

Formulations for a Switch Allocation Problem with Uncertainties

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ABSTRACT

Power distribution is an essential service for almost all human activities, including industry, commerce, services, habitation. The occurrence of failures in a power network can cause negative impacts from the customer's perspective and also economic losses for the utility given that an amount of energy is not distributed and accounted for. This work has the objective of proposing methodologies to improve power distribution network reliability. Guided by this objective, the switch allocation problem is investigated under a combinatorial optimization perspective. Switches are devices that isolate failure, restraining them from propagating throughout the network. The switch allocation problem consists in finding the best locations to install a set of switches in order to minimize failures impacts in the network. In this work, the switch allocation problem is addressed considering uncertainty in the input data regarding the system failure rates and the power demand of costumers. This uncertainty is incorporated within a formulation of a two-stage stochastic problem.

KEYWORDS. power distribution systems, switch allocation, switch relocation, integer linear programming

OR in Energy, Oil and Gas, Combinatorial Optimization, Statistics and Probabilistic Models

1. Introduction

The distribution of electric power is an indispensable service for most human activities, being one of the main forces that push the economy and the overall progress in society. To reach its destination, the energy generated by power plants requires a robust distribution infrastructure, composed of reliable, sustainable and efficient *Power Distribution Systems*.

A power distribution system consists of a set of components subject to failure: cables, poles, circuit breakers, switches, transformers, capacitors and voltage regulators. During a failure, part of the network is disconnected from the substation until a maintenance team identifies the source of the failure and repairs the damaged components (de Assis et al. [2015]). If some consumers are left without power, it can result in fines for the distributor and will negatively impact the system reliability.

One method with the potential to improve the reliability of the power distribution system is the switch allocation. The switches isolate defective sections in order to minimize *Energy Not Supplied* (ENS). The ideal strategy would be to allocate switches across all system, but this is impractical due to switch costs. The *Switch Allocation Problem* (SAP), consists of selecting the best way to allocate a certain number of switches in a power distribution system, with the objective of reaching optimal trade-offs of reliability and investments. SAP belongs to the family of combinatorial optimization problems (de Assis et al. [2015]). Benavides et al. [2013] and Miu et al. [1998] demonstrated that SAP is NP-Hard, however there are several heuristics that obtain good quality non-trivial solutions (Levitin et al. [1995] Ramírez-Rosado e Bernal-Agustín [2001]).

Celli e Pilo [1999] proposed an exact solution based on dynamic programming, but it is only applicable in small instances. de Assis et al. [2015] explains that exact methodologies can be suitable for local optimization of the distribution system, in other words, they can be used in portions of the network. de Assis et al. [2015] also proposed a memetic algorithm to achieve heuristic solutions for SAP. Usberti et al. [2014] proposed a MILP model (*Mixed Integer Linear Programming*) to solve SAP.

Real power distribution networks can present different types of uncertainties in the data, for example, failure rates, average duration of failures and demanded loads. One way to get around these uncertainties is to design possible future scenarios where values are estimated from data with uncertainties. This work considers a switch allocation problem with uncertainties in the data. The problem involves decisions that are made in two stages: the first stage (represents the present time) is deterministic, and the second stage (represents the future) is stochastic, with uncertainties distributed into a discrete set of possible scenarios. In the second stage it is possible to reallocate a limited number of switches for each scenario. The objective is to obtain a solution with the least expected ENS over the two stages. We call this problem *Stochastic Switch Allocation Problem*.

The objective of this work is to develop mathematical models and solution methodologies for the *Stochastic Switch Allocation Problem*. The motivation for this work is to provide planning tools that enable a more reliable distribution of energy, reducing the impact of interruptions for consumers, the distributor and society.

2. Problem Description

In graph terminology, a radially operated distribution system can be represented as an oriented tree $G(V, A)$ rooted at the substation (Ahuja et al. [1988]). The sets V and A are composed of the nodes and arcs of the tree, respectively. Each node $i \in V$ represents a load point or branch of the network at which it is associated with a pair (θ_i, l_i) . The θ_i is the average failure time (hours/year) and l_i is the average load consumed (kW). The arcs $(i, j) \in E$ where $i, j \in V$ represent the network cables, which are oriented in the same direction as the electric current. The set of

descendant nodes of i is represented by V_i . The accumulated load of a node i is the sum of the loads of the descending nodes of i (Eq. 1).

$$\tilde{l}_i = \sum_{j \in V_i} l_j \quad (1)$$

When a node fails, itself and other nodes may be interrupted through a cascading effect. This happens because of the propagation of short circuits generated by the defective nodes. Given the nodes $i, j \in V$, when i fails it affects j in two situations:

- Situation 1: i is ancestor of j ;
- Situation 2: i is not an ancestor of j , and there are no switches in $path(LCA(i, j), i)$, where $LCA(i, j)$ is the *Lowest Common Ancestor* of i and j .

