

A One-Commodity Flow Formulation for the Risk-Constrained Cash-in-Transit Vehicle Routing Problem

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RESUMO

Neste artigo, consideramos o Problema de Roteamento de Veículos de Transporte de Valores com Restrição de Risco (RCTVRP). Neste problema, cada cliente é visitado por um único veículo, onde é recolhida uma determinada quantia em valores. O problema considera ainda o conceito de risco total de uma rota. Ao se deslocar entre dois pontos, este risco é acrescido da distancia entre os dois pontos ponderada pela quantia total carregada pelo veículo no momento. O objetivo é então definir o roteamento que minimiza a distância total percorrida com a restrição que o risco total de cada rota não deve ultrapassar um limite máximo $\mathcal{T}$. Para o RCTVRP, propomos uma formulação baseada em fluxo, que é comparada a única formulação exata encontrada na literatura. Os experimentos computacionais mostram que a formulação proposta é mais rápida, sendo capaz de encontrar a solução ótima de diversas instâncias pela primeira vez.

PALAVRAS CHAVE. Roteamento de veículos, Transporte de valores, Formulação de fluxo.

Tópicos (L&T – Logística e Transportes, PM – Programação Matemática, OC – Otimização Combinatória)

ABSTRACT

In this paper, we consider the Risk-constrained Cash-in-Transit Vehicle Routing Problem (RCTVRP). In RCTVRP, each customer is visited by a vehicle that picks up an amount of cash. The problem also considers the concept of total risk of a route. When moving between two points, that risk is increased by the distance between the points multiplied by the total value carried by the vehicle. The objective of the problem is to minimize the total distance traveled with the constraint that the total risk of each route is limited by $\mathcal{T}$. We propose a one-commodity flow formulation for the problem and compares it to the only formulation found in the literature. The computational experiments showed that our method was faster, finding several optimal solutions by the first time.

KEYWORDS. Vehicle routing. Cash-in-transit. One-commodity flow formulation.

Paper topics (L&T – Logistics and Transportation, PM – Mathematical Programming, OC – Combinatorial Optimization)

1. Introduction

Over the last decades, Vehicle Routing Problems (VRPs) have been widely studied by a large number of researchers around the world. The great interest in these problems is because they can model several real life situations, especially those involving physical distribution. The first VRP was defined in [Dantzig e Ramser, 1959] and consists of designing optimal delivery routes from one depot to a given number of customers. Since then, a large number of variants has been proposed considering different conditions such that capacities, time windows, heterogeneous fleets, multiple depots, pickups and deliveries, etc.

The cash-in-transit (CIT) industry deals with the transportation of cash or valuable goods from one location to another. The locations include banks, automated teller machines, jewellers, and any location holding large amounts of cash. For CIT industry, an important issue is the security during the transportation, since the companies from this industry are exposed to risks such as robberies and armed assaults.

In the context of the CIT industry, a variant of the VRP has been addressed by [Talarico et al., 2015] called Risk-Constrained Cash-in-Transit Vehicle Problem (RCTVRP). The RCTVRP consists of designing a set of routes from a depot to a given number of customers minimizing the total traveled cost. Each customer is visited only once, and at each visit, the vehicle picks up an amount of cash. The problem assumes that the vehicles do not have limited capacities. However, single routes solutions are not allowed due to the risk constraints. In order to reduce the risk of robberies, the RCTVRP considers the risk of the cash being stolen during the transportation, which is proportional both to the time/distance travelled by the vehicle carrying the cash and the amount of cash being carried, is limited by a predefined constant called risk threshold.

[Talarico et al., 2015] also proposed four heuristics and six local search operators that were combined using two different metaheuristic structures: a multistart heuristic and a perturb-and-improve structure. Besides, the authors proposed a mixed integer linear programming formulation (MILP) and provided two set of instances (set R and V) with up to 200 customers and different risk levels for the problem. Later, [Talarico et al., 2017] presented a metaheuristic that combines ant colony heuristic and a variable neighborhood descent within a large neighborhood search framework. They also developed new benchmark instances (sets O and S) with up to 301 customers. Recently, [Radojičić et al., 2018] applied a fuzzy GRASP hybridized with path relinking methodology for solving the RCTVRP, which turned out to be superior to the algorithms presented in [Talarico et al., 2015] and [Talarico et al., 2017].

To the best of our knowledge, the only exact method for the RCTVRP is the MILP formulation proposed by [Talarico et al., 2015]. This formulation uses binary variables with three indexes and finds optimal solutions for instances with up to 20 customers spending more than 40 hours to solve the hardest instance.

In this paper, we propose for the RCTVRP a new integer programming model, which consists of a one-commodity flow formulation. Computational experiments have been conducted on four benchmark instance sets and revealed that the developed formulation was capable of outperforming the existing model both in terms of number of optimal solutions found and quality of the lower bounds.

The rest of the paper is organized as follows. Section 2 formally describes the problem considered in this study. Section 3 introduces the proposed formulation and we also provide the MILP developed by Talarico et al. [2015]. Section 4 reports and discusses the computational results. Finally, Section 5 contains the concluding remarks.

2. Problem description

The RCTVRP is defined on a directed graph $G = (V, A)$ where $V = \{0, \dots, n\}$ is the vertex set and A is the set of arcs. The vertex 0 corresponds to depot, whereas the set $V^+ = V \setminus \{0\}$ corresponds to the set of customers. A travel cost c_a is associated with each arc $a = (i, j) \in A$. For each customer $i \in V^+$, d_i denotes the amount of cash that should be collected by the vehicle during its visit; $d_0 = 0$. All vehicle starts empty from the depot with a risk index equal to zero. Each vehicle performs a single route, visiting a sequence of customers before returning to the depot where the collected cash is deposited. The risk index increases along the route and can be calculated by the product of the total amount of cash carried by the vehicle, which travels from node i to j , when it leaves i and the travel cost $c_a, a = (i, j)$. The global risk on each route cannot exceed the risk threshold $\mathcal{T}$. The goal of the RCTVRP consists of designing routes starting and ending at the depot where each customer must be visited exactly once. The objective is to minimize the total traveled cost along the selected routes.

