

Using Optimization to Design Public Urban Transportation Networks

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ABSTRACT

In an exceptionally urbanized world, public transport seems to be more important than ever as it contributes for more sustainable, accessible cities. In an attempt to assist both government agencies and private companies on constructing efficient transport networks, we propose a integer linear programming model to design a demand optimal line with respect to a budget constraint. This text explores particularities of street level transport but the model can be easily generalized for other modes. Preliminary results for grid networks of several sizes and densities were reported.

KEYWORDS. Line planning problem, network design, integer linear programming

Logistics and transport, combinatorial optimization, mathematical programming

1. Introduction

The importance of public transportation in the history of humankind is undeniable. Its development has favored cities to attain the level of urbanization they have today; it aids people to commute to work on a daily basis; it has “shortened” distances in a way that could not be thought of 200 years ago. Its relevance nowadays is reassured by the fact that the number of trips carried out on public transportation had an 18% increase between the years 2000 and 2015 [UITP, 2017]. Moreover, public transport is the answer to several problems large cities face in the present day, such as traffic congestion, air pollution and energy consumption [Kepaptsoglou e Karlaftis, 2009]. Therefore, it is important that public transit is adequately financed and projected in order to guarantee affordable and efficient mobility to citizens.

Network design, in the context of transportation engineering, is one of the stages in the process of projecting a public transit system. It comprises of deciding the itineraries of the different lines that compose the network of the system. As far as we are concerned, there is currently no consensus technical method or protocol which either private or public intuitions use to direct the design of transportation networks, leaving the process susceptible to be influenced by political agenda. The adoption of a method derived from a mathematical standpoint framed on technical knowledge on transportation would harbor this process from political pressure.

In the light of those facts, we propose an Integer Linear Programming (ILP) formulation to model the drafting of the itinerary of a public transit network line - sometimes referred in the literature as the line planning problem. The ILP formulation maximizes the demand served, that is, the number of trips carried out on the line.

1.1. Historical Overview on Public Transportation

This section is based on the first chapter of Vuchic’s book, Urban Transit Systems and Technology [Vuchic, 2007].

Transportation has played an important role in human agglomerations throughout history. From being a determining factor in city locations - cities were usually formed on seaside or by navigable rivers, where harbors could be built - to a limiting factor in city sizes - ineffective intraurban transport made difficult the distribution of food and other basic supplies.

But it was not until the 17th century that the first organized forms of public transportation systems emerged. Vehicles which could be hired for intraurban trips, known as coaches - the predecessors of modern taxis -, first appeared in London in 1600. In 1634, coach owners obtained permission to ply the streets for hire; by 1694 there were around 700 licensed coaches in London. In 1612 public-hire sedan chairs - a chair mounted on wooden poles carried by a pair of “chair-men” - first appeared in Paris; in 1634 they were introduced in London. In 1662, a public coach service started operating on fixed routes in Paris. Vehicles could carry eight passengers at a time and were horse-drawn. However, those forms of transport were accessible only to those of the upper classes as they were very costly.

Transport systems with higher capacity began to emerge in the end of the 18th century: horse-drawn omnibuses were operating in the outskirts of London as early as 1798. But it was in France the omnibus became more popular and where they were first used in the inner-city areas.

Shortly after, omnibuses services were created in the United States and other European cities. Subsequently, horse-drawn tramway - which was basically a omnibus on rails - services commenced in the US and later on in Europe. The low rolling resistance of the vehicle allowed for more efficient use of horsepower and a higher passenger capacity.

The first mechanized transport services appeared in the 19th century as early as 1833. However most of those fail to attain commercial success and public approval due to factors such as unreliability and being slow and uncomfortable. It was with the emergence of electric tramways that transport services with mechanized vehicles became successful. A regular electric tramway service initiated its operation in Cleveland in 1884 and several cities in the US had this service in the middle 1880s. In Europe, electric tramway services developed more slowly.

