

Benders decomposition approach for the multicrew scheduling and routing problem in road restoration

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ABSTRACT

This paper introduces a Benders decomposition approach to solve the multicrew scheduling and routing problem in road restoration. The problem consists of finding the schedule and route of heterogeneous crews that perform the restoration of damaged nodes in a network affected by extreme events. Computational experiments are conducted using benchmark instances to compare the performance of the proposed Benders decomposition and a compact formulation of the problem. The results indicate that the new Benders decomposition approach performs better than the compact MIP model.

KEYWORDS. Decomposition method. Network repair. Humanitarian logistics.

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1. Introduction

After extreme events such as natural disasters, some affected areas may require commodities, evacuation of people, rescue and medical assistance. However, some components of the transportation infrastructure such as roads, bridges, and tunnels can be damaged or blocked interrupting the distribution and/or evacuation operations and making it impossible the fast rescue and medical assistance in the affected areas (demand nodes). Therefore, a critical subset of the damaged components (damaged nodes) along the network must be repaired to reach the demand areas and effectively perform the response operations. In this context, the heterogeneous multi-crew road restoration problem (MCSRP) consists of determining the paths that must be used to quickly reach the demand nodes from a central depot, the allocation of the crews to repair the damaged nodes, the sequence in which the damaged nodes must be repaired by the crews, and the route that must be used by each crew to repair the damaged nodes.

The crews differ one from another in the time required to restore the damaged nodes, in the travel time, and in the set of damaged nodes that each crew can restore. However, the consideration of multiple heterogeneous crews in the problem has been neglected in the literature due mainly to the complexity involved in such consideration. In fact, the single crew scheduling and routing problem (SCSRP) is already challenging due to the scheduling and routing decisions that must be integrated [Maya-Duque et al., 2016; Moreno et al., 2019]. The SCSRP have been recently addressed by Maya-Duque et al. [2016] and Moreno et al. [2019, 2020b]. Maya-Duque et al. [2016] proposed a non-linear formulation for the SCSRP, and developed a dynamic programming algorithm and a GRASP-based metaheuristic to solve the problem. Moreno et al. [2019] linearized the model proposed by Maya-Duque et al. [2016] and obtained results for some small instances. For larger instances, the authors developed a branch-and-Benders-cut (BBC) algorithm that explores a Benders decomposition of the problem. Moreno et al. [2020b] proposed a hybrid BBC approach to solve the SCSRP. They hybridized the BBC with a simulated annealing meta-heuristic. Although the authors obtained satisfactory results, their solution methods are not directly applied to the MCSRP.

Some variants of the problem have been addressed in the literature considering multiple crews. Akbari e Salman [2017b] and Akbari e Salman [2017a] introduced the synchronization of homogeneous crews to recover the connectivity of a network affected by extreme events, but without integrate the definition of paths to connect the depot with the demand nodes. The objective was to minimize the completion time of the restoration. The same problem was addressed by Akbari e Salman [2017a] with a different objective function consisting of maximizing the network components connected to the depot node. Morshedlou et al. [2018] also considered multiples homogeneous crews and focused in the restoration of gas, water, and electric power infrastructure networks, assuming that no disruption affects the transportation network. They did not define paths between the depot and the demand nodes. To the best of our knowledge, there is a lack of studies considering heterogeneous crews in road restoration problems. Furthermore, there is not exact solution method developed to solve the MCSRP. In this study, we contribute to the literature by proposing a Benders decomposition approach to solve the MCSRP with heterogeneous crews.

The remainder of the paper is organized as follows. In Section 2, we provide the description of the MCSRP. Section 3 presents a compact formulation for the MCSRP, while section 4 presents the Benders decomposition approach. Section 5 discusses the computational results. We close with some concluding remarks and potential future research directions in Section 6.

2. Problem description

The *Multi-Crew Scheduling and Routing Problem* (MCSRP) can be defined on an undirected graph $G = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of undirected arcs. The graph

G represents a damaged network affected by extreme events. The subset $\mathcal{V}^d \subset \mathcal{V}$ characterizes the demand nodes while the subset $\mathcal{V}^r \subset \mathcal{V}$ contains the collection of damaged nodes. Furthermore, there may be transshipment nodes, which represent the intersection of two or more arcs. There is one depot (node 0) that is a supply node to be connected with the demand nodes and from which the crews initially depart to repair the damaged nodes. Figure 1(a) shows an example of a graph G representing a damaged network.