The example in Figure 1(a) is given to explain the calculation of the global route risk. The values on the nodes denote the demand associated to each customer, while those on arcs represent their travel cost. Values of R_g and R_i correspond, respectively, to the global route risk and the risk when the vehicle arrives at node i . The index risk at node 2 is zero because the vehicle travels empty along the arc $(0,2)$; the risk index at node 3 is equal to $R_3 = R_2 + (d_0 + d_2)c_{23} = 15$; at node 1 is $R_1 = R_3 + (d_0 + d_2 + d_3)c_{31} = 25$; finally, the global route risk when the vehicles returns to depot is $R_g = R_1 + (d_0 + d_2 + d_3 + d_1)c_{10} = 46$. Route is feasible if $R_g \leq \mathcal{T}$. A single giant route, which visits all customers, is avoided by the risk constraints and not by vehicle capacities. Also, note that the global risk of a route is generally not the same as the global route risk of the reversed one (see Figure 1(b)).

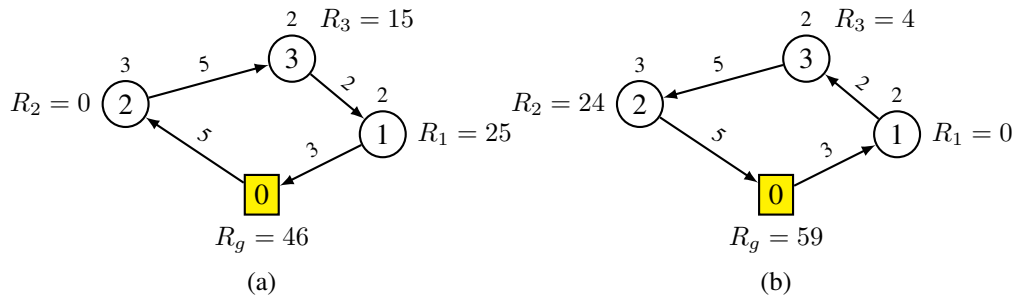


Figure 1: Risk calculation example

3. Mathematical formulations for the RCTVRP

This section presents the mathematical formulations addressed to the RCTVRP. For convenience's sake, we start by describing the MILP introduced by [Talarico et al., 2015], and afterward, we present the proposed formulation.

3.1. Literature MILP formulation

Let s and t represent depot at the start and the end of a route, respectively. Thus, the formulation is defined on the directed graph $G' = (V', A')$, where $V' = V^+ \cup \{s, t\}$ and $A' = \{(V^+ \times V^+) \cup (\{s\} \times V^+) \cup (V^+ \times \{t\})\}$. In the worst case, each route contains only one customer; hence the number of routes cannot be larger than n . Thus, the indexes of routes can be defined on the routes set $N = \{1, \dots, n\}$. Two families of variables are defined from each node: R_i^r stands

for the risk index for the vehicle when it arrives at node $i \in V'$ along the route $r \in N$, while D_i^r represents the amount of cash carried by the vehicle when it leaves node $i \in V'$ along route $r \in N$.

The binary variable x_{ij}^r is equal to 1 if the $(i, j) \in A'$ is traversed by the vehicle along the route $r \in N$ and 0, otherwise. From now on, we refer that formulation as *LF*. The LF follows.

$$\min \sum_{r \in N} \sum_{(i,j) \in A'} c_{ij} x_{ij}^r \quad (1a)$$

Subject to:

$$\sum_{j \in V^+} x_{sj}^r = \sum_{i \in V^+} x_{is}^r \quad \forall r \in N \quad (1b)$$

$$\sum_{j \in V^+} x_{sj}^1 = 1 \quad (1c)$$

$$\sum_{i \in V^+} x_{it}^r \geq \sum_{j \in V^+} x_{sj}^{r+1} \quad \forall r \in N \setminus \{n\} \quad (1d)$$

$$\sum_{r \in N} \sum_{j \in V' \setminus \{s\}} x_{ij}^r = 1 \quad \forall i \in V^+ \quad (1e)$$

$$\sum_{h \in V' \setminus \{t\}} x_{hj}^r - \sum_{k \in V' \setminus \{s\}} x_{jk}^r = 0 \quad \forall j \in V^+, \forall r \in N \quad (1f)$$

$$D_s^r = 0 \quad \forall r \in N \quad (1g)$$

$$D_j^r \geq D_i^r + d_j - (1 - x_{ij}^r)M_1 \quad \forall (i, j) \in A', \forall r \in N \quad (1h)$$

$$0 \leq D_i^r \leq M_1 \quad \forall i \in V', \forall r \in N \quad (1i)$$

$$R_s^r = 0 \quad \forall r \in N \quad (1j)$$

$$R_j^r \geq R_i^r + c_{ij}D_i^r - (1 - x_{ij}^r)M_2 \quad \forall (i, j) \in A', \forall r \in N \quad (1k)$$

$$0 \leq R_i^r \leq \mathcal{T} \quad \forall i \in V', \forall r \in N \quad (1l)$$

$$x_{ij}^r \in \{0, 1\} \quad \forall (i, j) \in A', \forall r \in N \quad (1m)$$