It was also in the 19th century that occurred the development of high-speed railway transit modes. Suburban railways, with vehicles based on steam engines, had a first large-scale development in London; the first line started operating in 1838. In 1883, an electric interurban railway - high-speed trains powered by electricity typically connecting a group of nearby cities - was established in Northern Ireland. The rapid-transit or metro occurrence in London (the Metropolitan Line) was inaugurated in 1863 and operated with steam powered locomotives; 30 years later the first electric powered vehicles started operating also in London.

The 20th century saw further development of electric tramway technology as well as the invention of new transport modes: the motorbus (based on an internal combustion engine), which would soon replace the horse omnibus and the trolleybus, an electric powered vehicle.

1.2. Related Work

Laporte et al. [2007] proposed a model based on linear programming to design networks of rapid-transit modes - which are commonly segregated and independent of the street network. The model is rather complex taking into account travel demand, how this demand is distributed between private and public transport, the decision of constructing both stations and alignments connecting stations - as well as the cost of constructing these.

Borndörfer et al. [2007] proposed a column-generation approach to network design tied to the problem of finding the frequencies of each line (this combination is known as the line planning problem) in order to satisfy a given travel demand. They detected two objectives: minimizing operating costs for the company and minimizing traveling times for passengers. A multi-commodity flow model was introduced and results for the data of the city of Postdam, Germany, were reported.

Bussieck et al. [2004] also investigated the line planning problem. They used both non-linear and mixed integer optimization to produce models for the problem and considered two objectives: minimizing operating costs and passenger convenience, which is modeled as a minimum for the frequency. Results for practical data of the Dutch Railways were presented.

Ángel Marín e Jaramillo [2009] presented a model for urban rapid transit network design, which comprises of the location of stations and of the alignments between stations. The model aims at maximizing travel demand supplied under cost constraints and it also takes into account that users can choose their transportation modes and trips. As the model cannot be solved efficiently by

a Branch and Bound due to the large number of variables and constraints, algorithms based on Benders decomposition were proposed. The two methods were compared using fabricated networks.

Zarrinmehr et al. [2016] proposed a multi-objective bi-level optimization model for the network design problem. The model takes into consideration (maximizing) transit ridership, (minimizing) agency operating cost, system performance and an elastic demand. The authors conceived a greedy algorithm to solve the model.

Our contribution We propose a generalized model which can adapt to support transport modes other than interurban trains and should be easier to solve when compared to the existing ones. The proposed model, rather than fixating at a specific transport mode, investigates a still unexplored proof of concept with respect to finding optimal network designs regarding user demand.

This paper is organized as follows. Section 2 presents a formal definition of this paper approach to the line planning problem and introduces the proposed ILP formulation of the problem. In Section 3 the experiments are described and its results are discussed. Section 4 presents the concluding remarks.

2. Problem Description and Model

This section considers the problem of modeling street level transport - which runs on streets, and therefore are constrained by the street network. Nevertheless the model presented in this section can be easily adapted to support underground and elevated transport modes.

2.1. Transport-Related Concepts

2.1.1. Origin-Destination Matrix

An *origin-destination matrix* summarizes information about trips a given population makes on a daily bases. Usually, these matrices represent a given period of time, for instance, the one hour period between 13:00 and 14:00, and weekdays and weekends usually are represented separately. Let $M = m_{ij}$ be an $n \times n$ origin-destination matrix for a given period, then $m[i][j]$ is the travel demand - the number of daily trips - from location i to location j , and $m[j][i]$ is the travel demand from location j to location i , $i, j \in \mathbb{Z}$, $i, j \leq n$. Note that $m[i][j]$ is not necessarily equals to $m[j][i]$; $m[i][i]$ is defined as 0.

2.1.2. Line of a Transportation System

A *line of a transportation system* is a fixed itinerary between two fixed locations through which vehicles of the given system drive with a predefined frequency. In this text we shall refer to a line of a transportation system simply as line.

2.1.3. Street Network

The *street network* of a given area is the network formed by the streets encompassed in this area. Also referred as road network.