The switches are devices that isolate faults in a power distribution network. The role of the switches is to open when a short circuit passes through it. However, the number of switches is limited and it is not economically reasonable to allocate a switch in every arc of a distribution system. Thereby, the SAP is a problem with the objective to choose the best arcs to allocate switches as to minimize the *ENS* (de Assis et al. [2015]). Let the set of arcs that have allocated switches be denoted as A_s , where $A_s \subseteq A$.

The interruption flow f_{ij} from node j to node i (reverse arc orientation) is the expected time that the i node will be interrupted due to faults originating at the $k \in V_j$ nodes (Usberti et al. [2014]). The calculation of the interruption flow f_{ij} for an arc (i, j) any can be represented by the Equation 2.

$$f_{ij} = \begin{cases} 0 & \text{if } (i, j) \in A_s \\ \theta_j + \sum_{(j,k) \in A} f_{jk} & \text{c.c.} \end{cases} \quad (2)$$

The *Energy Not Supplied* (ENS) is an index to evaluate quantitatively how reliable a distribution system is. The ENS is not a deterministic value but an expectation of a probability distribution (Billinton e Jonnavithula [1996]). According to Usberti et al. [2014] it is possible to express the *ENS* by the interruption flows, according to Equation 3:

$$END = \sum_{(i,j) \in A} (\tilde{l}_i - \tilde{l}_j) f_{ij} + \sum_{i \in V} \theta_i \tilde{l}_i \quad (3)$$

The *Stochastic Switch Allocation Problem* contains two stages. The first is the present stage, which is the current scenario of a distribution system and denoted by s^0 . The second is the future stage which is a set F containing the possible future scenarios for the distribution system. Each possible future scenario $s \in F$ has a probability of realization p^s .

The ENS of each scenario $s \in S$ is represent by ENS^s and can be calculated as shown in Equation 4. The END_{total} is the total expected energy not supplied from both stages (Equation 5).

$$END^s = \sum_{(i,j) \in A} (\tilde{l}_i^s - \tilde{l}_j^s) f_{i,j}^s + \sum_{i \in V} \theta_i^s \tilde{l}_i^s : \forall s \in S \quad (4)$$

$$END_{total} = END^0 + p^s \sum_{s \in F} END^s \quad (5)$$

It is assumed that switches allocated in the present will remain in the same arcs of the future scenarios, except for a limited number of reallocations, given by parameter r . Thus, each future scenario can reallocate up to r switches if necessary to improve reliability.

In summary, the *Stochastic Switch Allocation Problem* can be described as follows: given a distribution network, a set of scenarios, a limited amount of switches and a limited number of reallocations, the objective is to search for the best arcs to allocate switches and the best reallocations in future scenarios so as to minimize the END_{total} .

3. MILP Formulations

The following MILP formulation is proposed for the stochastic SAP. We denote k the number of switches, r the number of reallocations and M is a sufficiently large value. The formulation includes the following decision variables: $f_{ij}^s \in \mathbb{R}^+$ is the interruption flow; x_{ij}^s denotes if an arc $(i, j) \in E$ has a switch (1) or not (0); z_{ij}^s denotes if an arc $(i, j) \in A$ has changed its state in future scenario $s \in F$ with respect to the present.

$$\begin{aligned} & (StochasticSAP) \\ & MIN \quad ENS_{total}, \end{aligned} \tag{6}$$

subject to

$$\sum_{\forall (i,j) \in A} x_{ij}^s \leq k \quad \forall s \in S, \tag{7}$$

$$\sum_{\forall (i,j) \in A} z_{ij}^s \leq 2r \quad \forall s \in F, \tag{8}$$

$$z_{ij}^s + x_{ij}^s \geq x_{ij}^0 \quad \forall s \in F, \tag{9}$$

$$z_{ij}^s + x_{ij}^0 \geq x_{ij}^s \quad \forall s \in F, \tag{10}$$

$$f_{ij}^s \geq \theta_i^s + \sum_{(j,k) \in A} f_{jk}^s - Mx_{ij}^s \quad \forall (i,j) \in A, \forall s \in S, \tag{11}$$

$$f_{ij}^s \in \mathbb{R}^+ \quad \forall (i,j) \in A, \forall s \in S, \tag{12}$$

