

A computational experiment with low-cost heuristics for profile reduction

L. M. Silva, J. A. B. Bernardes, S. L. Gonzaga de Oliveira

Universidade Federal de Lavras - UFLA

Lavras, MG

{liberio, juniorbernardes, sanderson}@ufla.br

ABSTRACT

Previous publications reviewed methods that compute symmetric permutations to reduce the profile of sparse matrices. Based on this experience, this paper, with the support of extensive experiments, evaluates six low-cost heuristics for profile reduction. The heuristics evaluated are Sloan's algorithm and five variants of this method. Among the algorithms appraised, the Hu-Scott and MPG algorithms yielded the highest number of best profile results when applied to symmetric and nonsymmetric sparsity patterns, respectively.

KEYWORDS. Profile reduction. Sparse matrices. Graph labeling.

1. Introduction

In scientific computing, the solution of linear systems composed of sparse matrices is one of the most significant computational kernels. The solution of sparse linear systems is a step in many real-world and applied problems in science and engineering. In a numerical simulation, this is generally the step that requires a high computing time. Thus, the investigation to reduce the computational cost of solving linear systems composed of sparse matrices is a relevant subject in numerical mathematics because of its significance in many numerical simulations [Gonzaga de Oliveira et al., 2018, 2019b].

The matrix profile reduction favors a sparse direct solver for solving systems of linear equations [Gonzaga de Oliveira et al., 2019a]. Thus, practitioners use heuristics for profile reduction to achieve low computational costs for solving sparse systems of linear equations. The profile reduction is also crucial for increasing the effectiveness of data structures to represent matrices, such as applications that use the skyline data structure [Felippa, 1975].

Heuristics for profile reduction place nonzero coefficients of a sparse matrix as close as possible to the main diagonal. Let $A = [a_{ij}]$ be an $n \times n$ symmetric matrix associated with a connected undirected graph $G = (V, E)$, where V and E are sets of vertices and edges, respectively. The profile of a matrix A is defined as $profile(A) = \sum_{i=1}^n [\beta_i]$, where $\beta_i = i - \min_{1 \leq j \leq i} [j \mid a_{ij} \neq 0]$ and $a_{ii} \neq 0$.

The profile minimization problem is a well known $\mathcal{NP}$ -hard [Lin e Yuan, 1994] computational search problem investigated for over a half-century. As previously mentioned, the reason is that this problem is associated with a broad range of scientific and engineering application areas. Thus, researchers have proposed several heuristic methods for reordering the rows and columns of a sparse matrix to reduce its profile [Gonzaga de Oliveira et al., 2018, 2019b].

This paper concentrates on the family of applications based on sparse matrix factorization and related problems so that the experiments conducted in this study analyze six low-cost heuristics for profile reductions [Gonzaga de Oliveira et al., 2018, 2019b]. The profile reductions yielded by a reordering algorithm are not directly proportional to the reduction of the execution time of a linear system solver. Moreover, the objective is to minimize the total computing time of the simulation, including the preprocessing time required by the reordering heuristic, at least when only a single linear system is to be solved. Therefore, a vertex labeling algorithm must perform at low-cost [Gonzaga de Oliveira et al., 2018, 2019b].

This paper evaluates Sloan's [Sloan, 1989], MPG [Medeiros et al., 1993], NSloan [Kumfert e Pothen, 1997], Sloan-MGPS [Reid e Scott, 1999], Hu-Scott [Hu e Scott, 2001], and NMPG [Gonzaga de Oliveira et al., 2019b] heuristics. The methods compute symmetric permutations for matrix profile reduction.

In Section 2, the heuristics evaluated in this computational experiment are described. In Section 3, the tests conducted are reported. In Section 4, the experimental results are presented. Finally, In Section 5, the conclusions are addressed.

2. Related work

Sloan [1989] proposed an important heuristic for profile reduction. Sloan's heuristic employs two weights in its priority scheme in order to label the vertices of the instance: w_1 , associated with the distance $d(v, e)$ from the vertex v to a pseudo-peripheral (target end) vertex e that belongs to the last level of the level structure rooted at a starting vertex s , and w_2 , associated with the degree of each vertex. The priority function $p(v) = w_1 \cdot d(v, e) - w_2 \cdot (deg(v) + 1)$ employed in Sloan's heuristic presents different scales for both criteria. The value of $deg(v) + 1$ ranges from 1 to $m + 1$

(where $m = \max_{v \in V} [deg(v)]$ is the maximum degree found in the graph $G = (V, E)$), and $d(v, e)$ ranges from 0 (when $v = e$) to the eccentricity $\ell(e)$ (of the target end vertex e) [Gonzaga de Oliveira e Chagas, 2014].