2.1.4. Street Segment

A *street segment* is a section of a given street. In this text, we use street segment to refer to a section of street between any two consecutive intersections of the street.

2.2. Problem Description

Suppose one wishes to plan a public transport system for a given urban area for which the origin-destination matrix, the street network, and the cost of implementing the system infrastructure for each road segment are known. Their goal is to find the line that **maximizes** the demand served while respecting a **budget (cost) constraint** B . A demand is considered served if the line pass by both the origin and destination location. This problem can be modeled as a **binary linear program**.

2.3. Model

Let $R = (V, A)$ be the weighted graph representing the road network of the given urban area, with each arc representing the direction of a road segment, each vertex representing a road intersection, and each weight c_{ij} denoting the cost to realize the system infrastructure on that road segment. We construct $R' = (V', A')$ such that $V' = V \cup \{s, t\}$ and $A' = A \cup \{(s, i), c_{si} = 0 : \forall i \in V\} \cup \{(i, t), c_{it} = 0 : \forall i \in V\}$, where s and t are artificial source and target locations, respectively (an example is presented in Figure 1), so we do not have to select those beforehand - instead we allow the model to determine these.

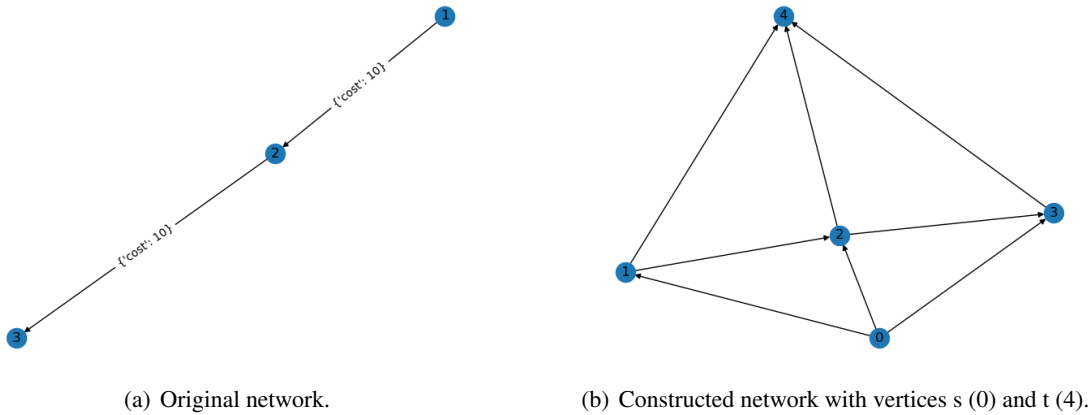


Figure 1: Example of original and constructed network.

Let T be the origin-destination matrix, a square matrix of dimension $|V|^2$ where t_{ij} is the travel demand from vertex i to vertex j , $i, j \in V, i \neq j$. Then we formulate a binary linear program that receives R' and T as inputs, and outputs the subgraph $L = (V_L, A_L)$, $L \subset R'$ with adjacency matrix $X = (x_{ij}), x_{ij} \in \{0, 1\}, i, j \in V$, which maximizes the demand served by meeting ($y_{ij} = 1$) or not ($y_{ij} = 0$) demands $t_{ij}, i, j \in V, i \neq j$. The formulation is as follows.

$$x_{ij} = \begin{cases} 1, & \text{if } (i, j) \in A_L. \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if demand } t[i][j] \text{ is served.} \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{Max } \sum_{i,j \in V} y_{ij} d_{ij}$$

Subject to

$$\sum_{i,j \in V} x_{ij} c_{ij} \leq B \quad (\text{budget constraint})$$

In order for L to be connected we must guarantee that for every arc that reaches a given vertex i , $i \notin s, t$, there is an arc that leaves i , that is, the in-degree of i equals its out-degree. For s and t the out-degree and the in-degree, respectively, must be equal to 1. Hence, we add the following constraints.