$$x_{ij}^s \in \{0, 1\} \quad \forall (i,j) \in A, \forall s \in S, \tag{13}$$

$$z_{ij}^s \in \{0, 1\} \quad \forall (i,j) \in A, \forall s \in F, \tag{14}$$

$$\tag{15}$$

The objective function (6) minimizes the ENS_{total} . Constraints (7) ensure that to each scenario $s \in S$ is allocated at most k switches. Constraints (8) ensure that each scenario $s \in F$ has at most r reallocations (or $2r$ state changes). Constraints (9) and (10) ensure that z_{ij}^s will be 1 if x_{ij}^0 is different of x_{ij}^s . Constraints (11) ensure that the interruption flow f_{ij} will be 0 if arc $(i, j) \in E$ contains a switch, otherwise it will have the value predicted by Equation (2).

4. Computational Experiments

4.1. Benchmark of Instances

The computational experiments used the benchmark of five distribution networks available in the literature (Kavasseri e Ababei [accessed June 30, 2020]). The instances are denominated as: *bus_32_1.pos*, *bus_83_11.pos*, *bus_135_8.pos*, *bus_201_3.pos* and *bus_873_7.pos*. Table 1 shows the instances sizes.

Table 1: Instances

Instance	Number of nodes	Number of edges
<i>bus_32_1.pos</i>	33	32
<i>bus_83_11.pos</i>	94	83
<i>bus_135_8.pos</i>	143	135
<i>bus_201_3.pos</i>	204	201
<i>bus_873_7.pos</i>	880	873

To generate the stochastic data for the experiments, each network was associated to two possible future scenarios, and in each future scenario the failure times of each node was derived by multiplying the present failure time by a random value, uniformly drawn from the range $[0.5, 1.0]$. Each instance was executed considering each $k \in \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$ and each $r \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

4.2. Computational Settings

The MILP formulations was implemented using solver Gurobi, set with a ten minutes time limit for each instance. It was used a PC under Ubuntu 18.04, Intel Core i7 2.50GHz processor and 8,0 GB of RAM. A time limit of 300 seconds was given for each instance.

Table 2: Computational results summary

k	r	<i>bus_32_1.pos</i>			<i>bus_83_11.pos</i>			<i>bus_135_8.pos</i>			<i>bus_201_3.pos</i>			<i>bus_873_7.pos</i>		
		ENS	GAP	time	ENS	GAP	time	ENS	GAP	time	ENS	GAP	time	ENS	GAP	time
5	0	1853.4	0.00	0	1084.8	0.00	0	10811.9	0.00	0	6641.0	0.00	2	1644730.8	0.00	4
	1	1852.0	0.00	1	1084.8	0.00	1	10795.4	0.00	11	6595.2	0.09	300	1641714.6	0.01	300
	2	1852.0	0.00	1	1084.8	0.00	1	10795.4	0.00	62	6595.2	0.14	300	1641181.9	0.00	300
	3	1852.0	0.00	1	1084.8	0.00	1	10795.4	0.00	74	6595.2	0.15	300	1641181.9	0.01	300
	4	1852.0	0.00	1	1084.8	0.00	0	10795.4	0.00	74	6595.2	0.15	300	1641181.9	0.00	300
	5	1852.0	0.00	1	1084.8	0.00	0	10795.4	0.00	76	6595.2	0.13	300	1641181.9	0.00	292
10	0	600.3	0.00	0	685.2	0.00	0	7452.6	0.00	0	4040.2	0.00	3	1221082.0	0.00	300
	1	590.6	0.00	17	677.0	0.00	3	7382.7	0.00	17	3972.5	0.17	300	1220549.4	0.07	300
	2	589.2	0.00	119	670.7	0.00	16	7370.9	0.00	150	3971.6	0.21	300	1220526.7	0.09	300
	3	589.1	0.15	300	669.0	0.00	35	7369.7	0.00	214	3971.6	0.22	300	1220526.7	0.08	300
	4	589.1	0.17	300	668.6	0.00	54	7369.7	0.02	300	3971.6	0.22	300	1220786.5	0.08	300
	5	589.1	0.19	300	668.6	0.00	60	7369.7	0.01	300	3971.6	0.22	300	1226762.5	0.08	300

k – number of switches.

r – number of reallocations.

ENS – energy not supplied ($kWh/year$).

$Gap = \cdot (UB - LB) / UB$.

$time$ – computational time in seconds.