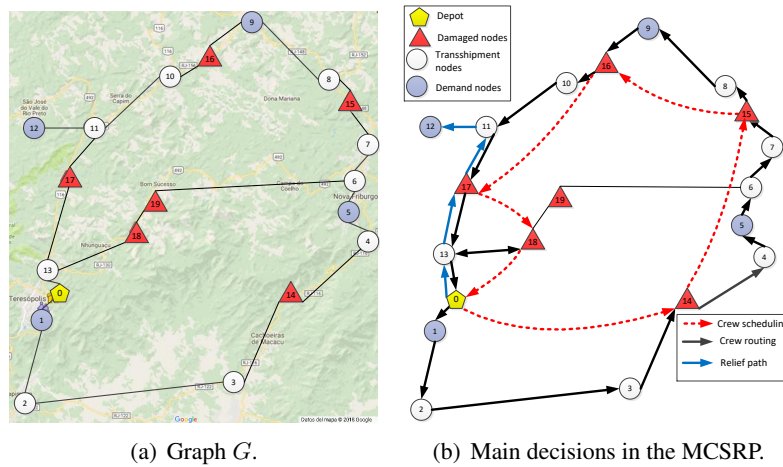


Figura 1: Example of a graph G and main decisions in the MCSRP

Multiple heterogeneous crews are available to perform the restoration activities. The crews are initially located in the central depot. Basically, the MCSRP consists of determining (i) the paths to connect the depot with the demand nodes (relief path decisions), (ii) the damaged nodes that must be repaired by the crews (allocation decisions), (iii) the schedule of the crews to repair the damaged nodes (scheduling decisions), and (iv) the route of the crews to repair the damaged nodes and return to the depot (routing decisions). Figure 1(b) shows an example of the main decisions considered in the MCSRP, highlighting the schedule of a crew (red dashed lines), the route of a crew (black arrows), and a relief path from the depot to demand node 12 (blue arrows).

The relief path decisions correspond to determine the sequence of nodes and arcs used to connect the depot with the demand nodes. The allocation decisions determine the damaged nodes that need to be repaired as well as the crews that must repair them. The scheduling decisions define the sequence in which the crews will repair the damaged nodes. It is assumed that more than one crew cannot simultaneously repair a damaged node. Thus, if a crew arrives at a damaged node while another crew is performing the restoration, then it has to wait until the damaged node be repaired. The routing decisions determine the routes to be used by the crews to repair the damaged nodes and return to the depot. Nodes and edges can be traversed multiples times by the crews. A damaged node is repaired the first time it is visited by the crew allocated for its restoration.

3. Compact formulation

In this section, we present a compact formulation for the MCSRP from the literature [Moreno et al., 2020a]. The sets, parameters and variables used in the model are formally defined as follows.

Sets

$\mathcal{V}$	All nodes.
$\mathcal{V}^r \subset \mathcal{V}$	Damaged nodes.
$\mathcal{V}_0^r = \mathcal{V}^r \cup \{0\}$	Damaged nodes including the source node 0 (depot).
$\mathcal{V}^u \subset \mathcal{V}$	Undamaged nodes ($\mathcal{V}^u = \mathcal{V} / \mathcal{V}^r$).
$\mathcal{V}^d \subset \mathcal{V}^u$	Demand nodes.
$\mathcal{E}$	Arcs.
$\mathcal{E}_i \subseteq \mathcal{E}$	Arcs incident to node $i \in \mathcal{V}$.
$\mathcal{R} = \{1, \dots, \mathcal{V}^r \}$	Positions at which an already repaired damaged node can be visited in a path between two damaged nodes.
$\mathcal{K}$	Available crews.
$\mathcal{K}_i \subseteq \mathcal{K}$	Crews able to repair the damaged node $i \in \mathcal{V}^r$.

Parameters

d_i	Demand of node $i \in \mathcal{V}^d$.
t_e	Travel time associated to the arcs used in the relief paths.
δ_{ki}	Repair time of crew $k \in \mathcal{K}_i$ at node $i \in \mathcal{V}^r$.
τ_{ke}	Travel time of crew $k \in \mathcal{K}$ on arc $e \in \mathcal{E}$.
ρ_{kij}	Shortest travel time of crew k between nodes $i \in \mathcal{V}$ and $j \in \mathcal{V}$ without using damaged nodes.
ℓ_e	Length of arc $e \in \mathcal{E}$.
l_i^d	Maximum distance allowed between node 0 and demand node $i \in \mathcal{V}^d$.
M	Sufficiently large number.