The objective function (1a) aims to minimize the total travel cost. Constraints (1b) ensure that each route starts and ends at the depot. Constraint (1c) imposes that the first route must exist and starts at the depot. Constraints (1d) ensure that the route $r + 1$ cannot exist unless the route r also exists (routes are numbered consecutively). Constraints (1e) indicate that each customer must be visited exactly once. Due to the Constraints (1f) in each route r , the vehicle leaves the customer j if and only if it has previously entered it. The Constraints (1g)-(1l) states the proper values for the cumulative variables R_i^r and D_i^r . Subroutes are prevented due to the increasing values of those variables along the route. Note that Constraints (1h) and (1k) use the sufficiently large numbers M_1 and M_2 . [Talarico et al., 2015] suggest to set $M_1 = \sum_{i \in V'} d_i$ and $M_2 = \mathcal{T}$. However, that constant $M_2 = \mathcal{T}$ is not large enough, as it may prohibit some feasible solutions. Indeed, on the assumption of $x_{ij}^r = 0$, the right hand side of (1k) can be nonnegative and leads to falsely not satisfying Constraints (1l). To avoid this, we suggest replacing M_2 by $M_{ij} = \mathcal{T} + M_1 c_{ij}, \forall (i, j) \in A'$.

3.2. One-commodity flow formulation

Now we present a MILP formulation for the RCTVRP. Our formulation is based on classical one-commodity formulation for CVRP [Gavish e Graves, 1981; Gouveia, 1995].

The OCF makes uses of the same graph defined in section 2. For each vertex $i \in V$, $\delta^-(i)$ and $\delta^+(i)$ denote the arcs entering and leaving i , respectively. Let q_a be total demand carried by the vehicle when it crosses the arc $a \in A$. Let t_a be equal to the risk index before traveling the arc $a \in A$. The binary variable x_a is equal to 1 if the arc $a \in A$ is covered by some route and 0, otherwise. The proposed OCF is as follows.

$$\min \sum_{a \in A} c_a x_a \quad (2a)$$

Subject to:

$$\sum_{a \in \delta^-(i)} x_a = 1 \quad \forall i \in V^+ \quad (2b)$$

$$\sum_{a \in \delta^+(i)} x_a = 1 \quad \forall i \in V^+ \quad (2c)$$

$$\sum_{a \in \delta^-(i)} q_a + d_i = \sum_{a \in \delta^+(i)} q_a \quad \forall i \in V^+ \quad (2d)$$

$$d_i x_a \leq q_a \leq \sum_{k \in V} d_k - d_j x_a \quad \forall a = (i, j) \in A \quad (2e)$$

$$\sum_{a \in \delta^-(i)} t_a + \sum_{a \in \delta^-(i)} c_a q_a = \sum_{a \in \delta^+(i)} t_a \quad \forall i \in V^+ \quad (2f)$$

$$t_a \leq \mathcal{T} - \left(\sum_{b \in \delta^+(j)} q_b \right) c_{j0} - q_a c_a \quad \forall a = (i, j) \in A \setminus \{(i, 0), (0, j)\} \quad (2g)$$

$$t_a \leq (\mathcal{T} - d_i c_a - (d_i + d_j) c_{j0}) x_a \quad \forall a = (i, j) \in A \setminus \{(i, 0), (0, j)\} \quad (2h)$$

$$t_{j0} + c_{j0} q_{j0} \leq \mathcal{T} x_{j0}, \quad \forall j \in V^+ \quad (2i)$$

$$t_{0j} = 0 \quad \forall j \in V^+ \quad (2j)$$

$$x_a \in \{0, 1\} \quad \forall a \in A \quad (2k)$$

$$q_a \geq 0 \quad \forall a \in A \quad (2l)$$

$$t_a \geq 0 \quad \forall a \in A \quad (2m)$$

The objective function (2a) aims to minimize the total travel cost. Degree Constraints (2b) and (2c) ensure that each customer is visited exactly once. Constraints (2d) and (2e) set the values for the variables q_a , where (2d) ensure that q_i units of cash are collected at customer i and (2d) provides the bounds for the cumulative demand along the route. Constraints (2f)–(2i) define the risk index values of the variables t_a . The Constraints (2g) and (2h) define the risk index upper bounds of each arc along the route and also ensure that the route must end at the depot. Those constraints assume that the costs c_a satisfy the triangular inequality and the adaptation for the general case would be simple: c_{j0} would have to be replaced by the cost of the shortest path from j to 0, which would no longer necessarily be the direct path. Constraints (2h) impose that the global route risk is limited by $\mathcal{T}$. Moreover, Constraints (2g) set the risk at the start of each route equal to zero. Finally, Constraints (2k)–(2m) define the domain of the variables. The increasing values of the risk index variables along the route prevent subroutes.

4. Computational Results

The proposed model was coded with JuMP [Dunning et al., 2017] for mathematical optimization in Julia 1.2. All experiments were executed in single thread of a Intel Core i7-3770 machine with a 3.40GHz CPU and 16GB of RAM running Linux Ubuntu 16.04 operating system. CPLEX 12.9 was used as MILP solver. To evaluate the performance of our model, we had reimplemented the LF formulation [Talarico et al., 2015] under the same computational environment.

Computational experiment was performed on four benchmark instance sets. The sets R and V suggested by [Talarico et al., 2015] contains 180 e 70 instances, respectively. The former was randomly generated with up to 20 customers and different risk levels, whereas the latter was generated based on 14 instances from the VRP library varying between 22 and 301 customers. The sets S and O consists of 68 instances (34 each) proposed by [Talarico et al., 2017] including between 10 and 337 customers.

