non-linear composite seems to be less compensatory than the second one.

Several scholars have proposed ordered clustering strategies to partition the set of countries included in the HDI ranking, based on non-compensatory multi-criteria decision analysis (MCDA) approaches [De Smet, 2014]. Boujelben e De Smet [2016] propose an approach that integrates PROMETHEE and *K*-means, a popular greedy algorithm for partitioning a set of elements into a pre-defined number of clusters so as to minimize the distances to the cluster centroids (the cluster centers). The underlying idea of their approach is based on the notion of belief distance between the alternatives and the centroids. Chen et al. [2018] propose a PROMETHEE-based algorithm, which also integrates the *K*-means rationale, to cluster countries in the HDI Report. They found that the ordered clustering is highly consistent with the HDI ranks. De Smet et al. [2012] propose an exact multi-criteria algorithm to find HDI ordered categories, based on valued preference degrees. They show the consistency between the obtained ordered partition and the HDI ranking. Monteiro et al. [2018] propose a supervised classification approach based on the ELECTRE TRI method in which the HDI categories are defined a priori by using fixed category boundaries. Different from other proposals using the outranking-based models, these authors consider that there is no imprecise data in HDI information sources.

However, two types of imprecision have been highlighted by authors analyzing the quality of information used in the HDI calculation. Firstly, the weighting process assigning equal importance levels to dimensions in HDI has been criticized and methods to deal with have been proposed [Zheng e Zheng, 2015]. Secondly, the importance of considering imprecise data in constructing composite indicators has been remarked [Cherchye et al., 2011]. There are multiple indicators used for different kind of indexes, for instance, the Environmental Performance Index, the Internal Market Index, the Human Development Index, or the Technology Achievement Index. However, for some indicators only interval information is available, within what a true value is believed to lie. Wolff et al. [2011] analyzed three sources of data error in HDI calculation and classification: measurement error due to data revisions, data error due to formula updating and misclassification due to inconsistent categories cut-off values. Actually, measurement error from data updating impacts the rough data, the indices normalization and the HDI calculation. They found that from 11% up to 34% of all countries could be interpreted as misclassified in the development categories. These authors also found that on average the expected absolute deviation is nine rank positions. These results mean that interpretation of HDI ranks must be very careful. Moreover, methods applied to HDI, which are based on the outranking relation, as for instance those using the PROMETHEE or the ELECTRE TRI methods, consider stable parameters. However, this choice seems to be inadequate since the imprecision introduced by data updating and inconsistent categories cut-off values. Thus, parameter values could be supposed to be uncertain, something that has not been yet considered in the HDI clustering literature.

Essentially, MCDA clustering processes integrating K -means lie in two features [De Smet e Eppe, 2009; Lolli et al., 2014; Panapakidis e Christoforidis, 2018; Chen et al., 2018]. First, the definition of an indifference-based metric that measures how indifferent is an alternative to a central alternative, or centroid, representing a category. Thus, an alternative is assigned to the cluster where the most indifferent centroid is found. Second, once all the alternatives have been assigned, the centroids are updated, using the information regarding the alternatives in their respective groups. This process continues until a stop condition is satisfied.

In this article, a stochastic approach for clustering in the HDI problem is proposed, which integrates an outranking-based classification process with K -means, such that: (1) building clusters based on an assignment rule proposed in the ELECTRE TRI-C sorting method [Almeida et al., 2008; Almeida-Dias et al., 2012] is used in the process, extending rules proposed in sorting methods to clustering; (2) uncertain thresholds and criteria weights are considered such that robustness analysis is performed. Thus, the proposed approach integrates a Monte Carlo Simulation process in which sets of parameter values are passed as input to the clustering algorithm. To the best of our knowledge, this is the first approach that deals with these two problems.

The article is organized as follows. In Section 2 basics of the clustering approach are presented. Section 3 presents the clustering approach. In Section 4 countries included in the HDI 2018 report are clustered by following the proposed approach. Results are presented and discussed. Limitations of this approach and final conclusions are described in Section 5.

2. Notation

2.1. Outranking relation

Most of non-compensatory approaches for computing the HDI clustering use the global outranking relation to model imprecise information regarding the evaluation of alternatives. Outranking was early proposed in Elimination Et Choix Traidusant la Realite (ELECTRE) methods [Figueira et al., 2010] and further became an essential feature in Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) methods Brans e Vincke [1985]. The outranking relation helps to model global assertions from kind “the alternative b is at least as good as the alternative a ”, based on information collected at the level of each criteria. Non-compensatory methods based on the outranking relation could lead to higher computational complexity. However, problems exist in which compensation among criteria is not allowable. We place the present article in that kind of situations and propose a clustering model based on the outranking relation, as defined in the ELECTRE family of methods [Figueira et al., 2010].