Decision variables

Q_i	Binary variable equal to 1 if node $i \in \mathcal{V}^r$ is repaired.
W_{ik}	Binary variable equal to 1 if node $i \in \mathcal{V}^r$ is repaired by crew k .
X_{ij}	Binary variable equal to 1 if node $j \in \mathcal{V}_0^r$ is repaired immediately after node $i \in \mathcal{V}_0^r$.
P_{eij}	Binary variable equal to 1 if arc $e \in \mathcal{E}$ is used either in the path from node $i \in \mathcal{V}_0^r$ to node $j \in \mathcal{V}^r$ or in the path from node $j \in \mathcal{V}^r$ to node $i \in \mathcal{V}^r$ with $i < j$.
N_{lij}	Binary variable equal to 1 if node $l \in \mathcal{V}^r$ is used either in the path from node $i \in \mathcal{V}_0^r$ to node $j \in \mathcal{V}^r$ or in the path from node $j \in \mathcal{V}^r$ to node $i \in \mathcal{V}^r$ with $i < j$.
R_{lhj}	Binary variable equal to 1 if node $l \in \mathcal{V}^r$ is the h th damaged node visited in the path to node $j \in \mathcal{V}^r$.
Y_{ej}	Binary variable equal to 1 if arc $e \in \mathcal{E}$ is used in the path from node 0 to node $j \in \mathcal{V}^d$.
V_{lj}	Binary variable equal to 1 if node $l \in \mathcal{V}$ is used in the path from node 0 to node $j \in \mathcal{V}^d$.
T_{lhj}^s	Time at which the damaged node $l \in \mathcal{V}^r$ in the position $h \in \mathcal{R}$ is visited in the path to node $j \in \mathcal{V}^r$ (arrival time).
T_{lhj}^w	Waiting time at the damaged node $l \in \mathcal{V}^r$ visited in the position $h \in \mathcal{R}$ in the path to node $j \in \mathcal{V}^r$.
Z_i^r	Restoration time of damaged node $i \in \mathcal{V}_0^r$.
Z_i^d	Accessibility time of demand node $i \in \mathcal{V}^d$.

The model for the MCSRP is formulated as follows:

$$\begin{aligned} \min \quad & \sum_{i \in \mathcal{V}^d} d_i Z_i^d, \\ \text{s.t.} \quad & \end{aligned} \tag{1}$$

Relief paths constraints (2) – (6),
Restoration time calculation (7) – (12),
Assignment and scheduling constraints (13) – (20),
Routing constraints (21) – (25),
Synchronization constraints (26) – (31),
Domain of the decision variables (32) – (41).

The objective function (1) minimizes the weighted sum of the accessibility time of the demand nodes. The other constraints are defined as follows.

Relief path constraints: Constraints (2) define the accessibility time for the demand nodes. Constraints (3)-(5) define the relief paths between the depot and the demand nodes. For a given relief path $0 - i$, there is one arc incident to node 0 (constraints (3)), one arc incident to node i (constraint (4)) and two arcs incident for each node l in the middle of this path (constraints (5)). Constraints (6) prohibit the use of relief paths whose total distance is greater than the maximum distance allowed.

$$Z_i^d \geq Z_j^r - M(1 - V_{ji}), \forall i \in \mathcal{V}^d, j \in \mathcal{V}^r, \quad (2)$$

$$\sum_{e \in \mathcal{E}_0} Y_{ei} = 1, \forall i \in \mathcal{V}^d, \quad (3)$$

$$\sum_{e \in \mathcal{E}_i} Y_{ei} = 1, \forall i \in \mathcal{V}^d, \quad (4)$$

$$\sum_{e \in \mathcal{E}_l} Y_{ei} = 2V_{li}, \forall j \in \mathcal{V}^d, l \in \mathcal{V} \setminus \{0, i\}, \quad (5)$$

$$\sum_{e \in \mathcal{E}} \ell_e Y_{ei} \leq l_i^d, \forall i \in \mathcal{V}^d. \quad (6)$$

Restoration time calculation: Constraints (7)-(8) define the restoration time when no damaged nodes are visited in crew path $i - j$, or when there is no waiting time associated with the visited damaged nodes. These constraints also prevent subtours. Constraints (7) are defined for $i < j$ while constraints (8) are defined for $i > j$. Similarly, constraints (9) define the restoration time when there is waiting time associated with the damaged nodes visited in a given crew path $i - j$. Constraints (10) evaluate the arrival time of crew k at the first damaged node l visited in crew path $i - j$. Similarly, constraints (11) define the arrival time of crew k at the damaged node l visited in position $h > 1$ based on the departure time of node v visited in position $h - 1$. Finally, constraints (12) compute the waiting time of the crew in damaged node l visited in position h .

$$Z_j^r \geq Z_i^r + \sum_{e \in \mathcal{E}} \tau_{ke} P_{eij} + \delta_{kj} - M(2 - X_{ij} - W_{kj}), \forall k \in \mathcal{K}, i \in \mathcal{V}_0^r, j \in \mathcal{V}^r : i < j, \quad (7)$$

$$Z_j^r \geq Z_i^r + \sum_{e \in \mathcal{E}} \tau_{ke} P_{eji} + \delta_{kj} - M(2 - X_{ij} - W_{kj}), \forall k \in \mathcal{K}, i \in \mathcal{V}^r, j \in \mathcal{V}^r : i > j, \quad (8)$$

$$Z_j^r \geq \sum_{l \in \mathcal{V}^r} (T_{lhj}^w + T_{lhj}^s + \rho_{klj} R_{lhj}) + \delta_{kj} - M(1 - W_{kj} + \sum_{l \in \mathcal{V}^r} R_{(h+1)lj}), \quad (9)$$

$$T_{1lj}^s \geq Z_i^r + \sum_{k \in \mathcal{K}} \rho_{kil} W_{kj} - M(2 - X_{ij} - R_{1lj}), \forall i \in \mathcal{V}_0^r, j \in \mathcal{V}^r, l \in \mathcal{V}^r, \quad (10)$$

$$T_{lhj}^s \geq \sum_{p \in \mathcal{V}^r} (T_{(h-1)pj}^w + T_{(h-1)pj}^s + R_{(h-1)pj} \rho_{kpl}) - M(2 - R_{lhj} - W_{kj}), \quad (11)$$

$$T_{lhj}^w \geq Z_l^r - T_{lhj}^s - M(1 - R_{lhj}), \forall l \in \mathcal{V}^r, j \in \mathcal{V}^r, h \in \mathcal{R}. \quad (12)$$