The MPG [Medeiros et al., 1993], NSloan [Kumfert e Pothén, 1997], and Sloan-MGPS [Reid e Scott, 1999] heuristics are based on Sloan's heuristic [Sloan, 1989]. Specifically, the Sloan-MGPS heuristic is Sloan's heuristic with the starting and target end vertices given by a variant of the procedure used in the GPS algorithm.

The MPG heuristic employs two max-priority queues: t contains vertices that are candidate vertices to be labeled, and q contains vertices belonging to t and also vertices that can be inserted to t . Similarly to Sloan's heuristic (and its variations), the current degree of a vertex is the number of adjacencies to vertices that neither have been labeled nor belong to q . A main loop performs three steps. First, a vertex v is inserted into q in order to maximize a specific priority function. Second, the current degree $cdeg(v)$ of each vertex $v \in t$ is observed: the algorithm labels a vertex v if $cdeg(v) = 0$, and the algorithm removes from t a vertex v (i.e., $t \leftarrow t - \{v\}$) if $cdeg(v) > 1$. Third, if t is empty, the algorithm inserts into t each vertex $u \in q$ with priority $p_u \geq p_{max}(q) - 1$ where $p_{max}(q)$ returns the maximum priority among the vertices in q . The priority function in the MPG heuristic is $p(v) = d(v, e) - 2 \cdot cdeg(v)$.

Kumfert e Pothén [1997] normalized the two criteria used in Sloan's algorithm. Thus, the Normalized Sloan (NSloan) heuristic uses the priority function $p(v) = w_1 \cdot d(v, e) - w_2 \cdot \lfloor d(s, e)/m \rfloor \cdot (deg(v) + 1)$.

Concerning Sloan's, NSloan, and Sloan-MGPS heuristics, this computational experiment set the two weights as described in the original papers. When the authors recommended more than one pair of values in the original papers, a previous publication determined the pair of values that yielded the best profile results [Gonzaga de Oliveira et al., 2018]. Thus, the two weights were assigned as $w_1 = 1$ and $w_2 = 2$ for Sloan's and Sloan-MGPS heuristics, and as $w_1 = 2$ and $w_2 = 1$ for the NSloan heuristic.

The NMPG heuristic is basically a hybrid of the MPG and NSloan heuristics. Specifically, the heuristic is based on the MPG heuristic together with the normalized scheme proposed by Kumfert e Pothén [1997]. The heuristic uses the priority function $p(v) = d(v, e) - 2 \cdot d(s, e) / \max_{v \in V} [deg(v)] \cdot (cdeg(v))$. The Hu-Scott heuristic [Hu e Scott, 2001] is a multilevel Sloan algorithm that employs a maximal independent vertex set for coarsening the adjacency graph of the matrix and Sloan-MGPS heuristic on the coarsest graph.

3. Description of the tests

This computational experiment employed the C++ programming language to implement five low-cost heuristics for profile reduction (Sloan's, MPG, NSloan, Sloan-MGPS, and NMPG heuristics). The implementations of these five heuristics for profile reductions appraised in this study employed binary heaps to code the priority queues (although the original Sloan's algorithm [Sloan, 1989] used a linked list). A previous publication [Gonzaga de Oliveira et al., 2018] showed the testing and calibration carried out to evaluate the implementations with the ones used by the original proposers of the four low-cost heuristics (Sloan's, MPG, NSloan, and Sloan-MGPS) to ensure that the codes employed in this study were comparable to the formerly proposed algorithms.

This computational experiment executed the Hu-Scott heuristic implemented in the routine MC73 contained in the HSL library [STFC, 2020]. This paper also uses the Hager exchange method implemented in the routine MC67 included in this library. We employed the Fortran programming language to use these routines.

The workstations used in the execution of the simulations featured an Intel® Core™ i7-4770 (CPU 3.40 GHz, 8 MB Cache, 12 GB of main memory DDR3 1.6 GHz) (Intel; Santa Clara, CA, United States). We performed three serial runs for each matrix.

The experiments used matrices contained in the SuiteSparse matrix collection [Davis e Hu, 2011]. The profile reduction depends on the choice of the initial ordering, and this paper considers the original labeling given in the instance contained in the matrix collection.

The nonsymmetric matrix A_u is added to its transpose matrix A_u^T ($A_u + A_u^T$). Subsequently, the implementations compute the profile of the resulting symmetric matrix [Gonzaga de Oliveira et al., 2018].

4. Results and analysis

In this section, we report the results delivered by several low-cost heuristics for profile reductions. The tables in this section show the matrix's name and size, the value of the initial profile of the matrix (profile₀), and the average values of profile yielded by the heuristics. In Sections 4.1 and 4.2, we show the results achieved by the heuristics when applied to symmetric and nonsymmetric matrices, respectively.

4.1. Symmetric matrices

In this section, we show the results yielded by several heuristics for profile results when applied to symmetric matrices. Tables 1 and 2 show that the MPG heuristic yielded 29 best profile results when applied to 65 symmetric matrices. Figure 1 shows the running times of five heuristics for profile reduction when applied to symmetric matrices.

