A model-checking based approach: Diagnosability Verification^{*}

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Abstract: In this work we propose a model checking approach to deal with the problem of the diagnosability verification. In this approach we consider a transition system (TS) which models the normal and the faulty behavior of the system. We propose a method to test the diagnosability property by using a linear temporal logic (LTL) formula. If a transition system does not satisfy our proposed LTL-formula, then it is not diagnosable. Our approach can be carry out in SPIN which is a tool used for formal verification of models. One advantage of SPIN is that it can handle a large state space which can be useful for diagnosability verification of complex system. Finally we present a simple example to illustrate our approach.

Keywords: Fault Diagnosis, Model-Checking, Transition Systems, Linear Temporal Logic, Discrete Event Systems.

1. INTRODUCTION

Failures are inherent to any real system, to ensure the safety of operation they should be diagnosed as soon as possible to avoid permanent damage to the system, low quality productions or even personal injuries.

It have been extensively applied in Continuous Variable Systems (CVS), and Discrete event Systems (DES) domains. In the context of CVS, the diagnoser infer the system conditions based on actual measurements of its variables and their expected value based on simulated models. In the later, the diagnoser infer the system condition based on observation of events.

However just a few works tries to explore this concept in a model checking paradigm Jiang and Kumar (2004, 2006); Cassez (2012). In Jiang and Kumar (2004) the system is modeled as a Transition System and the normal behavior is expressed as a linear temporal logic (LTL) formula. Failures are treated as violations of the normal behavior and are considered as permanent. In Jiang and Kumar (2006), it is extended to the case of repeated failures, it means, failures that can occur more than once. One of the main disadvantages of these approaches is due to the complexity of the construction of the Buchi Automata that recognizes the LTL formula, and despite the fact that the approach can be quite general in describing faults, it is difficult to express the normal behavior with LTL for complex systems.

Model checking is an automated technique that, given a finite-state model of a system and a formal property,

systematically checks whether this property holds for that model (Baier and Katoen, 2008). The main idea is to examines all possible system scenarios in a systematic manner, in order to show that a given system model truly satisfies a certain property. SPIN and NUSMV which are tools used for formal verification of models and can handle state spaces up to 10^{476} states for specific problems. Due to this fact, it has been successfully applied in several interesting study cases, as the analysis of IEEE standards for in-house communication of domestic appliances, identification of errors in an execution module of the Deep Space 1 spacecraft controller and others.

In this work we propose a model checking approach to deal with the problem of the diagnosability verification. In this approach we consider a transition system which models the normal and the faulty behavior of the system. We propose a method to test the diagnosability property by using an LTL formula. Here, in contrast with Jiang and Kumar (2004), we consider a unique LTL formula. If a transition system does not satisfy LTL-formula, then it is not diagnosable. Our approach can be carry out in SPIN being suitable to handle complex system. Finally we present a simple example to illustrate of our approach.

2. MODEL CHECKING PRELIMINARIES

In this section we present the necessary concepts of model checking for our proposed approach, the reader should refer to Clarke et al. (2018); Baier and Katoen (2008) for more details.

^{*} This work is financed in part by CNPq, CAPES and FAPERJ.

2.1 Transition Systems

A transition system TS is a tuple $(S, Act, \rightarrow, I, AP, L)$, where S is a set of states, Act is a set of actions, $\rightarrow \subseteq S \times Act \times S$ is a transition relation, $I \subseteq S$ is a set of initial states, AP is a set of atomic propositions, and $L : \rightarrow 2^{AP}$.

For $s \in S$ and $\alpha \in Act$ the set of direct α - *successors* of s is defined as: $Post(s, \alpha) = \{s' \in S \mid s \xrightarrow{\alpha} s'\}$, $Post(s) = \bigcup_{\alpha} Post(s, \alpha)$.

A $TS = (S, Act, \rightarrow, I, AP, L)$ is *action - deterministic* if $|I| \leq 1$ and $|Post(s, \alpha)| \leq 1$ for all $s \in S$ and $\alpha \in Act$.

A *finite* execution fragment ρ of a TS is an alternating sequence of states and actions ending with a state $\rho = s_0\alpha_1s_1\alpha_2 \dots \alpha_n s_n$ such that $s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$ for all $0 \leq i \leq n$.

A *infinite* execution fragment ρ of a TS is an alternating sequence of states and actions ending with a state $\rho = s_0\alpha_1s_1\alpha_2s_2\alpha_3 \dots$ such that $s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$ for all $0 \leq i$.

Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP_i, L_i)$, $i = 1, 2$ be transition systems and $H \subseteq Act_1 \cap Act_2$ with $\tau \notin H$. The transition system $TS_1 \parallel_H TS_2$ is defined as: $TS_1 \parallel_H TS_2 = (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, I_1 \times I_2, AP_1 \cup AP_2, L)$, where $L((s_1, s_2)) = L_1(s_1) \cup L_2(s_2)$, and where the transition relation $\rightarrow$ is defined as: (Baier and Katoen (2008) chapter 2).

- if $\alpha \notin H$ and $s_1 \xrightarrow{\alpha} s'_1$ then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s'_1, s_2 \rangle$
- if $\alpha \notin H$ and $s_2 \xrightarrow{\alpha} s'_2$ then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s'_2 \rangle$
- if $\alpha \in H$ and $s_1 \xrightarrow{\alpha} s'_1$ and $s_2 \xrightarrow{\alpha} s'_2$ then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s'_1, s'_2 \rangle$

The operator $\parallel_H$ defines the handshaking between two transition systems.

A finite path fragment $\hat{\pi}$ of TS is a finite state sequence $s_0s_1 \dots s_n$ such that $s_i \in Post(s_{i-1})$ for all $0 < i \leq n$, where $n \geq 0$. An infinite path fragment π is an infinite state sequence $s_0s_1s_2 \dots$ such that $s_i \in Post(s_{i-1})$ for all $i > 0$.

The following notations is adopted for infinite path fragment $\pi = s_0s_1 \dots$. For $j \geq 0$, let $\pi[j] = s_j$ denote the j th state of π and $\pi[\cdot \dots j] = s_0s_1 \dots s_j$ denotes the j th prefix of π . Similarly $\pi[j \dots] = s_js_{j+1} \dots$ denotes the j th suffix of π . These notions are defined analogously for finite paths. Besides, for finite path $\hat{\pi} = s_0s_1 \dots s_n$, let $last(\hat{\pi}) = s_n$ denote the last state of $\hat{\pi}$, and $len(\hat{\pi}) = n$ denote the length of $\hat{\pi}$.

A *maximal* path fragment is either a finite path fragment that ends in a terminal state, or an infinite path fragment. A path fragment is called *initial* if it starts in an initial state $s_0 \in I$. A path of transition system TS is an initial, maximal path fragment.

Let $Path(TS)$ denote the set of all paths in TS , and $Path_{fin}(TS)$ the set of all initial, finite path fragments of TS .

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system without terminal states. The trace of the infinite path fragment $\pi = s_0s_1 \dots$ is defined as $trace(\pi) = L(s_0)L(s_1) \dots$. $\mathcal{L}_\omega(\mathcal{A}) = \{\sigma \in \Sigma^\omega \mid \text{there exists an accepting run for } \sigma \text{ in } \mathcal{A}\}$.

The trace of the finite path fragment $\hat{\pi} = s_0s_1 \dots s_n$ is defined as $trace(\hat{\pi}) = L(s_0)L(s_1) \dots L(s_n)$.

We also propose the following definitions: Let $Runs(TS)$ denote the set of all runs of TS , and $Runs_{inf}$ the set of all infinite runs of TS . The string of a run $\rho = s_0\alpha_1s_1\alpha_2s_2 \dots$ is defined as $string(\rho) = \alpha_1\alpha_2 \dots$. The projection of a string $s = \beta_1\beta_2 \dots \alpha_1 \dots \alpha_2 \dots \alpha_n \dots$, with respect a set of observed actions Act_o is defined as $P_o(s) = \alpha_1\alpha_2 \dots \alpha_n$, where $\beta_i \notin Act_o$ and $\alpha_i \in Act_o$ for all $i \geq 1$. The projection erases all non observable actions of a given string.

Inspired from Cassandras and Lafortune (2008), we define the unobservable reach for a transition system $TS = (S, Act, \rightarrow, I, AP, L)$ with respect to a set of observable actions $Act_o \subseteq Act$ as:

$$UR(s) = \{p \in S : \exists \rho = s\alpha_1s_1\alpha_2s_2 \dots p$$

$$[\forall i > 0, \alpha_i \in Act_{uo} \text{ and } s_i \xrightarrow{\alpha_i} s_{i+1} \in \rightarrow]\} \cup \{s\}$$

In words the unobservable reach $UR(s)$, of each state $s \in S$ in a partially observed transition system is the set of all reachable states from s with a succession of transitions with unobservable actions plus the state s itself.

This definition can be extended to a set of states $P \subseteq S$ by $UR(P) = \bigcup_{p \in P} UR(p)$.

With these definitions we can define the observer of a Transition System $TS = (S, \rightarrow, Act, s_0, AP, L)$ with respect a set of observable actions $Act_o \subseteq Act$ as another Transition System $TS_o = (S_o, \rightarrow_o, Act_o, s_{o0}, \{\}, \{\})$ which is computed using algorithm 13, which is adapted from Cassandras and Lafortune (2008). Basically TS_o models all runs of TS that are observable with Act_o .