Let $A = \{a_1, a_2, \dots, a_n\}$ be a set of actions (alternatives); let $F = \{g_1, g_2, \dots, g_m\}$ be a coherent family of criteria used to evaluate each action and $W = \{w_j \mid 0 \leq w_j \leq 1, \sum_{j=1}^m w_j = 1\}$ the criteria weights. The vector of performances of an action $(g_1(a), g_2(a), \dots, g_m(a))$ is called the evaluation profile.

The outranking relation, interpreted as the assertion “ b is at least as good as a ”, can be measured on each criterion in terms of the partial direct concordance index, defined as follows:

$$c_j(b, a) = \begin{cases} 0 & \text{if } g_j(a) - g_j(b) > p_j, \\ 1 & \text{if } g_j(a) - g_j(b) \leq q_j, \\ \frac{g_j(b) - g_j(a) + p_j}{p_j - q_j} & \text{otherwise,} \end{cases} \quad (1)$$

where p_j, q_j are two imprecision parameters called direct preference threshold and direct indifference threshold, respectively. Similarly, let us consider the assertion “ a is at least as good as b ”, by using inverse thresholds p'_j, q'_j , such that a partial inverse concordance index may be defined as

$$c_j^{inv}(a, b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) > p'_j, \\ 1 & \text{if } g_j(b) - g_j(a) \leq q'_j, \\ \frac{g_j(a) - g_j(b) + p'_j}{p'_j - q'_j} & \text{otherwise.} \end{cases} \quad (2)$$

A detailed explanation regarding the properties that direct and inverse thresholds must satisfy can be found in Roy et al. [2012]. The partial direct and inverse concordance indices allow to compute the global concordance indices as follows:

$$\sigma_D(b_h, a) = \sum_{j=1}^m w_j c_j(b_h, a), \quad (3)$$

$$\sigma_I(a, b_h) = \sum_{j=1}^m w_j c_j^{inv}(a, b_h). \quad (4)$$

2.2. Outranking-based classification

Let $C_h, (h = 1, \dots, H; H \geq 2)$ be a set of ordered categories such that $C_H \succ C_{H-1} \succ \dots \succ C_1$, where the relation $\succ$ means that for any C_i, C_j such that $i > j$, an element $a_i \in C_i$ is not worse than any element $a_j \in C_j$. Conversely, a_j is worse than a_i , in terms of preferences. A special set of actions $B = \{b_1, \dots, b_H\}$ is defined, in which b_h represents the centroid of the category C_h .

Let us define the following trapezoidal fuzzy number (TrFN) [Ban et al., 2011]:

$$\mu_j(a, b_h) = \min\{c_j(b_h, a), c_j^{inv}(a, b_h)\}, \quad (5)$$

which can be interpreted as a fuzzy indifference relation, constructed from two outranking relations [Perny e Roy, 1992]. Conversely, deconstructing $\mu_j(a, b_h)$ into the two outranking relations $c_j(b_h, a)$ and $c_j^{inv}(a, b_h)$, the credibility indices (3) and (4) may be calculated and the following assignment rules can be defined [Pereira et al., 2018]:

1. Descending Rule. Let $\lambda \in [0.5, 1]$ be a minimum credibility level. Decrease h from $H + 1$ until the first t such that $\sigma_I(a, b_t) \geq \lambda$:
 - (a) If $t = H + 1$, assign a to C_H .
 - (b) If $t = 0$, assign a to C_1 .
 - (c) For $0 < t < H + 1$,
if $\min\{\sigma_I(a, b_t), \sigma_D(b_t, a)\} > \min\{\sigma_I(a, b_{t+1}), \sigma_D(b_{t+1}, a)\}$ then assign a to C_t ;
otherwise, assign a to C_{t+1} .
2. Ascending Rule. Let $\lambda \in [0.5, 1]$ be a minimum credibility level. Increase h from 0 until the first t such that $\sigma_D(b_t, a) \geq \lambda$:
 - (a) If $t = 1$, assign a to C_1 .
 - (b) If $t = H + 1$, assign a to C_H .
 - (c) For $0 < t < H + 1$,
if $\min\{\sigma_I(a, b_t), \sigma_D(b_t, a)\} > \min\{\sigma_I(a, b_{t-1}), \sigma_D(b_{t-1}, a)\}$ then assign a to C_t ;
otherwise, assign a to C_{t-1} .