$$\sum_{k \in V} x_{ki} = \sum_{l \in V} x_{il} \quad \forall i \in V$$

$$\sum_{i \in V} x_{si} = 1$$

$$\sum_{i \in V} x_{it} = 1$$

A demand t_{ij} is served ($y_{ij} = 1$) if and only if both i and j are in the path formed by L , that is, there is an arc $a \in A_L$ that leaves vertex $i \in V_L$ and an arc $b \in A_L$ that reaches vertex $j \in V_L$. Thus, we add the following constraints.

$$y_{ij} \leq \sum_{k \in V} x_{ik}$$

$$y_{ij} \leq \sum_{k \in V} x_{jk}$$

In order to ensure that the path described by L does not go through the same vertex more than once, that is, there are no subcycles in L , we must add constraints to remove subcycles.

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \quad \forall S \subseteq V$$

Note that these constraints also guarantee that L has a single connected component.

3. Experiments

3.1. Computational Environment

All experiments were run on a headless machine with two Intel Xeon CPU E5-2630 v4 2.20GHz processors, with 10 cores each, running Ubuntu 18.04.3; Gurobi v9.0 - through its Python interface - was used as an integer linear programming solver.

3.2. Preliminary Experiments

3.2.1. Instance

The instance selected for our preliminary experiments was introduced by Laporte et al. [2007] and is presented in Figure 2. We shall, hereafter, refer to it as Laporte network.

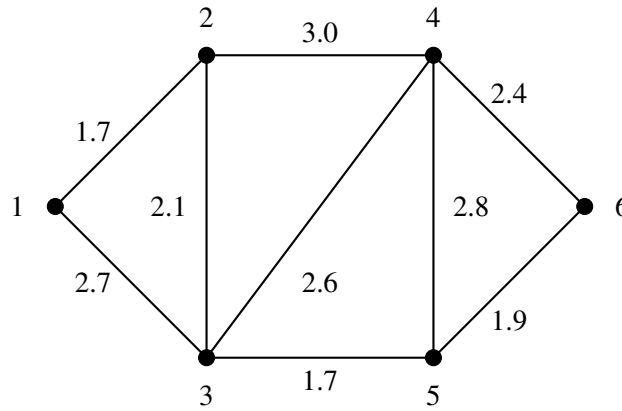


Figure 2: The graphic representation of the Laporte network.

Since our model expects a directed graph, we modified the original Laporte network graph to suit our needs; each edge $\{i, j\}$ in the original graph was replaced by two arcs (i, j) and (j, i) .

3.2.2. Results and Discussion

We ran our model on the modified Laporte network for different values of C (budget constraint), $C \in [0, 42]$, $C \in \mathbb{Z}$. The cost of the entire network is 41.8. Figure 3 shows a plot of the demand served as function of C . Figure 4 shows some of the outputted graphs.