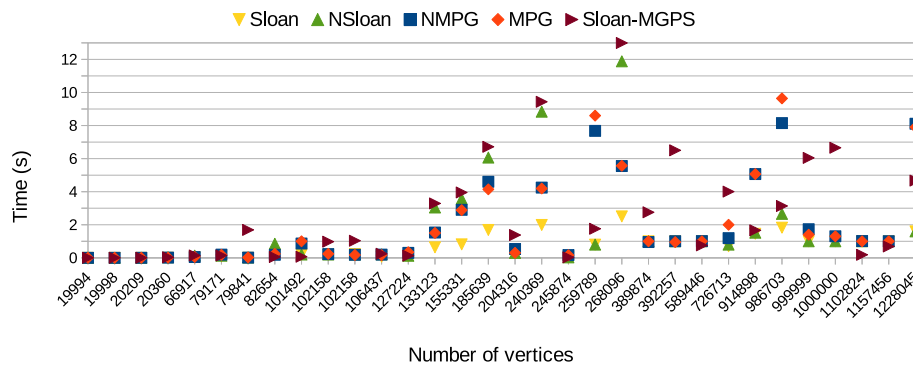


Figura 1: Execution times (in seconds) required by five heuristics for profile reductions when applied to several symmetric matrices.

Tables 1 and 2 show that the MPG heuristic yielded the highest number of best profile results. The last line of Table 2 shows the average time of the heuristics executed with the matrices contained in Tables 1 and 2. The tables reveal that the NSloan and NMPG heuristics achieved the second-best profile results. These two heuristics delivered 17 out of 65 best results. On average, the NSloan heuristic was more than 2x faster than the MPG and NMPG heuristics.

Since the MPG and NSloan heuristics yielded the best results in Tables 1 and 2, this study evaluated these two algorithms against the Hu-Scott heuristic. Thus, Tables 3 and 4 show the profile results yielded by the Hu-Scott, MPG, and NSloan heuristics when applied to several matrices arising from four application areas. Consistent with the current literature [Gonzaga de Oliveira et al., 2019b], the tables show that the Hu-Scott heuristic dominated the MPG and NSloan heuristics concerning the output quality of profile results at higher (lower) execution costs than the NSloan (MPG) heuristic.

Tabela 1: Profile results given by five heuristics applied to 34 symmetric matrices (continued on Table 2).

Matrix	<i>n</i>	profile	MPG	NSloan	NMPG	Sloan	Sloan-MGPS
ecology1	100000	999000999	667165503	667165502	667165504	667165500	667165501
ecology2	99999	998998001	667163923	667163760	667163858	667163920	667163817
helm2d03	392257	61464329677	209222375	223850617	224484026	260825526	262670530
darcy003	389874	48023597183	115077692	114810453	115124860	115372965	115446062
BenElechi1	245874	153291019	145959826	152615098	153316119	150344408	148108894
Number of best results (2D/3D problems)		0	2	2	0	1	0
StocF-1465	1465137	37107991076	3520919748	2339866877	3450982506	1846724584	2871875992
parabolic_fm	525825	112865719021	247246848	224877312	247246848	224877312	224877312
cfd2	123440	156107322	139135583	155998157	150083801	146517254	143379605
shallow_water1	81920	35117727	21859689	21888615	21847423	22711370	22623633
shallow_water2	81920	35117727	22676946	22366777	22394205	22636868	22425207
Nr.best res. (Comp. fluid dynamic prob.)		0	1	2	1	2	1
dielFilterV2real	1157456	604156752	630724799	856987640	630724799	1056823384	885977265
dielFilterV3real	1102824	973239590	4238.842.207	145500032	4238842207	257987430	196216079
tmt-sym	726713	593531565	587943680	531552353	531461788	593016001	594383981
gsm_106857	589446	258828825	1632166341	1787576968	1642238909	1890706331	1766505643
offshore	259789	3588201815	1145103260	1492233779	1236394856	1402477634	1690266416
2cubes_sphere	101492	483241271	176804411	186800075	181545679	194105604	204698856
Number of best results (electromagn. prob.)		1	3	1	1	0	0
bone010	986703	8846266758	2231282192	1282743704	1471105979	1571237233	1687120778
boneS10	914898	6345023025	3742417527	3823790163	3742417527	3890992689	3952972833
boneS01	127224	331330356	302253322	330301515	320120227	308162745	320695596
filter3D	106437	260719523	87199353	95192166	86376100	97975438	98931960
t3dh	79171	250243367	158470211	160944302	160795113	160693484	164226809
t3dl	20360	14726265	14081961	44263732	14146144	14654390	14754577
gyro	17361	14326901	4732829	7560520	5948631	5705185	5705185
t2dah	11445	1393324	929182	899122	900835	918636	918636
Nr. of best results (model reduction prob.)		0	6	4	2	2	2
nlpktr80	1062400	578579377	124771691	141597370	1274332818	1324967187	1261421630
boyd2	466316	60887715752	1849407	8174639	1786292	15770830	19761127
c-big	345241	37614204546	1993355106	2746808649	1993355106	2839984857	2746808649
cont-300	180895	8218011158	72234116	72006365	72225552	72006625	72006151
boyd1	93279	1508966	691874	8000658	691874	71949054	8000658
Number of best results (optimization problems)		0	3	0	3	0	1
HTC_336_4438	226340	11394862618	322174699	478529651	347706694	426725923	459751621
HTC_336_9129	226340	11394862618	276984385	334744635	286258585	373900499	401625744
TSOPF_FS_b39.c30	120216	5420403013	78613261	3187932662	78613261	3183278644	3187932662
TSOPF_FS_b300.c3	84414	2656188354	327833850	1658435988	327833850	1687329666	1658435988
TSOPF_FS_b39.c19	76216	2177883513	48864399	1278896293	48864399	1278307884	1278896293
Number of best results (power network problems)		0	5	0	3	0	0