Essentially, the descending and the ascending rules evaluate how indifferent an alternative a is to every b_h . Thus, a is assigned to the class for which it is closest. In practice, these rules may assign an alternative to different classes. Therefore, in a particular application of the algorithm presented below, just one of them must be chosen.

2.3. K -means algorithm

K -means is a classical unsupervised method to find a partition of a set into a set of disjoint homogeneous groups, by minimizing the following objective function

$$\min \sum_{h=1}^K \sum_{i=1}^n I_{ih} \|a_i - b_h\|, \quad (6)$$

where $I_{ih} = 1$ if $a_i \in C_h$, 0 otherwise; and $\|a_i - b_h\|$ is the Euclidean distance between a_i and b_h . Classical K -means algorithm consists of three steps: (1) define A, K, b_h as inputs;

(2) assign elements of A into clusters; (3) update the cluster centers and return to step (2), until a stop condition is satisfied. Step (2) assigns an element a_i to the group where the Euclidean distance between the element and the respective cluster center is minimized. Step (3) updates the cluster centroids as follows

$$g_j(b_h) = \frac{1}{|C_h|} \sum_{a_i \in C_h} g_j(a_i) \quad j = 1, \dots, m. \quad (7)$$

Thus, the algorithm is iterated a number of times, or until the change of centroids is less or equal than a given tolerance.

3. Clustering approach

The approach proposed here assumes that $K \geq 2$ ordered clusters must be found: $C_K \succ \dots \succ C_1$. Similarly to the original K -means algorithm, an iterative constructive clustering process is performed until a stop condition is satisfied, as described in Figure 1.

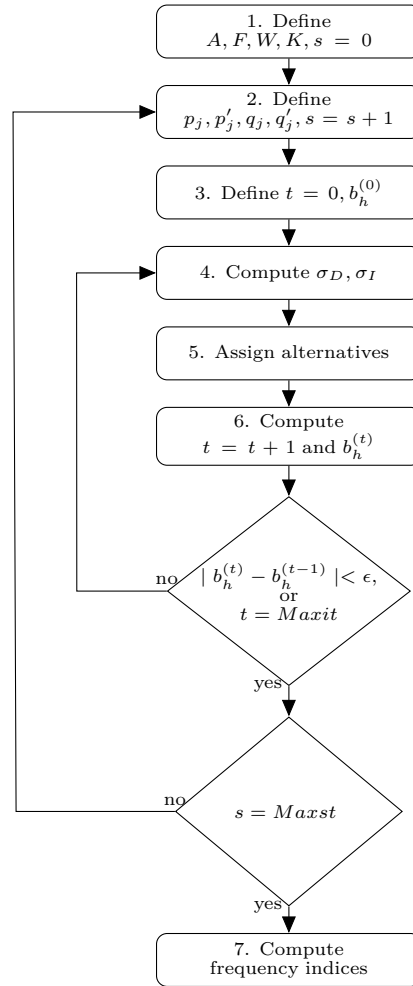


Figure 1: Outranking-based approach to ordered clustering

Initially, the set of alternatives, the family of criteria, the criteria weights, and the number of clusters are identified. Next, the imprecision parameters are set. Stochastic

preference and indifference thresholds are considered in this approach. Thus, each time that the Step 2 is performed, a new set of values for p_j, q_j, p'_j, q'_j ($j = 1, \dots, m$) are generated and used as input to the Step 3, where the new initial centroids are defined. Step 4 consists of computing the credibility indices, using the expressions (3) and (4). In Step 5, the alternatives are assigned into the classes, using the descending (or the ascending) rule proposed in Section 2.2. In Step 6, the evaluation profile of each new centroid in the iteration t and class C_h is computed as follows:

$$g_j(b_h^{(t)}) = \frac{1}{|C_h^{t,j}|} \sum_{a_i \in C_h^{t,j}} g_j(a_i) \quad j = 1, \dots, m, \quad (8)$$

where $C_h^{t,j} = \{a_i \in C_h \mid g_j(b_h^{(t)}) - p'_j \leq g_j(a_i) \leq g_j(b_h^{(t)}) + p_j\}$. Whenever $C_h^{t,j} = \emptyset$, then $g_j(b_h^{(t)}) = g_j(b_h^{(t-1)})$.