Assignment and scheduling constraints: Constraints (13) force the allocation of crews to damaged nodes that are repaired. Constraints (14),(15) state that damaged nodes used either in relief paths or in crew paths must be repaired. Constraints (16)-(18) define the schedule for the damaged nodes that must be repaired. Constraints (19) guarantee that if nodes i and j are considered in the same schedule ($X_{ij} = 1$), both are repaired by the same crew. Constraints (20) force the consideration of different crews for the different schedules.

$$Q_l = \sum_{k \in \mathcal{K}_l} W_{kl}, \forall l \in \mathcal{V}^r, \quad (13)$$

$$|\mathcal{V}^d|Q_l \geq \sum_{i \in \mathcal{V}^d} V_{li}, \forall l \in \mathcal{V}^r, \quad (14)$$

$$|\mathcal{V}^r|Q_l \geq \sum_{i \in \mathcal{V}_0^r} \sum_{\substack{j \in \mathcal{V}^r: \\ j > i}} N_{lij}, \forall l \in \mathcal{V}^r, \quad (15)$$

$$\sum_{\substack{i \in \mathcal{V}_0^r: \\ i \neq j}} X_{ij} = Q_j, \forall j \in \mathcal{V}^r, \quad (16)$$

$$\sum_{\substack{i \in \mathcal{V}_0^r: \\ i \neq l}} X_{il} - \sum_{\substack{j \in \mathcal{V}_0^r: \\ j \neq l}} X_{lj} = 0, \forall l \in \mathcal{V}_0^r, \quad (17)$$

$$\sum_{j \in \mathcal{V}^r} X_{0j} \leq |\mathcal{K}|, \quad (18)$$

$$W_{kj} \geq W_{ki} + X_{ij} - 1, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r, k \in \mathcal{K}, \quad (19)$$

$$\sum_{\substack{k' \in \mathcal{K}: \\ k' \neq k}} W_{k'j} \geq W_{ki} + X_{0i} + X_{0j} - 2, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r : i \neq j, k \in \mathcal{K}. \quad (20)$$

Routing constraints: Constraints (21)-(25) define the paths of the crews. For a crew traveling from $i \in \mathcal{V}_0^r$ to $j \in \mathcal{V}^r$, constraints (21),(23) force the use of an arc incident to node i , constraints (22),(24) force the use of an arc incident to node j , and constraints (25) force the use of two arcs incident to nodes l in the middle of the crew path $i - j$.

$$\sum_{e \in \mathcal{E}_0} P_{e0j} = X_{0j}, \forall j \in \mathcal{V}^r, \quad (21)$$

$$\sum_{e \in \mathcal{E}_j} P_{e0j} = X_{0j}, \forall j \in \mathcal{V}^r, \quad (22)$$

$$\sum_{e \in \mathcal{E}_i} P_{eij} = X_{ij} + X_{ji}, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \quad (23)$$

$$\sum_{e \in \mathcal{E}_j} P_{eij} = X_{ij} + X_{ji}, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \quad (24)$$

$$\sum_{e \in \mathcal{E}_l} P_{eij} = 2N_{lij}, \forall i \in \mathcal{V}_0^r, j \in \mathcal{V}^r, l \in \mathcal{V} \setminus \{i, j\} : i < j. \quad (25)$$

Synchronization constraints: Constraints (26)-(31) synchronize the arrival of crew k in damaged node l visited in crew path $i - j$. Here, i and j are damaged nodes repaired by crew k , while l is a damaged node used in path $i - j$. Thus, when crew k arrives at this node l , it either waits for node l to be repaired by another crew if this node is still damaged, or it can cross without waiting if l has been already repaired. A damaged node l cannot be visited more than once in the path to node j (constraints (26)). Also, a given crew cannot visit different damaged nodes simultaneously

(constraints (27)), i.e., in the same position h . Constraints (28) ensure that damaged nodes in path $i - j$ must be visited in consecutive positions. Constraints (29)-(31) link the allocation of damaged nodes to positions with the definition of the crew paths. Thus, only damaged nodes considered in the crew path $i - j$ ($N_{lij} = 1$) can be allocated to some position defined by the variable R_{lhj} .