Table 1 the number of instances and the minimum, average and maximum number of nodes of each instance set.

Table 1: Summary of the instance sets dimension

Instance set	#Inst	#Nodes		
		min	avg.	max
R	180	4	12.0	20
V	70	22	110.5	301
S	34	10	96.1	337
O	34	10	96.1	337

Table 2 summarizes the results obtained by the formulations. A time limit of 1 hour was imposed for each instance. For each benchmark set, the number of instances #Inst is given. Columns 3-4 indicate the number of optimal solutions found. The number of feasible solutions found, but non-optimal, are given in columns 5-6, whereas columns 7-8 show the average gaps (%) found by CPLEX. Finally, we also show the average CPU time, in seconds, spent by each model. The last row reports the overall results.

Table 2: Summary of results for the R, V, O and S instances

Instance set	#Inst	# Opt		# Feasible		Avg gap (%)		Avg time (s)	
		OCF	LF	OCF	LF	OCF	LF	OCF	LF
R	180	174	80	0	94	0.00	30.36	32.10	1983.06
V	70	10	0	31	16	11.68	66.55	2816.89	3600.00
O	34	14	2	11	8	5.76	40.24	1806.98	3162.53
S	34	9	1	13	10	8.12	51.68	2263.78	3276.91
Total	318	207	83	55	128				

The results obtained suggest that OCF outperforms, on average, the MILP formulation by [Talarico et al., 2015] over all the benchmark sets. The smaller average gaps illustrate that OCF managed to find better lower bounds for those instances where the optimal solution could not be determined. Moreover, in all sets, OCF was capable of finding the optimal solution of at least nine instances and a total of 207 optimal solutions. The model by [Talarico et al., 2015], on the other hand, could only solve 83 instances with up to 10 customers. The literature formulation also was dominated when evaluating the CPU time required by the solver.

The developed method was able to prove the optimality of 174 instances from set R (6 out of 180 are infeasible). However, it could not prove the optimality of at most instances from the sets V, S, and O. Despite that, it managed to find better lower bounds for those instances with an average gap of 3.06%. Detailed result Tables A.1-A.3 are provided in the Appendix.

5. Conclusions

In this work we proposed a exact formulation for the Risk-constrained Cash-in-Transit Vehicle Routing Problem (RCTVRP). The model consists of a one-commodity flow formulation (OCF). Computational experiments were carried out on 318 instances, divided into four benchmark sets, ranging between 4 and 337 customers. The results obtained by OCF were compared with the MILP from [Talarico et al., 2015]. The comparison shows that the proposed model outperforms the existing MILP and it was capable of solving 207 instances to optimality, whereas the latter could only prove the optimality of the 83 instances involving up to 10 customers. OCF also provided, on average, better results in terms of CPU time required to solve those instances to optimality and lower bounds for the unsolved instances.

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Appendix A. Detailed results tables

Complete results found by the formulations on all instances are shown in Tables A.1, A.2, and A.3. The instance name is presented in the first column. Moreover, the objective function value, the relative gap, computational time and the number of routes are given in columns Obj, Gap(%), t(s), and k , respectively. The symbol “–” indicates that the corresponding model failed to found a feasible solution and “INF” refers to infeasible instances.

Table A.1: Computational results on the set R instances

Table A.1: Computational results on the set R instances																	
Set R																	
OCF								LF									
Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)
4.1.1.0	343.05	0.00	3	<0.01	343.05	0.00	3	<0.01	12.5.1.0	INF.	–	–	–	INF.	–	–	–
4.1.1.5	343.05	0.00	3	<0.01	343.05	0.00	3	<0.01	12.5.1.5	995.93	0.00	6	0.11	995.93	59.83	6	3600
4.1.2.0	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	12.5.2.0	724.19	0.00	5	0.23	724.19	52.15	5	3600
4.1.2.5	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	12.5.2.5	696.98	0.00	4	2.85	696.98	50.78	4	3600
4.1.3.0	286.13	0.00	2	0.01	286.13	0.00	2	0.01	12.5.3.0	595.28	0.00	4	2.69	595.28	39.06	4	3600
4.3.1.0	343.05	0.00	3	<0.01	343.05	0.00	3	<0.01	12.7.1.0	INF.	–	–	–	INF.	–	–	–
4.3.1.5	343.05	0.00	3	<0.01	343.05	0.00	3	<0.01	12.7.1.5	911.37	0.00	6	0.22	911.37	48.75	6	3600
4.3.2.0	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	12.7.2.0	717.18	0.00	5	0.91	717.18	48.86	5	3600
4.3.2.5	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	12.7.2.5	656.27	0.00	4	6.55	656.27	47.99	4	3600
4.3.3.0	286.13	0.00	2	<0.01	286.13	0.00	2	0.01	12.7.3.0	563.44	0.00	3	2.41	570.28	39.19	3	3600
4.5.1.0	343.05	0.00	3	<0.01	343.05	0.00	3	<0.01	14.1.1.0	1026.92	0.00	9	0.13	1026.92	64.05	9	3600
4.5.1.5	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	14.1.1.5	836.74	0.00	6	0.83	836.74	64.79	6	3600
4.5.2.0	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	14.1.2.0	756.68	0.00	5	1.03	757.54	54.54	5	3600
4.5.2.5	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	14.1.2.5	549.4	0.00	4	1.63	562.67	44.21	4	3600
4.5.3.0	286.13	0.00	2	0.01	286.13	0.00	2	0.01	14.1.3.0	543.23	0.00	4	22.09	543.23	41.16	4	3600
4.7.1.0	343.05	0.00	3	<0.01	343.05	0.00	3	0.01	14.3.1.0	1026.92	0.00	9	0.08	1026.92	66.63	9	3600
4.7.1.5	332.93	0.00	2	<0.01	332.93	0.00	2	0.01	14.3.1.5	836.74	0.00	6	0.68	866.51	65.98	7	3600
4.7.2.0	286.13	0.00	2	<0.01	286.13	0.00	2	0.01	14.3.2.0	756.68	0.00	5	1.14	756.68	60	5	3600
4.7.2.5	286.13	0.00	2	0.01	286.13	0.00	2	0.02	14.3.2.5	549.4	0.00	4	1.54	549.4	49.11	4	3600
4.7.3.0	286.13	0.00	2	0.02	286.13	0.00	2	0.02	14.3.3.0	543.23	0.00	4	21.28	543.23	44.42	4	3600
6.1.1.0	594.64	0.00	4	<0.01	594.64	0.00	4	0.33	14.5.1.0	951.66	0.00	8	0.12	951.66	54.38	8	3600
6.1.1.5	594.34	0.00	4	<0.01	594.34	0.00	4	0.41	14.5.1.5	758.81	0.00	5	0.84	758.81	63.11	5	3600
6.1.2.0	438.51	0.00	3	0.05	438.51	0.00	3	0.34	14.5.2.0	549.4	0.00	4	0.87	591.74	44.64	5	3600
6.1.2.5	387.99	0.00	2	0.06	387.99	0.00	2	0.3	14.5.2.5	541.5	0.00	4	30.67	541.5	46.91	4	3600
6.1.3.0	387.68	0.00	2	0.08	387.68	0.00	2	0.3	14.5.3.0	482.56	0.00	3	7.09	518.84	41.54	3	3600
6.3.1.0	645.17	0.00	5	<0.01	645.17	0.00	5	0.48	14.7.1.0	812.17	0.00	6	4.04	812.17	59.64	6	3600