Roy et al. [2012] propose that a separability condition must be satisfied in classification problems in which a set of central reference alternatives is used: any two central reference alternatives must be separated enough such that the strong preference property holds, at the level of each criterion. We extend this property to the cases in which the direct and the inverse preference thresholds are used by proposing a strict separability condition:

$$g_j(b_{h-1}) + p_j \leq g_j(b_h) - p'_j, \quad (h = 2, \dots, H; j = 1, \dots, n). \quad (9)$$

If this condition is hold along the iterative process, the resulting cluster centers are ordered. The approach proposed satisfies this property.

In this approach, thresholds are stochastic variables where values are generated by a Monte Carlo Simulation process. Thus, the whole process is repeated *Maxst* times, once for each set of stochastic parameter values. Given a set of parameter values, the inner loop in Figure 1 iterates up to one out of two stop conditions applies: 1) the change of evaluation profiles is lower than a pre-defined ϵ value, or 2) the maximum number of iterations, *Maxit*, is reached.

In Step 7, the number of times that every alternative is assigned into each category is computed. Thus, a frequency profile is obtained for each alternative.

In the following section, this approach is applied to the HDI problem.

4. Clustering HDI countries

Data published in the 2018 HDI Report is retrieved to perform the analysis. There are 189 countries to be clustered. The family of criteria to be considered consists of the three indices: life expectancy (g_1), years of schooling (g_2) and GNI (g_3). These indices are computed using the formulae defined in the HDI Technical Notes [UNDP, 2019]. Initially, weights are considered to be equal: $w_j = 1/3$ for $j = 1, 2, 3$.

4.1. Initial settings

In order to have an idea about the number of clusters, the K -means Elbow Method was run on indices computed from HDI data. The Elbow Method is used to determine the optimal number of clusters in K -means clustering [Ketchen e Shook, 1996]. This gives four clusters, i.e. $K = 4$, which parallels the number of groups defined in the Human Development Report. In order to define an initial set of centroids, the K -means algorithm is run. The following initial centroids are found, which are used as input to our approach:

$$b_1 = (0.27, 0.25, 0.43), b_2 = (0.51, 0.39, 0.64), b_3 = (0.70, 0.53, 0.83), b_4 = (0.90, 0.71, 0.93).$$

For each criterion j , the minimum distance between consecutive centroids $m_j = \min_h \{g_j(b_h) - g_j(b_{h-1})\}$ is calculated, then $m_1 = 0.19, m_2 = 0.14, m_3 = 0.10$. Note that holding the separability condition means $g_j(b_{h-1}) + p_j \leq g_j(b_h) - p'_j$, or equivalently $g_j(b_{h-1}) + p_j + p'_j \leq g_j(b_h)$. Thus, to satisfy the strict separability condition, the following restriction must be defined $p'_j + p_j \leq m_j$ ($j = 1, 2, 3$).

Let us assume that p_j, q_j, p'_j, q'_j are uncertain. We may assume that stochastic variables $\xi_{p_j}, \xi_{q_j}, \xi_{p'_j}, \xi_{q'_j}$ correctly model each parameter. We assume that each stochastic variable is uniformly distributed in an interval $[\xi_m, \xi_M]$. If a stochastic interval is defined for each variable, as $[m_j - 0.1, m_j + 0.1]$ ($j = 1, 2, 3$), then the following linear equations system may be defined:

$$\begin{aligned} p_1, p'_1 &\in [0.090, 0.290], \\ p_2, p'_2 &\in [0.040, 0.240], \\ p_3, p'_3 &\in [0.000, 0.200], \\ q_1, q'_1 &\in [0.045, 0.145], \\ q_2, q'_2 &\in [0.020, 0.120], \\ q_3, q'_3 &\in [0.000, 0.100], \\ p_1 + p'_1 &\leq 0.19, \\ p_2 + p'_2 &\leq 0.14, \\ p_3 + p'_3 &\leq 0.10. \end{aligned}$$

A set of 3000 random values is generated for each parameter. Consequently, $Maxst = 3000$. The number of iterations is defined as $Maxit = 30$ because preliminary simulations showed that this is enough to reach stable centroids. The proposed approach is implemented in Python.

4.2. Results

In Table 1, four groups of alternatives are presented, the same defined in the HDI ranking. Thus, for instance, countries ranging from 1 (Norway) up to 62 (Seychelles) belong to the group of very high developed countries. Clusters built by the proposed approach are coded as C_1, C_2, C_3, C_4 and satisfy $b_1 \prec b_2 \prec b_3 \prec b_4$.