$$\sum_{h \in \mathcal{R}} R_{lhj} \leq 1, \forall j \in \mathcal{V}^x, l \in \mathcal{V}^x, \quad (26)$$

$$\sum_{l \in \mathcal{V}^x} R_{lhj} \leq 1, \forall j \in \mathcal{V}^x, h \in \mathcal{R}, \quad (27)$$

$$\sum_{l \in \mathcal{V}^x} R_{lhj} \leq \sum_{l \in \mathcal{V}^x} R_{(h-1)lj}, \forall j \in \mathcal{V}^x, h \in \mathcal{R} \setminus \{1\}, \quad (28)$$

$$\sum_{h \in \mathcal{R}} R_{lhj} \geq N_{lij} + X_{ij} - 1, \forall i \in \mathcal{V}^x, l \in \mathcal{V}^x, j \in \mathcal{V}^x : i < j, \quad (29)$$

$$\sum_{h \in \mathcal{R}} R_{lhi} \geq N_{lij} + X_{ji} - 1, \forall i \in \mathcal{V}^x, l \in \mathcal{V}^x, j \in \mathcal{V}^x : i < j, \quad (30)$$

$$\sum_{h \in \mathcal{R}} \sum_{j \in \mathcal{V}^x} R_{lhj} = \sum_{i \in \mathcal{V}_0^x} \sum_{j \in \mathcal{V}^x : j > i} N_{lij}, \forall l \in \mathcal{V}^x. \quad (31)$$

Domain of the decision variables: Constraints (32)-(41) impose the domain of the decision variables.

$$Q_j, W_{kj} \in \{0, 1\}, \forall j \in \mathcal{V}^x, k \in \mathcal{K}, \quad (32)$$

$$X_{ij} \in \{0, 1\}, \forall i \in \mathcal{V}_0^x, j \in \mathcal{V}_0^x, \quad (33)$$

$$R_{hij} \in \{0, 1\}, \forall i \in \mathcal{V}^x, j \in \mathcal{V}^x, h \in \mathcal{R}, \quad (34)$$

$$N_{lij} \in \{0, 1\}, \forall i \in \mathcal{V}_0^x, j \in \mathcal{V}^x, l \in \mathcal{V}, \quad (35)$$

$$P_{eij} \geq 0, \forall i \in \mathcal{V}_0^x, j \in \mathcal{V}^x, e \in \mathcal{E}, \quad (36)$$

$$V_{li} \in \{0, 1\}, \forall i \in \mathcal{V}^d, l \in \mathcal{V}, \quad (37)$$

$$Y_{el} \geq 0, \forall l \in \mathcal{V}^d, e \in \mathcal{E}, \quad (38)$$

$$T_{lhj}^s, T_{lhj}^w \geq 0, \forall l \in \mathcal{V}^x, j \in \mathcal{V}^x, h \in \mathcal{R}, \quad (39)$$

$$Z_i^x \geq 0, \forall i \in \mathcal{V}_0^x, \quad (40)$$

$$Z_j^d \geq 0, \forall j \in \mathcal{V}^d. \quad (41)$$

4. Solution approach

In this section, we present a logic-based Benders decomposition [Hooker e Ottosson, 2003] approach to solve the MCSRP, in which the master problem (MP) involves assignment, scheduling and relief path definition decisions and the subproblem (SP) considers relief path definition decisions and crew routing decisions. Both, MP and SP, include integer variables and therefore standard duality theory cannot be applied to derive cuts. The MP obtained from the Benders decomposition is solved by a single search tree, exploring the generation of cuts inside the tree during the branch-and-bound process. This strategy has been recently referred to as Branch-and-Benders-Cut (BBC) [Gendron et al., 2016] or Branch-and-Check algorithm (B&Ch) [Tran et al., 2016]. Usually, B&Ch is more common in the context of logic-based Benders decomposition approaches.

4.1. Logic-based Benders reformulation (LBBR)

Let θ_j^x be an auxiliary variable that represents the restoration time of damaged node j . Let $\beta_j(\bar{q})$ be a function that returns the optimal restoration time of damaged node $j \in \mathcal{V}^x$ given an assignment-scheduling solution $\bar{q} = (\bar{X}, \bar{W}, \bar{Q})$. Also, let $\mathcal{SP}$ be the set of assignment-scheduling

solutions for which a feasible crew routing decision can be defined. The logic-based Benders reformulation (LBRR) for the RCSRP is as follows:

$$(LBRR) \quad \min \sum_{i \in \mathcal{V}^d} d_i Z_i^d, \quad (42)$$

s.t.

$$\text{Relief paths constraints (3) – (6),} \quad (43)$$

$$\text{Assignment and scheduling constraints (13), (14), (16) – (20),} \quad (44)$$

$$\text{Domain of the decision variables (32), (33), (37), (38), (41),} \quad (45)$$

$$Z_i^d \geq \theta_j^r - M \cdot (1 - V_{ji}), \forall i \in \mathcal{V}^d, j \in \mathcal{V}^r; \quad (46)$$

$$\theta_j^r \geq \beta_j(\mathbf{q}), \forall j \in \mathcal{V}^r; \quad (47)$$

$$\mathbf{q} \in \mathcal{SP}; \quad (48)$$

$$\theta_j^r \geq 0, \forall j \in \mathcal{V}^r. \quad (49)$$

Constraints (46) define the accessibility time for the demand nodes. Constraints (47) updated variable $\theta_{j\Gamma}^r$ according to the optimal restoration time of damaged node j . Constraint (48) prohibits infeasible assignment-scheduling solutions for which it is not possible to find synchronized routes of the crews. Finally, constraints (49) define the domain of $\theta_{j\gamma}^r$ variables.