Continued on next page

Set R																	
OCF									LF								
Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)
6.3.1.5	594.34	0.00	4	<0.01	594.34	0.00	4	0.44	14.7.1.5	623.2	0.00	4	16.16	655.05	57.46	5	3600
6.3.2.0	445.44	0.00	3	0.03	445.44	0.00	3	0.41	14.7.2.0	512.57	0.00	3	34.45	512.57	38.46	3	3600
6.3.2.5	387.99	0.00	2	0.04	387.99	0.00	2	0.3	14.7.2.5	488.47	0.00	3	52.79	516.39	45.23	4	3600
6.3.3.0	387.68	0.00	2	0.09	387.68	0.00	2	0.32	14.7.3.0	427.96	0.00	2	36.95	449.61	34.76	3	3600
6.5.1.0	594.64	0.00	4	<0.01	594.64	0.00	4	0.44	16.1.1.0	1067.62	0.00	10	0.1	1067.62	65.91	10	3600
6.5.1.5	594.34	0.00	4	<0.01	594.34	0.00	4	0.52	16.1.1.5	790.19	0.00	6	0.81	809.61	62.59	7	3600
6.5.2.0	387.99	0.00	2	0.02	387.99	0.00	2	0.35	16.1.2.0	651.2	0.00	5	12.41	662.35	54.1	5	3600
6.5.2.5	387.99	0.00	2	0.06	387.99	0.00	2	0.32	16.1.2.5	565.46	0.00	4	7.61	611.33	49.43	6	3600
6.5.3.0	387.68	0.00	2	0.07	387.68	0.00	2	0.35	16.1.3.0	550.35	0.00	4	111.48	564.53	47.31	4	3600
6.7.1.0	534.96	0.00	3	0.01	534.96	0.00	3	0.41	16.3.1.0	1029.39	0.00	9	0.02	1029.39	66.08	9	3600
6.7.1.5	445.44	0.00	3	0.04	445.44	0.00	3	0.39	16.3.1.5	790.19	0.00	6	1.3	794.83	62	7	3600
6.7.2.0	362.88	0.00	2	0.06	362.88	0.00	2	0.27	16.3.2.0	650.56	0.00	5	10.73	660.94	54.34	5	3600
6.7.2.5	360.68	0.00	2	0.06	360.68	0.00	2	0.31	16.3.2.5	565.46	0.00	4	16.07	585	49.32	4	3600
6.7.3.0	358.63	0.00	2	0.07	358.63	0.00	2	0.27	16.3.3.0	550.35	0.00	4	76.78	588.28	47.71	5	3600
8.1.1.0	541.56	0.00	5	0.02	541.56	0.00	5	6.45	16.5.1.0	1014.01	0.00	9	0.04	1014.01	64.82	9	3600
8.1.1.5	487.53	0.00	4	0.03	487.53	0.00	4	4.57	16.5.1.5	853.95	0.00	7	4.37	855.89	62.6	7	3600
8.1.2.0	431.01	0.00	3	0.06	431.01	0.00	3	3.28	16.5.2.0	661.21	0.00	5	2.17	668.09	54.72	6	3600
8.1.2.5	416.9	0.00	3	0.13	416.9	0.00	3	4.68	16.5.2.5	593.59	0.00	5	18.36	618.29	50.37	6	3600
8.1.3.0	379.64	0.00	2	0.14	379.64	0.00	2	4.55	16.5.3.0	559.76	0.00	4	52.86	585.65	46.87	5	3600
8.3.1.0	658.32	0.00	6	<0.01	658.32	0.00	6	5.97	16.7.1.0	884.7	0.00	7	0.76	884.7	64.09	7	3600
8.3.1.5	487.53	0.00	4	0.01	487.53	0.00	4	4.52	16.7.1.5	655.12	0.00	6	4.98	706.68	55.41	8	3600
8.3.2.0	431.01	0.00	3	0.04	431.01	0.00	3	3.04	16.7.2.0	584.1	0.00	4	30.7	589.73	47.43	5	3600
8.3.2.5	416.9	0.00	3	0.16	416.9	0.00	3	4.2	16.7.2.5	520.38	0.00	4	71.7	540.7	42.87	4	3600
8.3.3.0	384.23	0.00	3	0.16	384.23	0.00	3	4.98	16.7.3.0	476.75	0.00	4	32.48	479.22	35.84	4	3600
8.5.1.0	574.23	0.00	5	0.01	574.23	0.00	5	3.85	18.1.1.0	INF.	-	-	-	INF.	-	-	-
8.5.1.5	487.53	0.00	4	0.02	487.53	0.00	4	3.84	18.1.1.5	1058.47	0.00	7	12.18	1102.66	67.91	10	3600
8.5.2.0	431.01	0.00	3	0.06	431.01	0.00	3	3.55	18.1.2.0	819.26	0.00	5	29.98	845.18	59.02	7	3600
8.5.2.5	416.9	0.00	3	0.13	416.9	0.00	3	4.46	18.1.2.5	734.73	0.00	4	306.77	755.09	52.89	4	3600
8.5.3.0	379.64	0.00	2	0.15	379.64	0.00	2	2.83	18.1.3.0	680	0.00	4	362.98	748.81	54.31	5	3600
8.7.1.0	485.04	0.00	4	0.01	485.04	0.00	4	3.04	18.3.1.0	INF.	-	-	-	INF.	-	-	-
8.7.1.5	416.9	0.00	3	0.05	416.9	0.00	3	3.25	18.3.1.5	1026.37	0.00	6	9.47	1085.83	68.49	9	3600