Given a row in a tabular structure, values indicate how frequently an alternative (country) is assigned into a cluster, regarding the $Maxst$ simulations. For instance, the alternative 1 in the group VERY HIGH DEVELOPMENT, 99% of times is assigned into C_4 , whilst 1%, 0%, and 0% of times it is assigned into C_3, C_2, C_1 , respectively. Colors help to recognize these frequencies: the more intense a color, the higher the frequency an alternative reaches in a cluster.

As observed in the VERY HIGH DEVELOPMENT group, the alternatives 1 up to 34, 39 and 42 reach high frequencies in C_4 . The alternatives 35 up to 62 have some weak arguments to be allocated in C_4 or C_3 (except 39 and 42, as mentioned). Note that there are nine alternatives for which the arguments to be assigned into C_4 are strong, equalling 1.0 (i.e., 100% of times these alternatives are assigned into this category).

Table 1: Ordered clustering vs HDI development groups

VERY HIGH DEVELOPMENT					HIGH DEVELOPMENT					MEDIUM DEVELOPMENT					LOW DEVELOPMENT				
Country	C ₁	C ₂	C ₃	C ₄	Country	C ₁	C ₂	C ₃	C ₄	Country	C ₁	C ₂	C ₃	C ₄	Country	C ₁	C ₂	C ₃	C ₄
1	0.00	0.00	0.01	0.99	63	0.00	0.03	0.89	0.08	117	0.00	0.21	0.78	0.01	154	0.53	0.40	0.08	0.00
2	0.00	0.00	0.00	1.00	64	0.00	0.17	0.81	0.02	118	0.07	0.28	0.63	0.03	155	0.47	0.52	0.01	0.00
3	0.00	0.00	0.02	0.98	65	0.01	0.11	0.82	0.06	119	0.02	0.31	0.65	0.02	156	0.36	0.56	0.08	0.00
4	0.00	0.00	0.00	1.00	66	0.01	0.16	0.76	0.06	120	0.16	0.62	0.22	0.00	157	0.32	0.57	0.11	0.00
5	0.00	0.00	0.06	0.94	67	0.00	0.20	0.76	0.04	121	0.29	0.19	0.50	0.02	158	1.00	0.00	0.00	0.00
6	0.00	0.00	0.00	1.00	68	0.03	0.21	0.27	0.48	122	0.00	0.38	0.60	0.02	159	0.80	0.20	0.00	0.00
7	0.00	0.00	0.01	0.99	69	0.01	0.10	0.56	0.33	123	0.10	0.65	0.25	0.00	160	0.36	0.64	0.00	0.00
8	0.00	0.00	0.02	0.98	70	0.00	0.00	0.74	0.26	124	0.15	0.39	0.45	0.00	161	0.68	0.32	0.00	0.00
9	0.00	0.02	0.13	0.86	71	0.00	0.07	0.88	0.05	125	0.02	0.67	0.30	0.00	162	0.34	0.59	0.07	0.00
10	0.00	0.00	0.04	0.96	72	0.00	0.01	0.46	0.53	126	0.23	0.42	0.35	0.00	163	0.35	0.65	0.00	0.00
11	0.00	0.00	0.01	0.99	73	0.04	0.27	0.67	0.02	127	0.26	0.47	0.27	0.00	164	1.00	0.00	0.00	0.00
12	0.00	0.00	0.02	0.98	74	0.02	0.26	0.68	0.03	128	0.15	0.26	0.57	0.01	165	0.85	0.15	0.00	0.00
13	0.00	0.00	0.00	1.00	75	0.01	0.18	0.77	0.04	129	0.19	0.51	0.30	0.00	166	0.61	0.39	0.00	0.00
14	0.00	0.00	0.00	1.00	76	0.04	0.22	0.70	0.05	130	0.18	0.69	0.13	0.00	167	0.33	0.67	0.00	0.00
15	0.00	0.00	0.00	1.00	77	0.09	0.23	0.62	0.06	131	0.31	0.49	0.20	0.00	168	0.83	0.17	0.00	0.00
16	0.00	0.00	0.00	1.00	78	0.03	0.26	0.64	0.08	132	0.29	0.53	0.18	0.00	169	0.52	0.48	0.00	0.00
17	0.00	0.01	0.09	0.90	79	0.08	0.23	0.59	0.10	133	0.12	0.74	0.14	0.00	170	0.47	0.53	0.00	0.00
18	0.00	0.00	0.16	0.84	80	0.05	0.22	0.67	0.06	134	0.33	0.54	0.13	0.00	171	0.86	0.14	0.00	0.00
19	0.00	0.00	0.09	0.91	81	0.00	0.06	0.92	0.02	135	0.26	0.47	0.27	0.00	172	0.38	0.56	0.06	0.00