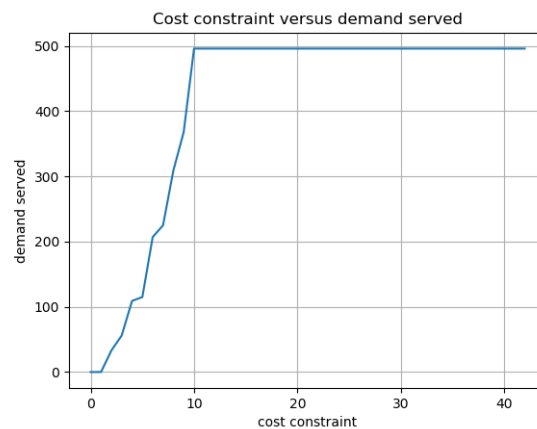
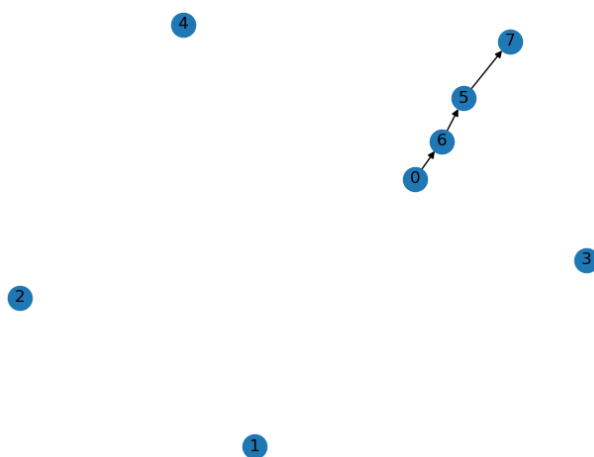
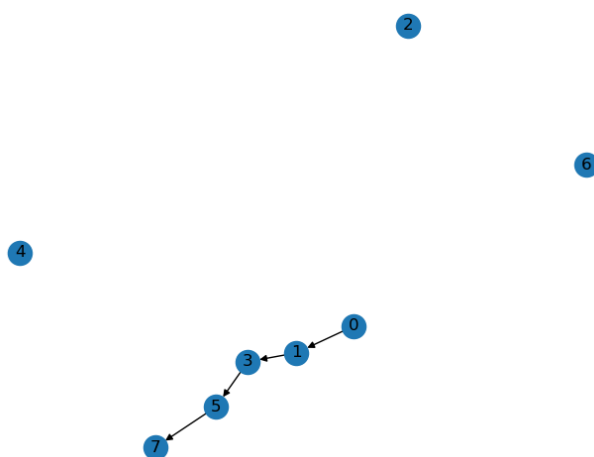


Figure 3: Demand served as a function of the budget constraint.

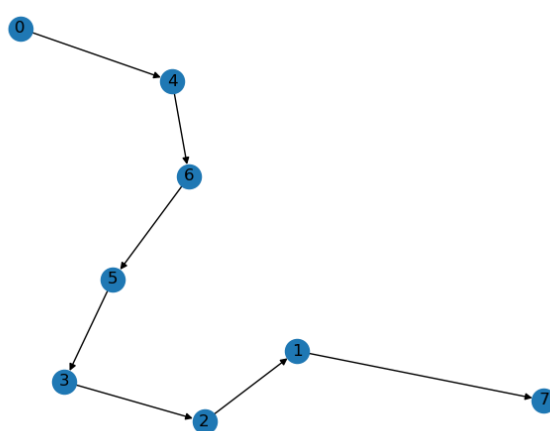
Note that the demand served reaches its maximum value for $C = 10$. Another way to state this result is that the Laporte network admits a Hamiltonian path with minimum cost of $C = 10$. The relationship between the proposed network design problem and Hamiltonian paths shall be further explored in the course of this research project in order to formalize a proof of NP-hardness.



(a) Optimal network for $C = 2$.



(b) Optimal network for $C = 5$.



(c) Optimal network for $C = 10$.

Figure 4: Optimal networks for selected values of C .

3.3. Experiments with Grid Networks

3.3.1. Instances

For this experiment we created 20 different grid networks. First, 4 grid graphs, with dimensions 4×4 , 6×6 , 8×8 , 10×10 , were generated. Let us call these grids germinal grids. Then, for each germinal grid, edges were randomly removed to produce grids with densities 20%, 40%, 60%, 80%, and 100% (which is the germinal grid itself). We shall call those density grids. For a given dimension, each density grid derives directly from the germinal grid, that is, an instance with 80% density does not necessarily contain the instance of same dimensions and density of 60%, and so forth. All grids are directed and parallel edges have the same cost.

The costs of the edges are generated based on an uniform distribution and are drawn as follows. Let C_1 be a circle of radius 0.2 and center $(0, 0)$ in the Cartesian plane. Let C_2 be a circle with radius 0.2 and center $(0.6, 0)$. We draw a point $p_1 = (x_1, y_1)$ from C_1 and a point $p_2 = (x_2, y_2)$ from C_2 by sampling x_1 , y_1 and y_2 from the uniform distribution $[-0.2, 0.2)$ and x_2 from the uniform distribution $[0.4, 0.8)$. We, then, calculate the Euclidean distance $dist$ between p_1 and p_2 , multiply $dist$ by 1000 and attribute this value to the cost c_e of edge $e = (u, v)$ and the cost $c_{e'}$ of the parallel edge $e' = (v, u)$, where v and u are vertices of a given germinal grid. This process is repeated for each pair of parallel edges of the given grid. Note that if an edge e is in both a germinal grid and the derived density grid, then its cost c_e is the same in both grids.