Tabela 2: Profile results given by five heuristics applied to 24 symmetric matrices (continued from Table 1).

Matrix	n	profile ₀	MPG	NSloan	NMPG	Sloan	Sloan-MGPS
Plan_1565	1564794	17382449070	1619717833	2291626930	2795155807	3157801849	1452112402
af_shell10	1508065	3957707605	3618259697	3274691763	3274634535	3616987955	3616310505
Hook_1498	1498023	28125827298	1842983874	2250120654	1836709023	2543531980	2145278763
Geo_1438	1437960	27532332804	2595689927	2771466182	2589434394	2721717656	2110215523
Serena	1391349	77321314974	414882067	405923733	781955109	2654141420	414555183
Idoor	952203	127150863374	3335730024	3255104118	3356204302	3212710382	3301259943
audikw_1	943695	390218762952	2288601671	2931452981	2268039486	3062047799	3007224410
Emilia_923	923136	11536969011	2014136821	2052356578	1987745527	2059091505	1992318292
apache2	715176	1717106409	1430724488	1293455779	1353405262	1608315598	1530095066
Fault_639	638802	8421729249	1815975305	2062701059	1890058787	1914488060	1848341753
inline_1	503712	57452838418	961019105	1033619470	961019105	1070547018	1034142515
af_0_k101	503625	429524175	414879983	405939717	416879983	414597075	414599867
af_1_k101	503625	429524175	414851742	405952350	416851742	414502900	414705092
af_2_k101	503625	429524175	414808925	405952692	416808925	414698308	414593800
af_3_k101	503625	429524175	414808925	405938383	416808925	414599867	414564025
af_4_k101	503625	429524175	414808925	405953358	416808925	414599867	414651083
af_5_k101	503625	429524175	414882067	405923733	416882067	414551383	414555183
msdoor	415863	23550593343	629079463	669487465	630079463	667873655	684130050
bmw3_2	227362	1371669955	168955002	152372231	169955002	150186067	162287388
Number of best results (structural problems)		0	3	7	2	2	0
thermal2	1228045	53657335080	584643264	587246981	586063633	593981740	609179419
thermomech_dM	204316	10671293780	30825327	30939618	30796587	31984980	32276780
thermomech_TC	102158	2667823445	15412663	15446340	15346513	16021130	16109532
thermomech_TK	102158	2667823445	15504258	15493278	15398294	15906569	16109532
thermal1	82654	175625317	10403099	10226717	10263788	10677120	10561255
ted_B	10605	165915	83187	83232	83202	83232	83202
ted_B_unscaled	10605	165915	83187	83217	83202	83187	83202
Number of best results (thermal problems)		0	3	1	3	1	0
Ga41As41H72	268096	6188064837	718290905	764051495	723851369	1096938863	759438297
Si87H76	240369	5137473252	3364251744	3437223650	3365881775	3605125637	3424589623
Si41Ge41H72	185639	3333819775	2471261300	2574121434	2496961517	2676005370	2644947535
SiO2	155331	2482485963	1719601610	1746580017	1719326824	1787790031	1750423028
Ga19As19H42	133123	1899298868	1491076475	1553526817	1481688086	1594127792	1518400362
Nr.best res.(theor./quantum chemistry prob.)		0	3	0	2	0	0
Total number of best results		1	29	17	17	8	4
Average time, in seconds		—	1.9	0.8	1.9	0.7	2.5

Tabela 3: Profile results given by the Hu-Scott, MPG, and NSloan heuristics applied to 15 symmetric matrices (Profile·10⁶) (continued on Table 4).