20	0.00	0.00	0.01	0.99	82	0.08	0.22	0.64	0.06	136	0.15	0.73	0.12	0.00	173	0.71	0.29	0.00	0.00
21	0.00	0.00	0.37	0.63	83	0.01	0.21	0.75	0.03	137	0.18	0.47	0.34	0.01	174	0.84	0.16	0.00	0.00
22	0.00	0.00	0.00	1.00	84	0.02	0.20	0.74	0.04	138	0.29	0.61	0.11	0.00	175	0.86	0.14	0.00	0.00
23	0.00	0.00	0.04	0.96	85	0.08	0.25	0.63	0.04	139	0.44	0.56	0.00	0.00	176	0.52	0.48	0.00	0.00
24	0.00	0.00	0.03	0.97	86	0.02	0.19	0.72	0.06	140	0.33	0.59	0.09	0.00	177	0.66	0.34	0.00	0.00
25	0.01	0.13	0.25	0.61	87	0.00	0.28	0.71	0.01	141	0.18	0.58	0.23	0.00	178	0.70	0.30	0.00	0.00
26	0.00	0.00	0.00	1.00	88	0.00	0.08	0.88	0.05	142	0.23	0.67	0.10	0.00	179	0.82	0.18	0.00	0.00
27	0.00	0.02	0.26	0.72	89	0.08	0.23	0.65	0.03	143	0.20	0.69	0.11	0.00	180	0.85	0.15	0.00	0.00
28	0.00	0.03	0.15	0.82	90	0.04	0.25	0.67	0.04	144	0.84	0.16	0.00	0.00	181	1.00	0.00	0.00	0.00
29	0.00	0.09	0.26	0.65	91	0.13	0.22	0.59	0.06	145	0.39	0.56	0.05	0.00	182	1.00	0.00	0.00	0.00
30	0.00	0.00	0.01	0.99	92	0.00	0.31	0.67	0.02	146	0.31	0.52	0.17	0.00	183	1.00	0.00	0.00	0.00
31	0.00	0.00	0.27	0.73	93	0.10	0.38	0.51	0.01	147	0.28	0.61	0.11	0.00	184	1.00	0.00	0.00	0.00
32	0.00	0.07	0.23	0.70	94	0.02	0.53	0.45	0.01	148	0.31	0.47	0.22	0.00	185	0.48	0.52	0.00	0.00
33	0.00	0.00	0.09	0.91	95	0.04	0.41	0.54	0.02	149	0.41	0.59	0.00	0.00	186	1.00	0.00	0.00	0.00
34	0.00	0.00	0.07	0.93	96	0.01	0.22	0.75	0.02	150	0.34	0.66	0.00	0.00	187	1.00	0.00	0.00	0.00
35	0.00	0.11	0.85	0.05	97	0.00	0.41	0.57	0.01	151	0.56	0.44	0.00	0.00	188	1.00	0.00	0.00	0.00
36	0.00	0.19	0.73	0.08	98	0.08	0.27	0.60	0.04	152	0.67	0.33	0.00	0.00	189	1.00	0.00	0.00	0.00
37	0.01	0.14	0.36	0.49	99	0.00	0.21	0.79	0.00	153	0.33	0.53	0.14	0.00					
38	0.00	0.00	0.89	0.11	100	0.05	0.31	0.64	0.01										
39	0.00	0.00	0.14	0.86	101	0.02	0.49	0.47	0.01										
40	0.02	0.18	0.26	0.55	102	0.02	0.31	0.66	0.00										
41	0.03	0.27	0.67	0.04	103	0.01	0.22	0.75	0.02										
42	0.00	0.07	0.24	0.69	104	0.16	0.27	0.56	0.02										
43	0.02	0.18	0.74	0.05	105	0.00	0.16	0.81	0.04										
44	0.00	0.00	0.90	0.09	106	0.02	0.51	0.46	0.01										
45	0.02	0.16	0.72	0.10	107	0.01	0.59	0.41	0.00										
46	0.00	0.02	0.66	0.32	108	0.06	0.81	0.12	0.00										
47	0.01	0.13	0.79	0.07	109	0.00	0.54	0.46	0.00										
48	0.00	0.07	0.38	0.55	110	0.10	0.34	0.54	0.01										
49	0.00	0.01	0.93	0.06	111	0.08	0.47	0.44	0.02										
50	0.00	0.00	0.84	0.16	112	0.00	0.24	0.75	0.01										
51	0.00	0.01	0.89	0.10	113	0.02	0.70	0.29	0.00										
52	0.00	0.01	0.90	0.09	114	0.02	0.40	0.55	0.02										
53	0.00	0.02	0.89	0.09	115	0.10	0.64	0.26	0.00										
54	0.00	0.05	0.89	0.06	116	0.12	0.43	0.44	0.02										
55	0.00	0.00	0.62	0.38															
56	0.00	0.07	0.50	0.43															
57	0.12	0.24	0.60	0.03															
58	0.03	0.21	0.28	0.47															
59	0.09	0.23	0.25	0.43															
60	0.00	0.11	0.87	0.02															
61	0.00	0.18	0.78	0.04															
62	0.01	0.14	0.71	0.14															