Given a solution $\bar{\mathbf{q}}$, $\beta_j(\bar{\mathbf{q}}) = \bar{Z}_j^{r*}$, where $\bar{Z}_j^{r*}$ is equal to the value of variable Z_i^r in the optimal solution of the following subproblem (SP).

$$(SP) \quad \min \sum_{i \in \mathcal{V}^d} d_i Z_i^d, \quad (50)$$

s.t.

$$\text{Relief paths constraints (2) – (6),} \quad (51)$$

$$\text{Routing and synchronization constraints (25) – (28),} \quad (52)$$

$$\text{Domain of the decision variables (34) – (41),} \quad (53)$$

$$Z_j^r \geq Z_i^r + \sum_{e \in \mathcal{E}} \tau_{ke} P_{eij} + \delta_{kj}, \forall k \in \mathcal{K}, i \in \mathcal{V}_0^r, j \in \mathcal{V}^r : i < j, \bar{X}_{ij} = 1, \bar{W}_{kj} = 1, \quad (54)$$

$$Z_j^r \geq Z_i^r + \sum_{e \in \mathcal{E}} \tau_{ke} P_{eji} + \delta_{kj}, \forall k \in \mathcal{K}, i \in \mathcal{V}^r, j \in \mathcal{V}^r : i > j, \bar{X}_{ij} = 1, \bar{W}_{kj} = 1, \quad (55)$$

$$Z_j^r \geq \sum_{l \in \mathcal{V}^r} (T_{lhj}^w + T_{lhj}^s + \rho_{klj} R_{lhj}) + \delta_{kj} - M \left(\sum_{l \in \mathcal{V}^r} R_{(h+1)lj} \right), \quad (56)$$

$$T_{1lj}^s \geq Z_i^r + \sum_{k \in \mathcal{K}} \rho_{kil} \bar{W}_{kj} - M(1 - R_{1lj}), \forall i \in \mathcal{V}_0^r, j \in \mathcal{V}^r : \bar{X}_{ij} = 1, l \in \mathcal{V}^r, \quad (57)$$

$$T_{lhj}^s \geq \sum_{p \in \mathcal{V}^r} (T_{(h-1)pj}^w + T_{(h-1)pj}^s + R_{(h-1)pj} \rho_{kpl}) - M(1 - R_{lhj}), \quad (58)$$

$$T_{lhj}^w \geq Z_l^r - T_{lhj}^s - M(1 - R_{lhj}), \forall l \in \mathcal{V}^r : \bar{Q}_l = 1, j \in \mathcal{V}^r : \bar{Q}_j = 1, h \in \mathcal{R}, \quad (59)$$

$$\sum_{e \in \mathcal{E}_0} P_{e0j} = 1, \forall j \in \mathcal{V}^r : \bar{X}_{0j} = 1, \quad (60)$$

$$\sum_{e \in \mathcal{E}_j} P_{e0j} = 1, \forall j \in \mathcal{V}^r : \bar{X}_{0j} = 1, \quad (61)$$

$$\sum_{e \in \mathcal{E}_i} P_{eij} = 1, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \bar{X}_{ij} + \bar{X}_{ji} = 1, \quad (62)$$

$$\sum_{e \in \mathcal{E}_j} P_{eij} = 1, \forall i \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \bar{X}_{ij} + \bar{X}_{ji} = 1, \quad (63)$$

$$\sum_{h \in \mathcal{R}} R_{lhj} \geq N_{lij}, \forall i \in \mathcal{V}^r, l \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \bar{X}_{ij} = 1, \quad (64)$$

$$\sum_{h \in \mathcal{R}} R_{lhi} \geq N_{lij}, \forall i \in \mathcal{V}^r, l \in \mathcal{V}^r, j \in \mathcal{V}^r : i < j, \bar{X}_{ji} = 1, \quad (65)$$

$$\sum_{h \in \mathcal{R}} \sum_{j \in \mathcal{V}^r} R_{lhj} = \sum_{i \in \mathcal{V}_0^r} \sum_{j \in \mathcal{V}^r : j > i} N_{lij}, \forall l \in \mathcal{V}^r : \bar{Q}_l = 1, \quad (66)$$

$$\sum_{j \in \mathcal{V}^d} V_{lj} + \sum_{i \in \mathcal{V}_0^r} \sum_{j \in \mathcal{V}^r} (N_{lij} + N_{jil}) + \sum_{h \in \mathcal{R}} \sum_{j \in \mathcal{V}^r} (R_{lhj} + R_{jhl}) + \sum_{i \in \mathcal{V}_0^r} \sum_{e \in \mathcal{E}} P_{eil} \leq 0, \forall l \in \mathcal{V}^r : \bar{Q}_l = 0, \quad (67)$$

$$\sum_{l \in \mathcal{V}} N_{lij} + \sum_{e \in \mathcal{E}} P_{eij} \leq 0, \forall i \in \mathcal{V}_0^r, j \in \mathcal{V}^r : \bar{X}_{ij} = 0. \quad (68)$$