Lastly, the origin-destination matrix associated with the $n \times n$ germinal grid - and the set of derived density grids - had its elements sampled from the discrete uniform distribution $[1, 100]$ and has dimension $n^2 \times n^2$.

3.3.2. Results and Discussion

Tables 1-3 show the proposed methodology results for instances with a budget constraint (C) of 25%, 50% and 75% with respect to the network total cost. The model obtained optimal solutions for most (49 out of 60) instances. Lower and upper bounds were obtained for all instances, except one, namely the instance with 100 vertices, 100% density and 25% budget constraint. The results also show that instances composed of sparse graphs were solved more often than those composed of dense graphs, as all instances with density of 40% or less were solved to optimality.

With respect to optimality gaps, Figure 5 shows the average gap obtained by the methodology for every graph size and budget constraint. The figure clearly shows an exponential growth in the average gap with the increase of the graph size. For the instance in which no lower bound was attained, the optimality gap was assumed to be infinite. As for the instances budget, Figure 5 indicates that loose budget constraints proved to be the easier to solve, since all but one instance with 75% budget constraint were solved to optimality.

4. Final Remarks

This work proposed a new combinatorial optimization problem for the modeling of urban transportation networks. The model assumption is the existence of an origin-destination matrix, which consolidates the demand for transportation, a network portraying the possible candidate edges to form the transportation line, with their respective realization costs, and a limited budget.

Number of nodes	Results	Density				
		20%	40%	60%	80%	100%
16	LB	605	782	1,926	2,190	3,799
	UB	605	782	1,926	2,190	3,799
	Gap	0%	0%	0%	0%	0%
36	LB	970	3,767	8,088	11,588	18,581
	UB	970	3,767	8,088	11,588	22,326
	Gap	0%	0%	0%	0%	20.1550%
64	LB	2,391	3,002	23,647	37,720	54,944
	UB	2,391	3,002	23,647	60,509	96,674
	Gap	0%	0%	0%	60.4162%	75.9501%
100	LB	1,810	18,009	46,400	86,599	-
	UB	1,810	18,009	95,376	180,947	266,298
	Gap	0%	0%	105.5517%	108.9491%	-

Table 1: Test results for a budget constraint of 25% of the total cost of the network.

Number of nodes	Results	Density				
		20%	40%	60%	80%	100%
16	LB	984	2,379	4,420	6,911	10,459
	UB	984	2,379	4,420	6,911	10,459
	Gap	0%	0%	0%	0%	0%
36	LB	1,375	5,956	20,839	42,829	58,575
	UB	1,375	5,956	20,839	42,829	58,575
	Gap	0%	0%	0%	0%	0%
64	LB	3,133	3,002	55,835	127,169	200,134
	UB	3,133	3,002	55,835	137,879	205,621
	Gap	0%	0%	0%	8.4219%	2.7417%
100	LB	1,810	18,009	116,812	320,725	125,342
	UB	1,810	18,009	242,190	372,483	501,178
	Gap	0%	0%	107.3332%	16.1378%	299.8484%

Table 2: Test results for a budget constraint of 50% of the total cost of the network.

Number of nodes	Results	Density				
		20%	40%	60%	80%	100%
16	LB	984	2,379	4,974	11,926	13,342
	UB	984	2,379	4,974	11,926	13,342
	Gap	0%	0%	0%	0%	0%
36	LB	1,375	5,956	22,659	62,037	65,277
	UB	1,375	5,956	22,659	62,037	65,277
	Gap	0%	0%	0%	0%	0%
64	LB	3,133	3,002	55,835	192,993	206,005
	UB	3,133	3,002	55,835	192,993	206,005
	Gap	0%	0%	0%	0%	0%
100	LB	1,810	18,009	125,121	427,150	501,178
	UB	1,810	18,009	125,121	444,717	501,178
	Gap	0%	0%	0%	4.1126%	0%

Table 3: Test results for a budget constraint of 75% of the total cost of the network.