Algorithm 1: Computation of Observer

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1 function OBS( $TS = \{S, \rightarrow, Act, \{s_0\}, AP, L\}, Act_o$ )
2    $s_{o0} \leftarrow UR(s_0)$ 
3    $S_o \leftarrow s_{o0}$ 
4    $S'_o \leftarrow \emptyset$ 
5   while  $S'_o \subset S_o$  do
6      $S'_o \leftarrow S_o$ 
7     forall  $P \in S_o \setminus S'_o \wedge \alpha \in Act_o$  do
8       Let  $s_\alpha \in P$  be a state in  $P$ 
9       if exists an state  $q \in S$  such that  $s_\alpha \xrightarrow{\alpha} q \in \rightarrow$ 
10        then
11           $P' \leftarrow UR(\{s \in S : s_\alpha \xrightarrow{\alpha} s \in \rightarrow\})$ 
12           $S_o \leftarrow S_o \cup P'$ 
13          Add  $P \xrightarrow{\alpha} P'$  to  $\rightarrow_o$ 
14   return  $TS_o = (S_o, \rightarrow_o, Act_o, \{s_{o0}\}, \emptyset, \emptyset)$ 

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2.2 Nondeterministic Büchi Automaton

A nondeterministic Büchi automaton (NBA) is a tuple $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ where, Q is a finite set of States, Σ is an alphabet, $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function, $Q_0 \subseteq Q$ is a set of initial states, and $F \subseteq Q$ is a set of accept (or: final) states, called the acceptance set.

A run $\sigma = A_0A_1A_2 \dots \in \Sigma^\omega$ denotes an infinite sequence $q_0q_1q_2 \dots$ of states in $\mathcal{A}$ such that $q_0 \in Q_0$ and $q_i \xrightarrow{A_{i+1}} q_{i+1}$ for $i \geq 0$. Run $q_0q_1q_2 \dots$ is accepting if $q_i \in F$ for infinitely many index $i \in \mathbb{N}$. The accepted language of $\mathcal{A}$ is $\mathcal{L}_\omega(\mathcal{A}) = \{\sigma \in \Sigma^\omega \mid \text{there exists an accepting run for } \sigma \text{ in } \mathcal{A}\}$.

The size of $\mathcal{A}$ is denoted $|\mathcal{A}| = |Q| + \sum_{q \in Q} \sum_{A \in \Sigma} |\delta(q, A)|$, is defined as the number of states and transitions in $\mathcal{A}$

The transition function δ with the induced transition relation $\rightarrow \subseteq Q \times \Sigma \times Q$ which is given by

$$q \xrightarrow{A} p \text{ iff } p \in \delta(q, A)$$

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system without terminal states and $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ a nonblocking NBA. Then, $TS \otimes \mathcal{A}$ is the following transition system:

$$TS \otimes \mathcal{A} = (S \times Q, Act', \rightarrow', I', AP', L')$$

where $\rightarrow'$ is the smallest relation defined by the rule

- if $s \xrightarrow{\alpha} t$ and $q \xrightarrow{L(t)} p$ then $\langle s, q \rangle \xrightarrow{\alpha'} \langle t, p \rangle$

and where

- $I' = \{\langle s_0, q \rangle \mid s_0 \in I, \exists q_0 \in Q_0 : q_0 \xrightarrow{L(s_0)} q\}$
- $AP' = Q$ and $L' : S \times Q \rightarrow 2^Q$ is given by $L'(\langle s, q \rangle) = \{q\}$

2.3 Linear Temporal Logic

The syntax of LTL is formed according to the following rules:

$$\varphi ::= \text{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg \varphi \mid \bigcirc \varphi \mid \varphi_1 U \varphi_2$$

where $a \in AP$.

The until operator allows to derive the temporal modalities $\Diamond$ (“eventually”, sometimes in the future) and $\Box$ (“always”, from now on forever) as $\Diamond \varphi \equiv \text{true} U \varphi$ and $\Box \varphi \equiv \neg \Diamond \neg \varphi$. Intuitively, $\Diamond \varphi$ ensures that φ will be true eventually in the future, while $\Box \varphi$ is satisfied iff it is not the case that eventually $\neg \varphi$ holds, equivalently φ should hold from now on to forever.

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transitions system without terminal states, and let φ be an LTL-formula over AP. For infinite path fragment π of TS , the satisfaction relation is defined by $\pi \models \varphi$ iff $\text{trace}(\pi) \models \varphi$. For state $s \in S$, the satisfaction relation $\models$ is defined by $s \models \varphi$ iff $(\forall \pi \in \text{Paths}(s), \pi \models \varphi)$. TS satisfies φ , denoted by $TS \models \varphi$, if $\text{Traces}(TS) \subseteq \text{Words}(\varphi)$.