Matrix	<i>n</i>	profile ₀	Hu-Scott	t(s)	MPG	t(s)	NSloan	t(s)
gas_sensor	66917	69	63	0.3	73	0.1	69	0.1
t3dh	79171	250	154	1.0	161	0.2	158	0.1
filder3D	106437	261	69	0.7	87	4.1	95	0.4
boneS01	127224	331	246	1.2	302	4.9	330	0.5
boneS10	914898	6345	2147	9.7	3742	35.3	3824	3.8
bone010	986703	8846	188	2.2	2231	38.1	1283	4.1
Nr. best results (model red. problems)			6		0		0	
thermomech_TC	102158	2668	13	0.6	15	0.4	15	0.2
thermomech_TK	102158	2668	13	0.4	16	0.4	15	0.2
thermomech_dM	204316	10671	29	0.7	31	0.3	31	0.4
thermal2	1228045	53657	577	4.5	585	3.3	587	2.4
Nr. best results (thermal problems)			4		0		0	
shallow_water1	81920	35	20	1.3	22	1.5	32	0.0
shallow_water2	81920	35	21	1.3	22	1.4	32	0.0
cf2	123440	156	136	0.7	139	0.9	156	0.3
parabolic_fem	525825	112866	230	2.0	247	0.9	225	0.4
StocF-1465	1465137	37108	9607	8.0	3521	18.3	2340	2.3
Nr. best results (CFD problems)			3		0		2	

Tables 3 shows that the Hu-Scott heuristic yielded the highest number of best profile results when applied to symmetric matrices originating from three application areas. On the other hand, Table 4 shows that the MPG heuristic delivered the best profile results when applied to the highest matrix (Flan_1565) used in this study. The same table shows that the NSloan heuristic yielded the best profile results when applied to six similar matrices (af_*_k101). Thus, Tables 3 and 4 show that this heuristic yielded a higher number of best profile results than the MPG heuristic.

Table 4 also shows that the NSloan and MPG heuristics yielded in general much better profile results than the Hu-Scott algorithm when applied to matrices with sizes larger than 600,000 originating from structural problems. Table 5 shows that the Hu-Scott algorithm yielded better profile results than the MPG heuristic when executed together with the Hager exchange method implemented in the routine MC67 contained in the HSL library [STFC, 2020].

4.2. Nonsymmetric matrices

In this section, we show the results yielded by six heuristics. These algorithms compute symmetric permutations to reduce the profile of sparse matrices.

Tables 6 and 7 show the results yielded by five heuristics for profile reduction when applied to 53 nonsymmetric matrices arising from 11 application areas. In particular, Tables 7 shows that the MPG, NMPG, NSloan, Sloan-MGPS, and Sloan's heuristics did not reduce the profile of five nonsymmetric matrices arising from subsequent semiconductor device problems. Thus, these five symmetric permutations did not reduce the profile of 18 out of 53 nonsymmetric matrices, which is more than the number of best results given by the MPG heuristic (15). Among the heuristics evaluated in this study, Tables 6 and 7 show that the MPG heuristic provided the highest number of best results when applied to these nonsymmetric matrices. Figure 2 shows the running times of five heuristics for profile reduction when executed to nonsymmetric matrices.

Tabela 4: Profile results given by the Hu-Scott, MPG, and NSloan heuristics applied to 23 symmetric matrices (Profile·10⁶) arising from structural problems (continued from Table 3).

Matrix	<i>n</i>	profile ₀	Hu-Scott	t(s)	MPG	t(s)	NSloan	t(s)
pwtk	217918	57453	160	2.0	961	5.1	1034	0.2
hood	220542	13717	136	2.0	148	2.7	203	1.0
msdoor	415863	23551	423	3.9	629	5.2	629	2.0
af_0.k101	503625	430	417	2.9	415	4.8	406	1.7
af_1.k101	503625	430	417	7.2	415	4.2	406	1.7
af_2.k101	503625	430	417	3.1	415	4.5	406	1.7
af_3.k101	503625	430	417	3.1	415	4.5	406	1.7
af_4.k101	503625	430	417	3.0	415	4.5	406	1.7
af_5.k101	503625	430	417	3.0	415	4.5	406	2.6
inline_1	503712	57453	622	7.3	961	4.0	1034	3.8
af_shell3	504855	802	519	3.2	625	3.1	544	1.8
af_shell4	504855	802	519	3.2	625	3.1	526	1.1
af_shell7	504855	802	519	3.2	625	3.1	526	1.1
af_shell8	504855	802	519	3.4	625	3.1	515	2.1
Fault_639	638802	8422	5321	6.1	1816	5.89	2063	2.8
apache2	715176	1717	1444	6.4	1431	1.04	1293	1.3
Emilia_923	923136	11537	9491	8.6	2014	150.0	2052	4.2
audikw_1	943695	390219	5277	28.1	2289	46.4	2931	8.0
ldoor	952203	127151	1320	9.5	3336	65.09	3255	4.4
Serena	1391349	77321	18914	27.4	782	112.2	3876	8.3
Geo_1438	1437960	27532	16049	15.3	2596	90.85	2771	6.2
Hook_1498	1498023	28126	9195	24.6	2250	55.7	2250	6.1
Flan_1565	1564794	17382	7703	21.7	1620	172.19	2292	11.7
Nr. best results			8		7		9	
Total number of best results			21		7		11	
Average time, in seconds			6.1		22.8		2.4	