The group named HIGH DEVELOPMENT has an important number of countries that could be ambiguously assigned into C_4 , C_3 or C_2 . However, four groups of alternatives can be observed: 3 countries (68, 69 and 72), having arguments to be allocated into C_4 or C_3 ; about 30% of countries having arguments to be allocated into C_3 or C_2 ; about 60% of countries having arguments to be allocated into C_3 ; 3 countries (108, 113 and 115) allocated into C_2 . Thus, this group is mostly representative of C_3 .

The group named MEDIUM DEVELOPMENT has three groups of alternatives: those with arguments to be assigned into C_3 or C_2 ; the alternatives having arguments to be allocated into C_2 or C_1 ; countries allocated into C_1 . However, this group is mostly representative of C_2 .

The group named LOW DEVELOPMENT shows countries with arguments to be assigned into C_2 or C_1 , revealing a clear shift to the lower part of clusters. Note that ten alternatives have strong arguments to be assigned into C_1 , i.e. scoring 1.0 in that category. This group is mostly representative of C_1 .

Other approaches for clustering based on the outranking relation have not relieved in the uncertain nature of parameters, but only in the imprecise information available to evaluate countries, thus using deterministic thresholds [De Smet et al., 2012; De Smet, 2014; Fernandez et al., 2010; Chen et al., 2018]. However, there is evidence showing that imprecision parameters could be uncertain, which depends on multiple factors, changing on time. Wolff et al. [2011] found that the higher the development status of a country, the more precise are the reported data. This seems to encompass our results: countries in the upper part of very developed countries are strongly assigned to C_4 . These authors also found that when many countries are close to the group cut-off thresholds (quartiles), up to 45% of the developing countries are misclassified. Table 1 shows this phenomenon. For instance, 40% countries in the lower part of the VERY HIGH DEVELOPMENT group are classified in C_3 .

5. Conclusions

In this paper, a non-compensatory and stochastic multi-criteria clustering approach is applied to group countries included in the Human Development Index Report. Results obtained with this approach are compared to the deterministic 2018 HDI ranking. Findings show that uncertainty shapes results, but it depends on the variable in which randomness is considered. Our approach is based on the outranking relation, which uses two preferential parameters: the indifference and the preference thresholds, and the weights of criteria. We have found that uncertainty in thresholds leads to more ambiguity in outcomes than uncertainty in weights.

A limitation of our approach is the usage of K -means like algorithm because results are dependent on the initial set of cluster centers. To mitigate this drawback, we have used some heuristics to find the number and initial centroids. Anyway, future research is needed to consider this feature in contexts of stochastic variables.

References

- Almeida, J., Figueira, J., e Roy, B. (2008). Electre Tri-C: A multiple criteria sorting method based on central reference actions. CAHIER DU LAMSADE 274, Université Paris Dauphine.
- Almeida-Dias, J., Figueira, J., e Roy, B. (2012). A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-nC method. European Journal of Operational Research, 217(3):567–579.