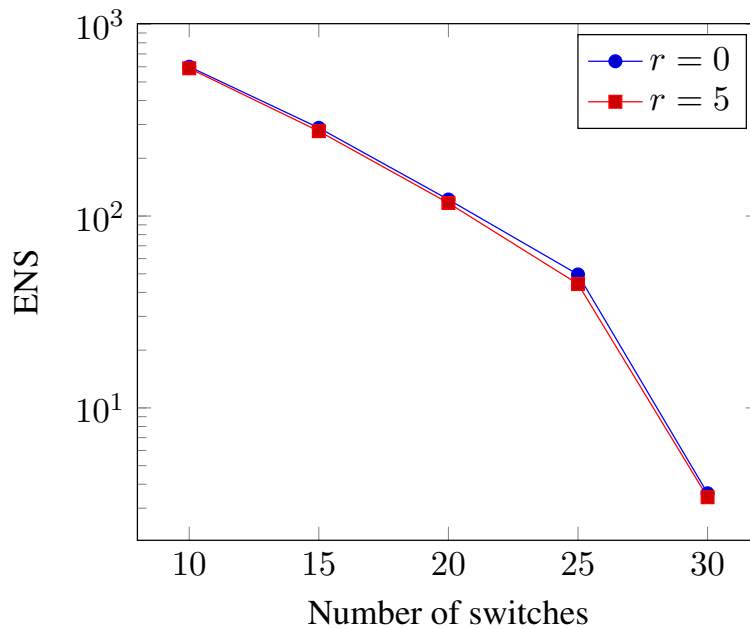


Figure 1: ENS of network *bus_32_1.pos* in function of the number of switches.

4.3. Results

Table 2 shows the results of the experiments for all instances with $k = 5$ and $k = 10$ available switches and number of reallocations r ranging from 0 to 5. The results show the effectiveness of the model which solved optimally 38 out of 60 instances, all in less than five minutes. The instances with a less amount of switches were easier to solve, given that their search space is smaller. It can be observed that increase in the number of switches from 5 to 10 had a substantial impact on the reduction of the ENS, for all instances. An ENS reduction, although not as significant, can also be observed by increasing the number of available reallocations.

Figure 1 shows how the number of switches impact the ENS of network *bus_32_1.pos* for two possible numbers of reallocations. It can be noticed that the ENS decreases consistently as more switches are allowed in the network. As previously observed in Table 2, the effect of the number of reallocation is small in comparison, but still some ENS reduction is possible through reallocation of switches. Figure 2 offers another perspective of the same conclusions, now focusing on network *bus_135_8.pos*. These minor impacts from reallocations should be further investigated by considering other types of stochastic distribution of failure times and also considering variations of the demand.

5. Final Remarks

This work proposed a new combinatorial optimization problem which offers a realistic framework for the decision making process of switch allocation in power distribution systems. The model hypothesis is that the interruption times and power loads are imbued with uncertainties. These uncertainties are addressed by formulating a two-stage stochastic optimization problem as an integer linear model. The proposed model was tested with a benchmark of instances from literature. The results have shown the model effectiveness in providing optimal solutions for most instances, showing the impacts on reliability that comes from a good allocation and reallocation of switches.

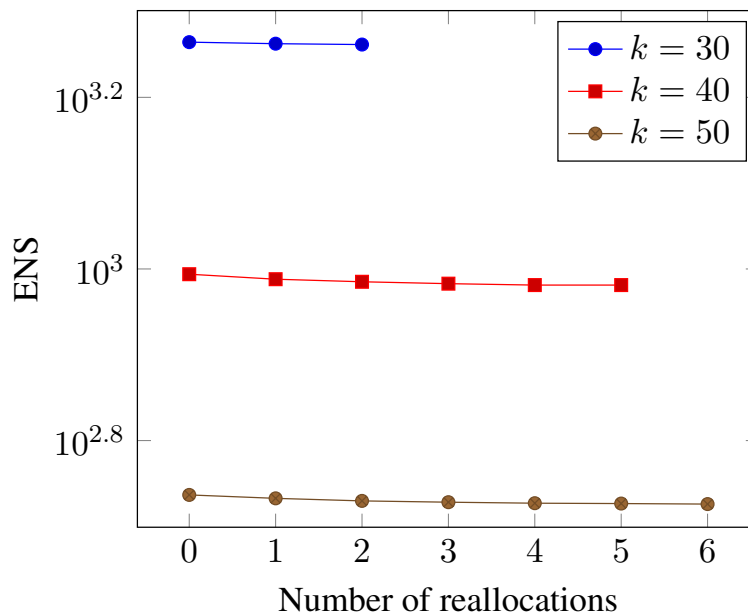


Figure 2: ENS of network *bus_135_8.pos* in function of the number of reallocations.

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