Tabela 5: Profile results given by the Hu-Scott and MPG heuristics applied to eight symmetric matrices (Profile·10⁶) joined with the Hager exchange method implemented in the routine MC67 contained in the HSL library.

Matrix	<i>n</i>	Profile	Hu-Scott	t(s)	MPG	t(s)
2cubes_sphere	101492	483	155	4	169	3
filter3D	106437	261	66	4	80	2
Ge99H100	112985	1190	1145	5	1190	9
shipsec8	114919	251	181	4	251	1
cf2	123440	156	133	5	134	6
thermomech_dM	204316	10671	28	7	30	7
offshore	259789	3588	517	11	1136	19
F1	343791	621	593	14	621	22

Tabela 6: Profile results given by five heuristics applied to 29 nonsymmetric matrices (continued on Table 7).

Matrix	n	profile ₀	MPG	NMPG	NSloan	Sloan-MGPS	Sloan
torso3	259156	3166854687	1319048251	1294858355	1446016469	1613538415	1600385811
stomach	213360	403651704	240657271	241928646	246381678	253851996	250416003
torso1	116158	475768888	379215921	439723220	468047294	483159182	468047368
torso2	115967	210074270	53470327	81850382	86565077	58536210	60554533
Baumann	112211	246888420	166292121	166300600	166320306	166317654	166454073
Nr.best res. (2D/3D problems)							
Hamle3(18)	1447360	2208802446	2055266938	1958522669	2499026239	1037033682	2833388808
ASIC_680k	682862	1339065039	3469835779	3469835779	171591805	171591805	467751384
ASIC_680ks	682712	621713	877118264	755410159	3977811276	392245317	1287358127
rajat29	643994	517154092	3154628368	3154628369	3860373414	3860373414	2691226452
rajat30	643994	121144239	1478393660	1478393660	1679866002	1679866001	2206370469
Nr.best res. (circuit simulation prob.)							
atmosmodl	1489752	2080864658	1958837670	1959169422	1956319409	1956123892	1962734303
atmosmodd	1270432	3162347142	1202622067	1203092812	1201457966	1201438914	1207777104
atmosmodj	1270432	1581173571	2751378430	2751195784	2748275797	2748190852	2751382492
RM07R	381689	3484377407	688258538	1032387809	1067986237	868087051	814426024
PR02R	161070	1026993602	635764281	553428957	567504519	554626308	575265319
Nr.best res. (Comp. fluid dyn. prob.)							
webbase-1M	1000005	942841652	2345182369	2312687587	2612641690	1949407714	942841652
cagel4	1505785	916010091	3486287624	3669349816	2109403746	1412559411	2186185323
cagel3	445315	3199074122	3141414501	2432114271	3500491639	2101188413	3599590821
language	399130	143307476	997384007	1818092688	2024291249	2143032080	1623905024
patents. main	240547	1409853171	3921471586	3899412751	3022817490	2884695781	3666646979
Nr.best res. (dir. weighted graph prob.)							
tmt. unsym	917825	1683217172	1191784885	1191784885	1183917828	3830768863	1276439432
Zhao1	33861	676917778	27424927	34686567	26371310	27616451	26740082
Zhao2	33861	676917778	27585519	33803426	26379621	27606278	26602999
li	22695	39546992	77458503	71688498	78436238	110599384	95904285
Nr.best res. (electromagnetics prob.)							
fd18	16428	79655410	58984202	58914599	57201133	61292217	56536460
xenon2	157464	777021255	232559601	235851219	236480145	241137475	235498730
xenon1	48600	133164235	105032309	107146407	107842926	107592150	105397026
viscorocks	37762	12552322	12713469	12688992	12888365	12904486	12798003
viscoplastic2	32769	603714548	46831716	46831716	40613590	40578110	37219223
Nr.best res. (materials prob.)							
		1	2	0	0	0	2

Tabela 7: Profile results given by five heuristics applied to 24 nonsymmetric matrices (continued from Table 6).