Constraints (54)-(66) are constraints of the original MCSRP with the fixed values $(\bar{X}, \bar{W}, \bar{Q})$ of the assignment-scheduling variables. These constraints apply for the damaged nodes allocated to some crew, i.e., $\forall i, j : \bar{X}_{ij} = 1, \forall k, j : \bar{W}_{kj} = 1, \forall j : \bar{Q}_j = 1$. We add constraints (67), (68) to prohibit routing decisions between damaged nodes that are not allocated to any crew, i.e., $\forall i, j : \bar{X}_{ij} = 0, \forall k, j : \bar{W}_{kj} = 0, \forall j : \bar{Q}_j = 0$. This way we avoid the generation of unnecessary routing and synchronization constraints (54)-(66) for damaged nodes that are not repaired by the crews.

4.2. Branch-and-check algorithm (B&Ch)

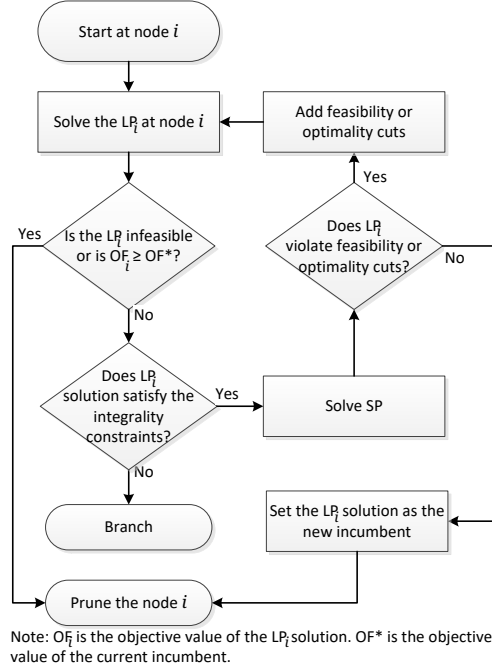
We develop a B&Ch to solve the LBBR model (42)-(49). Basically, we define a master problem (MP) by removing constraints (47), (48) from the original LBBR model and solve the MP by using a branch-and-bound method. The removed constraints are dynamically checked in the nodes of the branch-and-bound tree and optimality/feasibility cuts are added to the MP when these constraints are violated. Figure 2 shows a flowchart of the main steps carried out in B&Ch algorithm at each node of the branch-and-bound tree. At each node i , we solve the linear relaxation of the current MP, denoted by LP_i . If the LP_i is infeasible or the objective value of the LP_i solution (OF_i) is higher than or equal to the objective value of the current incumbent solution, then node i is pruned. Otherwise, integrality constraints are checked, and if the LP_i solution is not integer feasible, then branching is performed. If the LP_i solution is integer feasible, we solve the subproblem SP (50)-(68) to verify violation of constraints (47), (48). A constraint of the set of constraints (47) is violated for a given node j if $\bar{\theta}_j^r < \bar{Z}_i^{r*}$, where $\bar{\theta}_j^r$ is the value of variable θ_j^r in the LP_i solution. Constraint (48) is violated if the subproblem SP is infeasible. If no constraint is violated, then the LP_i solution is feasible for the original LBBR and is set as the new incumbent solution. Otherwise, the MP is modified by the addition of Benders cuts, LP_i is resolved, and the described steps are applied again.

4.3. Logic-based Benders Cuts

Every time an integer solution is found at the nodes of the branch-and-bound tree, the subproblem SP is solved and the corresponding feasibility/optimality Benders cuts are added to the MP. We rely on logic-based Benders cuts based on particular characteristics of the problem. Let $\bar{q} = (\bar{X}, \bar{W}, \bar{Q})$ be an assignment-scheduling solution for the MP. Let

$$\Theta_{\bar{q}}(q) = \sum_{i \in \mathcal{V}_0^r} \sum_{\substack{j \in \mathcal{V}_0^r : \\ \bar{X}_{ij}=1}} (X_{ij}-1) + \sum_{k \in \mathcal{K}} \sum_{\substack{j \in \mathcal{V}^r : \\ \bar{W}_{kj}=1}} (W_{kj}-1) + \sum_{\substack{j \in \mathcal{V}^r : \\ \bar{Q}_j=1}} (Q_j-1) - \sum_{i \in \mathcal{V}_0^r} \sum_{\substack{j \in \mathcal{V}_0^r : \\ \bar{X}_{ij}=0}} X_{ij} - \sum_{k \in \mathcal{K}} \sum_{\substack{j \in \mathcal{V}^r : \\ \bar{W}_{kj}=0}} W_{kj} - \sum_{\substack{j \in \mathcal{V}^r : \\ \bar{Q}_j=0}} Q_j.$$

Figura 2: Flowchart illustrating the main steps of the B&Ch algorithm in a given node i of the branch-and-bound.