8.7.2.0	384.23	0.00	3	0.12	384.23	0.00	3	1.21	18.3.2.0	819.26	0.00	5	20.92	919.69	60.11	5	3600
8.7.2.5	327.71	0.00	2	0.03	327.71	0.00	2	0.87	18.3.2.5	730.55	0.00	4	213.41	751.73	54.83	6	3600
8.7.3.0	327.71	0.00	2	0.11	327.71	0.00	2	0.71	18.3.3.0	680	0.00	4	456.98	750.11	53.43	6	3600
10.1.1.0	608.9	0.00	7	0.01	608.9	0.00	7	561.14	18.5.1.0	INF.	-	-	-	INF.	-	-	-
10.1.1.5	540.57	0.00	6	0.06	540.57	0.00	6	526.16	18.5.1.5	1059.65	0.00	7	9.1	1177.41	66.16	11	3600
10.1.2.0	466.03	0.00	4	0.17	466.03	0.00	4	252.62	18.5.2.0	816.79	0.00	5	20.84	914.81	61.14	7	3600
10.1.2.5	422.8	0.00	4	0.15	422.8	0.00	4	193.03	18.5.2.5	752.46	0.00	4	97.62	798.8	54.2	5	3600
10.1.3.0	417.33	0.00	3	0.74	417.33	0.00	3	552.62	18.5.3.0	619.77	0.00	4	21.86	642.61	45.37	6	3600
10.3.1.0	608.9	0.00	7	0.01	608.9	0.00	7	668.73	18.1.1.0	INF.	-	-	-	INF.	-	-	-
10.3.1.5	534.25	0.00	5	0.05	534.25	0.00	5	391.72	18.7.1.5	923.73	0.00	6	24.97	971.74	64.31	7	3600
10.3.2.0	466.03	0.00	4	0.24	466.03	0.00	4	191.9	18.7.2.0	782.82	0.00	4	47.32	826.97	58.04	7	3600
10.3.2.5	422.8	0.00	4	0.16	422.8	0.00	4	154.09	18.7.2.5	703.04	0.00	4	2170.42	807.25	56.91	9	3600
10.3.3.0	417.33	0.00	3	0.73	417.33	0.00	3	429.68	18.7.3.0	590.82	0.00	3	39.87	807.25	57.44	9	3600
10.5.1.0	608.9	0.00	7	<0.01	608.9	0.00	7	515.55	20.1.1.0	1254.04	0.00	13	0.03	1267.37	70.74	14	3600
10.5.1.5	481.99	0.00	5	0.03	481.99	0.00	5	143	20.1.1.5	1070.76	0.00	10	0.82	1073.92	68.49	10	3600
10.5.2.0	428.77	0.00	4	0.08	428.77	0.00	4	134.04	20.1.2.0	787.19	0.00	7	1.7	848.36	64.03	10	3600
10.5.2.5	421.57	0.00	4	0.21	421.57	0.00	4	276.57	20.1.2.5	741.46	0.00	6	65.61	828.81	60.72	10	3600
10.5.3.0	412.43	0.00	3	0.93	412.43	0.00	3	283.64	20.1.3.0	693.6	0.00	6	320.59	802.25	59.97	10	3600
10.7.1.0	576.47	0.00	6	0.02	576.47	0.00	6	679.17	20.3.1.0	1255.4	0.00	13	0.05	1265.89	73.68	14	3600
10.7.1.5	468.5	0.00	4	0.05	468.5	0.00	4	145.1	20.3.1.5	1070.76	0.00	10	0.87	1084.08	69.68	10	3600
10.7.2.0	424.08	0.00	4	0.16	424.08	0.00	4	104.69	20.3.2.0	785.37	0.00	7	3.67	907.47	64.45	10	3600
10.7.2.5	406.83	0.00	3	0.21	406.83	0.00	3	178.01	20.3.2.5	725.39	0.00	6	57.31	770.29	57.93	7	3600
10.7.3.0	406.83	0.00	3	0.75	406.83	0.00	3	190.17	20.3.3.0	618.39	0.00	5	17.75	767.62	58.65	10	3600
12.1.1.0	1347.19	0.00	10	<0.01	1347.19	47.44	10	3600	20.5.1.0	1252.49	0.00	13	0.04	1252.49	71.98	13	3600
12.1.1.5	1123.46	0.00	7	0.01	1123.46	62.41	7	3600	20.5.1.5	1070.76	0.00	10	0.53	1092.74	69.12	11	3600
12.1.2.0	840.46	0.00	5	0.61	840.46	57.42	5	3600	20.5.2.0	809.34	0.00	7	4.51	891.42	64.52	9	3600
12.1.2.5	714.43	0.00	4	2.25	721.84	46.88	4	3600	20.5.2.5	709.53	0.00	6	57.65	711.86	54.81	6	3600
12.1.3.0	594.74	0.00	4	2.18	594.74	42.98	4	3600	20.5.3.0	655.54	0.00	5	192.17	722.06	57.35	7	3600
12.3.1.0	1345.07	0.00	10	<0.01	1345.07	48.83	10	3600	20.7.1.0	1091.65	0.00	12	0.26	1091.65	68.64	12	3600
12.3.1.5	1129.26	0.00	7	0.01	1129.26	63.9	7	3600	20.7.1.5	871.25	0.00	8	9.82	936.51	66.15	11	3600
12.3.2.0	839.27	0.00	5	0.57	840.46	52.53	5	3600	20.7.2.0	722.98	0.00	6	23.17	856.82	62.86	9	3600