Matrix	n	profile ₀	MPG	NMPG	NSloan	Sloan-MGPS	Sloan
ABACUS_shell.ud	23412	5558085	4899985	5456845	5396311	5685675	5623196
chipcool0	20082	23086070	22850400	22850400	24067869	24027242	24728570
chipcool1	20082	23086070	22890347	22846321	24019823	24108725	24632043
inlet	11730	1191711	1838948	2157212	1537574	1806431	1814808
Nr.best res.(model reduction prob.)		1	2	2	0	0	0
hvd2	189860	147474412	29036805	28088564	20273329	20186759	20752717
TSOPF_RS_b39_c30	60098	2255240782	943421624	943728490	912016158	912244066	912401694
TSOPF_RS_b300_c3	42138	1077257833	576490011	576490011	445692127	445759624	447257140
TSOPF_RS_b2383	38120	702397199	326451954	326451954	326892736	326892736	327177750
TSOPF_RS_b2383_c1	38120	702397199	326451954	326451954	326892736	326892736	327177750
Nr.best res.(power network prob.)		0	2	2	2	1	0
para-9	155924	528273907	1607549185	1628003464	1577446601	1564544258	1609460349
para-8	155924	528273907	1628003464	1587994871	1583886850	1594530645	1646187318
para-7	155924	528273907	1633667638	1630856381	1660200731	1540494748	1583490187
para-10	155924	528273907	1607978337	1553232688	1573419197	1641151648	1646187318
para-6	155924	528273907	1570114929	1567969744	1616012221	1558881038	1605302167
Nr.best res.(semicond. dev. prob.)		5	0	0	0	0	0
Chebyshev4	68121	210479796	161628247	159919266	151574916	148978456	138289995
sme3Dc	42930	1786758025	80703045	81123523	86643549	86549702	92227931
sme3Db	29067	818765013	52139125	52593082	57838972	63395714	66117252
Nr.best res.(structural problems)		0	2	0	0	0	1
thermomech.dK	204316	1380383359	133138507	131764766	132935260	133792179	133563385
FEM_3D_thermal2	147900	482861538	311660469	324827176	294723853	311044562	316091402
epb3	84617	12144048	40267389	34798244	35736070	41339021	39958549
epb2	25228	10189084	14610049	14610049	14001473	14610791	14791475
FEM_3D_thermal1	17880	22890642	9033871	8992397	9268926	9158524	9120990
ted_A	10605	56534841	1326994	1329065	1397705	1398245	1404542
ted_A_unscaled	10605	56534841	1328029	1328029	1397975	1397751	1403907
Nr.best res.(thermal problems)		2	2	3	1	0	0
Total number of best results		18	15	9	7	6	4

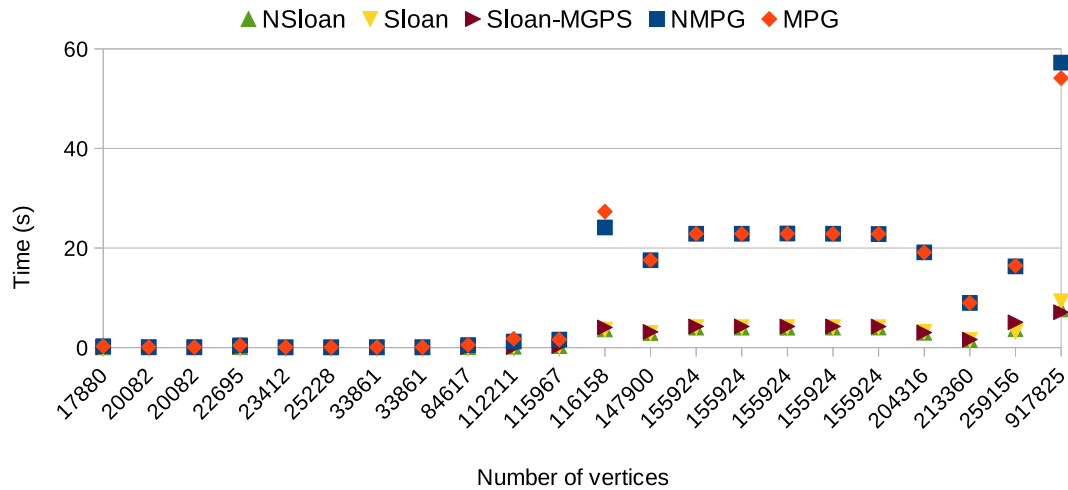


Figura 2: Execution times (in seconds) required by five heuristics for profile reductions when applied to several nonsymmetric matrices.

The Hu-Scott was designed for matrices with symmetric sparsity pattern [STFC, 2020]. Furthermore, a previous publication showed that the MPG heuristic provided in general better profile results than the Hu-Scott heuristic [Gonzaga de Oliveira et al., 2019a]. Table 8 partially reproduces these results.

Tabela 8: Profile results given by the MPG and Hu-Scott heuristics applied to eight nonsymmetric matrices.