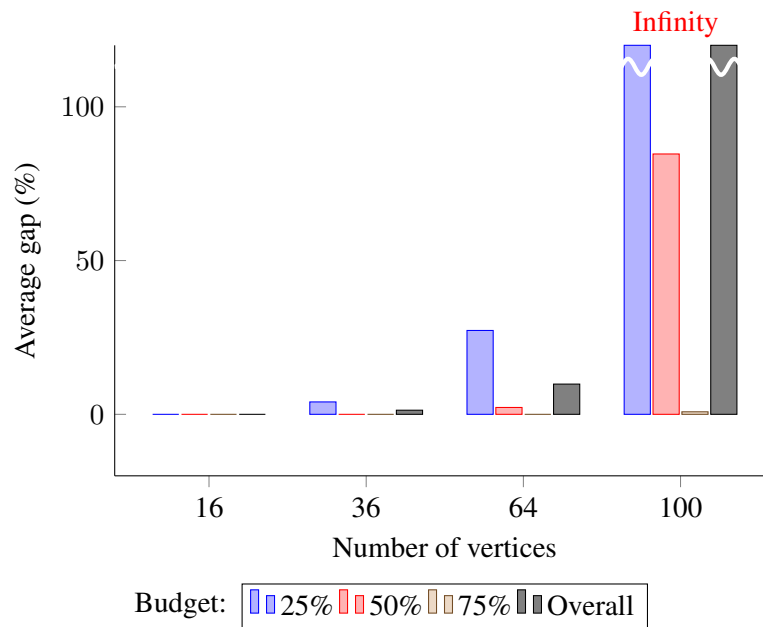


Figure 5: Average gaps obtained by the methodology with respect to the instances sizes and budgets.

An integer linear model is proposed for the new problem. The model is uncomplicated by the operational constraints which often appear in similar works bound by a specific transportation mode. Therefore, this work main contribution concerns the development of a generalized framework for the research on transportation network design for multiples modes of transportation. Moving forward, a promising line of investigation is to address the polyhedral aspects of the model in the search for facet defining inequalities.

References

- Borndörfer, R., Grötschel, M., e Pfetsch, M. E. (2007). A column-generation approach to line planning in public transport. *Transportation Science*, 41(1):123–132.
- Bussieck, M. R., Lindner, T., e Lübbecke, M. E. (2004). A fast algorithm for near cost optimal line plans. *Mathematical Methods of Operations Research*, 59(2):205–220.
- Kepaptsoglou, K. e Karlaftis, M. (2009). Transit route network design problem: Review. *Journal of Transportation Engineering*, 135:491–505.
- Laporte, G., Marín, Á., Mesa, J. A., e Ortega, F. A. (2007). An integrated methodology for the rapid transit network design problem. In *Algorithmic Methods for Railway Optimization*, p. 187–199. Springer Berlin Heidelberg. ISBN 978-3-540-74247-0.
- UITP (2017). Urban public transport in the 21st century. *uitp.org/*. Available: <http://www.uitp.org/urban-public-transport-21st-century> [Last access: 29 Jan 2020].
- Vuchic, V. R. (2007). *Urban Transit Systems and Technology*. Wiley. ISBN 9780471758235. URL <https://books.google.com.br/books?id=zFby0C3ohwQC>.

- Zarrinmehr, A., Saffarzadeh, M., Seyedabrishami, S., e Nie, Y. M. (2016). A path-based greedy algorithm for multi-objective transit routes design with elastic demand. *Public Transport*, 8(2): 261–293.
- Ángel Marín e Jaramillo, P. (2009). Urban rapid transit network design: accelerated Benders decomposition. *Annals of Operations Research*, 169(1):35–53.