Continued on next page

Set R																	
OCF					LF				OCF					LF			
Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)
12_3.2.5	714.43	0.00	4	1.42	714.43	52.7	4	3600	20_7.2.5	668.37	0.00	6	189.84	731.97	54.34	8	3600
12_3.3.0	602.45	0.00	4	2.48	602.45	39.39	4	3600	20_7.3.0	583.89	0.00	5	71.05	681.78	52.06	8	3600

Table A.2: Computational results on set V instances

Set V																		
OCF					LF				OCF					LF				
Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	
22_1.0	782.5	0.00	10	12	954.65	74	17	3600	101_2.0	2214.69	47.00	27	3600	—	—	—	—	
26_1.0	872.44	0.00	13	5.9	—	—	—	—	121_2.0	—	—	—	—	—	—	—	—	
30_1.0	656.03	10.00	5	3600	1273.47	77.15	16	3600	135_2.0	—	—	—	—	—	—	—	—	
36_1.0	953.57	0.00	16	25.8	—	—	—	—	151_2.0	—	—	—	—	—	—	—	—	
45_1.0	907.11	10.00	6	3600	—	—	—	—	200_2.0	—	—	—	—	—	—	—	—	
51_1.0	1442	1.00	23	3600	—	—	—	—	256_2.0	—	—	—	—	—	—	—	—	
72_1.0	461.58	37.00	10	3600	—	—	—	—	301_2.0	—	—	—	—	—	—	—	—	
101_1.0	2274.16	12.00	33	3600	—	—	—	—	22_2.5	442.45	0.00	5	734.54	540.53	56.99	9	3600	
121_1.0	—	—	—	—	—	—	—	—	26_2.5	536.38	6.00	6	3600	699.37	60.96	13	3600	
135_1.0	—	—	—	—	—	—	—	—	30_2.5	477.66	11.00	3	3600	1187.52	75.56	16	3600	
151_1.0	—	—	—	—	—	—	—	—	36_2.5	589.34	5.00	8	3600	—	—	—	—	
200_1.0	—	—	—	—	—	—	—	—	45_2.5	734.95	12.00	4	3600	—	—	—	—	
256_1.0	83956.13	0.00	255	7.15	—	—	—	—	51_2.5	811.13	11.00	11	3600	—	—	—	—	
301_1.0	—	—	—	—	—	—	—	—	72_2.5	275.33	22.00	5	3600	—	—	—	—	
22_1.5	578.19	0.00	7	206.99	667.09	64.34	12	3600	101_2.5	1538.54	33.00	21	3600	—	—	—	—	
26_1.5	680.85	0.00	9	230.67	904.19	72.38	17	3600	121_2.5	—	—	—	—	—	—	—	—	
30_1.5	514.34	6.00	4	3600	1236.25	76.53	16	3600	135_2.5	—	—	—	—	—	—	—	—	
36_1.5	749.84	3.00	12	3600	—	—	—	—	151_2.5	—	—	—	—	—	—	—	—	
45_1.5	782.05	12.00	5	3600	—	—	—	—	200_2.5	—	—	—	—	—	—	—	—	
51_1.5	1098.85	7.00	16	3600	—	—	—	—	256_2.5	—	—	—	—	—	—	—	—	
72_1.5	321.37	24.00	6	3600	—	—	—	—	301_2.5	—	—	—	—	—	—	—	—	
101_1.5	1847.67	22.00	25	3600	—	—	—	—	22_3.0	401.68	0.00	5	353.94	544.56	54.32	9	3600	
121_1.5	—	—	—	—	—	—	—	—	26_3.0	498.58	6.00	6	3600	637.41	59.64	11	3600	
135_1.5	—	—	—	—	—	—	—	—	30_3.0	477.66	12.00	3	3600	738.56	60.68	10	3600	
151_1.5	—	—	—	—	—	—	—	—	36_3.0	576.18	11.00	8	3600	1171.45	72.01	28	3600	
200_1.5	—	—	—	—	—	—	—	—	45_3.0	732.57	13.00	4	3600	—	—	—	—	
256_1.5	—	—	—	—	—	—	—	—	51_3.0	759.34	12.00	10	3600	—	—	—	—	
301_1.5	—	—	—	—	—	—	—	—	72_3.0	348	40.00	6	3600	—	—	—	—	
22_2.0	496.68	0.00	6	2029.21	516.28	54.75	7	3600	101_3.0	1302.92	27.00	16	3600	—	—	—	—	
26_2.0	566.46	0.00	7	285.91	735.09	65.21	12	3600	121_3.0	—	—	—	—	—	—	—	—	
30_2.0	489.92	6.00	3	3600	1003.81	70.39	13	3600	135_3.0	—	—	—	—	—	—	—	—	
36_2.0	664.41	6.00	10	3600	1097.08	69.83	26	3600	151_3.0	—	—	—	—	—	—	—	—	
45_2.0	735.57	10.00	4	3600	—	—	—	—	200_3.0	—	—	—	—	—	—	—	—	
51_2.0	906.71	10.00	13	3600	—	—	—	—	256_3.0	—	—	—	—	—	—	—	—	
72_2.0	341.73	34.00	6	3600	—	—	—	—	301_3.0	—	—	—	—	—	—	—	—	