If the assignment-scheduling solution $\bar{q}$ violates constraints (48), a valid feasibility cut for the problem is

$$\Theta_{\bar{q}}(q) \leq -1. \quad (69)$$

If the assignment-scheduling solution $\bar{q}$ violates any constraint of the set of constraints (47) for a given node j , a valid optimality cut for the problem is

$$\theta_j^x \geq \beta_j(\bar{q}) + \beta_j(\bar{q}) \cdot \Theta_{\bar{q}}(q). \quad (70)$$

5. Computational results

In this section, we evaluate the efficiency of the solution method in comparison to solve the compact model by a general purpose commercial solver. All the algorithms were coded in C++ programming language and run on a Linux PC with an AMD Opteron 6172 processor with 16.0 GB of RAM and a single thread. Benders cuts are added using the Callback classes available in the Concert Technology Library. The MCSRP model and the MP and SP subproblems were solved by CPLEX Optimization Solver 12.8. The stopping criterion on CPLEX was either the elapsed time exceeding the time limit of 3,600 seconds or the optimality gap being smaller than 10^{-4} . The methods were tested using the CS instance classes for the MCSRP presented in Moreno et al. [2020a]. CS instances are based on the 2011 megadisaster of the Serrana Region in Rio de Janeiro, Brazil.

We adapt the valid inequalities proposed for the compact formulation of the MCSRP [Moreno et al., 2020a] to the LBBR. Also, the simulated annealing (SA) metaheuristic proposed for the SCSRP in Moreno et al. [2020b] is adapted to the MCSRP. The main changes consist on the consideration of new local search operators regarding the multiple crews. Another relevant change is that

instead of using the feasibility and optimality check algorithms to verify the feasibility and cost of the solutions, we resort to the subproblem SP (50)-(68). Table 1 summarizes the solution methods used in the experiments.

Tabela 1: Solution strategies.

Strategy	Description
M-MCSR	MIP model (1)-(41).
B&Ch	LBBR model (42)-(49) solved by the branch-and-check algorithm.
SA	Simulated annealing metaheuristic.
SA-B&Ch	Warm-start the B&Ch using a solution from SA.

Table 2 shows for each approach the total number of instances (#ins), the number of instances for which the approach finished with feasible and optimal solutions (#feas, #opt), each one followed by the corresponding percentage with respect to the total number of instances (%feas, %opt), the average upper and lower bounds, the average relative gap in percentage, and the average elapsed time.

Tabela 2: Comparison of the solution approaches.

Solution method	#ins	#feas	%feas	#opt	%opt	Avg. upper bound	Avg. lower bound	Avg. gap (%)	Avg. time (seconds.)
M-MCSR	228	119	52.19	33	14.47	241,291	178,465	37.40	2,778
B&Ch	228	169	74.12	47	20.61	222,434	190,210	18.04	2,588
SA	228	228	100.00	NA	NA	210,622	NA	NA	1,089
SA-B&Ch	228	228	100.00	86	37.71	212,542	202,221	8.51	2,160

*NA: Not available.

The strategy M-MCSR found optimal solutions for only 14.47% of the cases (33 instances) and failed at finding feasible solutions in 47.81% of the cases (109 instances). The B&Ch algorithm increased the number of optimal solutions to 47, an increment of 42.42%, and was not able to provide a feasible solution in only 25.88% of the cases (59 instances). The SA was able to find feasible solutions for all tested instances. Also, on average, the upper bound obtained by SA was 5.31% smaller than in the B&Ch. Additionally, the SA metaheuristic improves the elapsed time by 57.92%, on average, in relation to B&Ch. Finally, providing an initial solution with SA to warm-start the B&Ch algorithm seems to be a good strategy, as SA-B&Ch reduced the upper bound by 4.44%, in relation to the B&Ch algorithm. Regarding the average elapsed time, SA-B&Ch is faster than B&Ch, but it has a higher average elapsed time than SA. Finally, regarding the gap and lower bound, the improvements were 52.82% and 6.31%, respectively, with respect to B&Ch.

6. Conclusion

This paper explored a Benders decomposition approach for the multicrew scheduling and routing problem in road restoration. As a key contribution, it developed the first exact solution approach based on decomposition for the problem considering multiple heterogeneous crews. The addressed problem is found in post-disaster situations where the damaged network must be repaired as quickly as possible. The integrated scheduling and routing decisions make it difficult the problem, for which commercial solvers cannot be efficiently used. The proposed decomposition method seems to provide good-quality solutions for the tested instances. Enhancement strategies such as the use of metaheuristic to warm-start the Benders decomposition and valid inequalities

were considered. These strategies improved the performance of the proposed approach. Future research should be focused on the study of additional enhancement strategies to solve large instance of the problem.

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