Matrix	n	$ E $	profile ₀	MPG	t(s)	Hu-Scott	t(s)
FA	10617	72176	34212487	12297127	0.12	14536286	0.35
coupled	11341	97193	35458355	1905438	0.02	1493472	0.06
cage10	11397	150645	14938283	8978703	0.06	9316795	0.05
fd15	11532	44206	26738656	15582864	0.13	14399331	0.12
sinc15	11532	551184	30494313	18327088	0.13	21726329	0.40
circuit_3	12127	48137	1015101	298785	0.11	811345	0.12
appu	14000	1853104	96358011	91330379	5.70	91649005	0.77
cage11	39082	559722	802478040	70618331	0.40	519218965	3.35
Nr. of best results	—	—	—	6	5	2	3

5. Conclusions

This computational experiment evaluated six existing low-cost heuristics for profile reduction (Sloan's, MPG, NSloan, Sloan-MGPS, Hu-Scott, and NMPG heuristics). These heuristics compute symmetric permutations to reduce the profile of sparse matrices.

The Hu-Scott heuristic is a multilevel Sloan algorithm. This study used the routine MC73 contained in the HLS library [STFC, 2020]. The results showed that this algorithm yielded the highest number of best results in experiments performed on symmetric matrices, including when executed in tandem with the Hager exchange method implemented in the routine MC67 contained in the same library. The results were different when using nonsymmetric matrices. The heuristics evaluated did not reduce the profile of 18 out of 53 nonsymmetric matrices used. Among the heuristics appraised, consistent with the current literature [Gonzaga de Oliveira et al., 2018], the

MPG heuristic yielded the highest number of best profile results. This heuristic delivered the best profile results when applied to 15 nonsymmetric matrices.

The next step in this work is to evaluate low-cost heuristics for profile reductions implemented using parallel libraries (e.g., OpenMP, Galois, and Message Passing Interface systems) and in GPU-accelerated computing. Similarly, regarding massively parallel computing, an evaluation of these heuristics implemented within the Intel[®] Math Kernel Library running on Intel[®] Xeon[®] Scalable processors is another future step of this investigation.

Referências

- Davis, T. A. e Hu, Y. (2011). The University of Florida sparse matrix collection. *ACM Transactions on Mathematical Software*, 38(1):1–25.
- Felippa, C. A. (1975). Solution of linear equations with skyline-stored symmetric matrix. *Computers and Structures*, 5(1):13–29.
- Gonzaga de Oliveira, Silva, L. M., e Abreu, A. A. A. M. (2019a). Efeitos de permutações simétricas no tempo de execução de um método direto para resolução de sistemas de equações lineares esparsos e assimétricos. In *Proc. of the LI Brazilian Symp. Operations Research (SBPO)*, Limeira.
- Gonzaga de Oliveira, S. L., Bernardes, J. A. B., e Chagas, G. O. (2018). An evaluation of low-cost heuristics for matrix bandwidth and profile reductions. *Comput. Appl. Math.*, 37(2):1412–1471.
- Gonzaga de Oliveira, S. L. e Chagas, G. O. (2014). *Introdução a heurísticas para redução de largura de banda de matrizes*. Sociedade Brasileira de Matemática Aplicada e Computacional, São Carlos-SP. Notas em Matemática Aplicada.
- Gonzaga de Oliveira, S. L., Osthoff, C., e Henderson, N. (2019b). An Experimental Analysis of Heuristics for Profile Reduction. In et al., M. S., editor, *Proceedings of 19th International Conference on Computational Science and Its Applications, ICCSA. LNCS*, volume 11619, p. 25–36, Saint Petersburg, Russia. Springer International Publishing.
- Hu, Y. e Scott, J. A. (2001). A multilevel algorithm for wavefront reduction. *SIAM Journal on Scientific Computing*, 23(4):1352–1375.
- Kumfert, G. e Pothen, A. (1997). Two improved algorithms for envelope and wavefront reduction. *BIT Numerical Mathematics*, 37(3):559–590.
- Lin, Y. X. e Yuan, J. J. (1994). Profile minimization problem for matrices and graphs. *Acta Mathematicae Applicatae Sinica*, 10(1):107–122.
- Medeiros, S. R. P., Pimenta, P. M., e Goldenberg, P. (1993). Algorithm for profile and wavefront reduction of sparse matrices with a symmetric structure. *Eng. Comput.*, 10(3):257–266.
- Reid, J. K. e Scott, J. A. (1999). Ordering symmetric sparse matrices for small profile and wavefront. *International Journal for Numerical Methods in Engineering*, 45(12):1737–1755.
- Sloan, S. W. (1989). A Fortran program for profile and wavefront reduction. *International Journal for Numerical Methods in Engineering*, 28(11):2651–2679.
- STFC (2020). The Science and Technology Facilities Council. HSL. A collection of Fortran codes for large scale scientific computation. <http://www.hsl.rl.ac.uk>. Accessed: April 2020.