Table A.3: Computational results on the instances from sets O and S

Set O									Set S								
OCF					LF				OCF					LF			
Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)	Inst.	Obj	Gap (%)	k	t(s)	Obj	Gap (%)	k	t(s)
O10	26.83	0.00	3	1.13	26.83	0.00	3	33.19	S10	19.98	0.00	3	0.13	19.98	0.00	3	45.82
O13	35.78	0.00	4	0.44	35.78	16.4	4	3600	S13	26.65	0.00	4	0.65	26.65	15.27	4	3600
O16	38.83	0.00	3	0.33	38.83	0.00	3	2792.02	S16	35.41	0.00	3	1.07	35.41	28.65	3	3600
O19	52.44	0.00	6	632.15	53.56	40.83	6	3600	S19	38.18	0.00	6	17.68	43.66	41.77	8	3600
O21	51.78	0.00	4	1.4	61.8	33.47	6	3600	S21	47.22	0.00	4	19.69	60.49	48.23	7	3600
O25	64.88	5.68	7	3600	79.2	56.56	11	3600	S25	49.72	4.18	7	3600	62.73	52.45	12	3600
O28	62.83	0.00	3	2.24	105.4	45.08	11	3600	S28	61.8	0.00	3	26.9	114.47	67.83	11	3600
O31	77.66	2.10	6	3600	116.9	60.43	15	3600	S31	70.82	7.22	6	3600	95.4	54.31	12	3600
O37	83.78	0.00	4	27.34	277.89	74	27	3600	S37	82.39	0.00	4	1510.62	297.35	83.89	28	3600
O40	86.83	0.00	3	10.42	335.28	75.58	25	3600	S40	98.59	0.00	3	137.91	398.91	83.72	24	3600
O41	101.98	8.50	8	3600	—	—	—	—	S41	89.79	10.59	8	3600	—	—	—	—
O49	101.29	4.58	12	3600	—	—	—	—	S49	81.62	9.98	13	3600	—	—	—	—
O52	110.83	0.00	3	91.36	—	—	—	—	S52	118.71	0.00	3	1288.25	879.67	92.33	43	3600
O53	115.78	0.00	4	110.73	—	—	—	—	S53	123.11	4.02	4	3600	—	—	—	—
O55	125.66	1.93	6	3600	—	—	—	—	S55	134.02	9.54	6	3600	—	—	—	—
O64	134.83	0.00	3	215.36	—	—	—	—	S64	151.87	8.35	3	3600	—	—	—	—
O69	147.78	0.00	4	609.22	—	—	—	—	S69	166.12	5.70	4	3600	—	—	—	—
O73	167.55	4.62	8	3600	—	—	—	—	S73	164.22	14.73	8	3600	—	—	—	—
O79	173.66	1.81	6	3600	—	—	—	—	S79	197.18	10.97	6	3600	—	—	—	—
O81	194.16	23.88	16	3600	—	—	—	—	S81	142.07	16.32	14	3600	—	—	—	—
O85	179.78	0.00	4	2488.89	—	—	—	—	S85	258.26	28.75	5	3600	—	—	—	—
O94	194.83	0.00	3	1383.39	—	—	—	—	S94	368.67	48.32	6	3600	—	—	—	—
O103	221.66	2.21	6	3600	—	—	—	—	S103	—	—	—	—	—	—	—	—
O105	1587.06	86.12	104	3600	—	—	—	—	S105	—	—	—	—	—	—	—	—
O125	—	—	—	—	—	—	—	—	S125	—	—	—	—	—	—	—	—
O127	269.67	2.58	6	3600	—	—	—	—	S127	—	—	—	—	—	—	—	—
O137	—	—	—	—	—	—	—	—	S137	—	—	—	—	—	—	—	—
O145	—	—	—	—	—	—	—	—	S145	—	—	—	—	—	—	—	—
O169	—	—	—	—	—	—	—	—	S169	—	—	—	—	—	—	—	—
O187	—	—	—	—	—	—	—	—	S187	—	—	—	—	—	—	—	—
O209	—	—	—	—	—	—	—	—	S209	—	—	—	—	—	—	—	—
O249	—	—	—	—	—	—	—	—	S249	—	—	—	—	—	—	—	—
O273	—	—	—	—	—	—	—	—	S273	—	—	—	—	—	—	—	—
O337	—	—	—	—	—	—	—	—	S337	—	—	—	—	—	—	—	—

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