- Ban, A., Brândas, A., Coroianu, L., Negrutiu, C., e Nica, O. (2011). Approximations of fuzzy numbers by trapezoidal fuzzy numbers preserving the ambiguity and value. *Computers and Mathematics with Applications*, 61:1379–1401.
- Berenger, V. e Chouchane, A. V. (2007). Multidimensional measures of well-being: Standard of living and quality of life across countries. *World Development*, 35(7):1259–1276.
- Boujelben, M. e De Smet, Y. (2016). A multicriteria ordered clustering algorithm to determine precise or disjunctive partitions. *International Journal of Multicriteria Decision Making*, 6(2):157–187.
- Brans, J. e Vincke, P. (1985). A preference ranking organisation method: (the PROMETHEE method for multiple criteria decision-making). *Management Science*, 31(6):647–656.
- Chen, L., Xu, Z., Wang, H., e Liu, S. (2018). An ordered clustering algorithm based on K-means and the PROMETHEE method. *International Journal of Machine Learning and Cybernetics*, 9(6):917–926.
- Cherchye, L., Moesen, W., Rogge, N., e Van Puyenbroeck, T. (2011). Constructing composite indicators with imprecise data: A proposal. *Expert Systems With Applications*, 38:10940–10949.
- De Smet, Y. (2014). An extension of PROMETHEE to divisive hierarchical multicriteria clustering. In *IEEE International Conference on Industrial Engineering and Engineering Management*, volume 10.1109/IEEM.2014.7058699. IEEE.
- De Smet, Y. e Eppe, S. (2009). Multicriteria relational clustering: The case of binary outranking matrices. In et al., M. E., editor, *EMO 2009, LNCS 5467*.
- De Smet, Y., Nemery, P., e Selvaraj, R. (2012). An exact algorithm for the multicriteria ordered clustering problem. *Omega*, 40:861–869.
- Fernandez, E., Navarro, J., e Bernal, S. (2010). Handling multicriteria preferences in cluster analysis. *European Journal of Operational Research*, 202:819–827.
- Figueira, J., Greco, S., Roy, B., e Slowinski, R. (2010). *Handbook of Multicriteria Analysis*, chapter ELECTRE Methods: Main Features and Recent Developments, p. 51–89. Number Vol. 103. Springer.
- Ketchen, D. e Shook, C. (1996). The application of cluster analysis in strategic management research: An analysis and critique. *Strategic Management Journal*, 17(6):441–458.
- Klugman, J., Rodríguez, F., e Choi, H. J. (2011). The HDI 2010: New controversies, old critiques. *The Journal of Economic Inequality*, 9(2):249–288.
- Lolli, F., Ishizaka, A., e Gamberini, R. (2014). New ahp-based approaches for multi- criteria inventory classification. *International Journal of Production Economics*, 156:62–74.
- Lozano, S. e Gutiérrez, E. (2009). Human development index: A non-compensatory assessment. *Cuadernos de Economía*, 28(50):223–235.

- Martinez, E. (2013). Social and economic wellbeing in europe and the mediterranean basin: Building an enlarged human development indicator. *Social Indicators Research*, 111: 527–547.
- Mazziotta, M. e Pareto, A. (2015). Comparing two non-compensatory composite indices to measure changes over time: a case study. *Statistika*, 95(2):44–53.
- Monteiro, R., Pereira, V., e Costa, H. (2018). A multicriteria approach to the Human Development Index classification. *Social Indicators Research*, 136:417–438.
- Natoli, R. e Zuhair, S. (2011). Rediscovering the Condorcet approach as an aggregation technique for progress measures. *Economic Papers*, 30(3):368–376.
- Noorbakhsh, F. (1998). The Human Development Index: some technical issues and alternative indices. *Journal of International Development*, 10:589–605.
- Panapakidis, I. e Christoforidis, G. (2018). Optimal selection of clustering algorithm via Multi-Criteria Decision Analysis (MCDA) for load profiling applications. *Applied Sciences*, 8(237):doi:10.3390/app8020237.
- Pereira, J., de Oliveira, E., e Gomes, L. F. A. M. (2018). Sorting retail locations in a large urban city by using ELECTRE TRI-C and trapezoidal fuzzy numbers. *Soft Computing*, <https://doi.org/10.1007/s00500-018-3068-2>.
- Perny, P. e Roy, B. (1992). The use of fuzzy outranking relations in preference modelling. *Fuzzy Sets and System*, (49):33–53.
- Roy, B., Figueira, J., e Almeida, J. (2012). Discriminating thresholds as a tool to cope with imperfect knowledge in multiple criteria decision aiding: Theoretical results and practical issues. *CAHIER DU LAMSADE 2012*, Université Paris Dauphine.
- UNDP (2019). HUMAN DEVELOPMENT REPORT 2019 Beyond income, beyond averages, beyond today: Inequalities in human development in the 21st century. Technical report, UNDP.
- Wolff, H., Chong, H., e Auffhammer, M. (2011). Classification, detection and consequences of data error: Evidence from the Human Development Index. *The Economic Journal*, 121(553):843–870.
- Zheng, B. e Zheng, C. (2015). Fuzzy ranking of human development: A proposal. *Mathematical Social Sciences*, 78:39–47.