2.4 LTL Model Checking

An LTL model-checking algorithm is a decision procedure which for a transition system TS and a LTL formula φ returns the answer “yes” if $TS \models \varphi$, and “no” (plus a counterexample) if $TS \not\models \varphi$. The counterexample consists of an appropriate finite prefix of an infinite path in TS where φ does not hold.

In the automata based model-checking approach proposed in Vardi and Wolper (1986), TS is assumed to be finite and to have no terminal states. This approach is based on the fact that each LTL formula φ can be represented by a nondeterministic Büchi automaton (NBA). The idea is to disprove $TS \models \varphi$ by searching for a path π in TS with $\pi \models \neg \varphi$. If such a path is found, a prefix of π is returned as an error trace. If no such path is encountered, it is concluded that $TS \models \varphi$.

The algorithm relies on the fact that $TS \models \varphi$ iff $\text{Traces}(TS) \subseteq \text{Words}(\varphi)$, or equivalently iff $\text{Traces}(TS) \cap \text{Words}(\neg \varphi) = \emptyset$.

Hence, for NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\neg \varphi)$ we have $TS \models \varphi$ iff $\text{Traces}(TS) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$. Thus, to check whether φ holds for TS one first constructs an NBA for the negation of the input formula φ and then checks for the intersection emptiness of $\text{Traces}(TS) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$.

3. MODEL CHECKING FRAMEWORK FOR DIAGNOSABILITY

In our framework we consider that the system failures are modeled as actions and we use the atomic proposition F to indicate that a failure has occurred in the system.

The Diagnosability concept proposed in Sampath et al. (1995) presents the required conditions for the construction of a diagnosis module called diagnoser, which is translated to the terminology of TS as below.

Definition 1. Let $TS = (S, Act, \rightarrow, AP, L)$ be the transition system that model the normal and faulty behavior of the plant, with observable actions $Act_o \subset Act$, and fault actions $Act_f \subset Act \setminus Act_o$. Then, TS is diagnosable with respect to Act_f and Act_o if and only if

$$\begin{aligned} (\forall \varrho_1 \in \text{Runs}_{inf}(TS) : \exists s_i \xrightarrow{\alpha_{i+1}} s_{i+1}, \alpha_{i+1} \in Act_f) \Rightarrow \\ (\forall \varrho_2 \in \text{Runs}(TS) : \nexists p_i \xrightarrow{\beta_{i+1}} p_{i+1}, \beta_{i+1} \in Act_f) \\ (P_o(\text{string}(\varrho_1)) \neq P_o(\text{string}(\varrho_2))) \end{aligned}$$

In words, the plant is diagnosable w.r.t to Act_f and Act_o , if and only if for all arbitrary faulty long run ϱ_1 of TS , and all normal runs ϱ_2 of TS , $(P_o(\text{string}(\varrho_1)) \neq P_o(\text{string}(\varrho_2)))$, it means that there is no uncertainty between a possible faulty run and a possible normal run of TS .

For verification of diagnosability of systems we split the sytem behavior in two parts, the normal behavior and the faulty behavior as below.

Definition 2. The Normal behavior of a transition system $TS = (S, Act, \rightarrow, \{s_0\}, AP, L)$ with respect a set of failure actions Act_f is a transition system:

$$TS_N = (S', Act \setminus Act_f, \rightarrow', \{s_0\}, AP = \emptyset, L = \emptyset)$$

where,

- $S' = \{p \in S : \exists \varrho = s_0 \alpha_1 s_1 \alpha_2 s_2 \dots p \mid \forall i > 0, \alpha_i \in Act \setminus Act_f \text{ and } s_i \xrightarrow{\alpha_{i+1}} s_{i+1} \in \rightarrow\} \cup \{s_0\}$;
- $\rightarrow'$ is the smallest relation defined by the rule:
• If $s \xrightarrow{\alpha} p \in \rightarrow$ and $\alpha \in Act \setminus Act_f$ then: $s \xrightarrow{\alpha} p \in \rightarrow'$

By definition, TS_N models all the possible non faulty behaviors of TS . The idea behind the construction of TS_N , is to perform a reachability analysis in TS , whose transitions with actions $\alpha \in Act_f$ are not considered.

Definition 3. The Faulty behavior of a transition system $TS = (S, Act, \rightarrow, \{s_0\}, AP, L)$ with respect a set of failure actions Act_f is a transition system:

$$TS_F = (S', Act, \rightarrow', \{s_0\}, AP = \{F\}, L)$$

where,

- $S' = \{p \in S : \exists \varrho = s_0 \alpha_1 s_1 \alpha_2 s_2 \dots p \mid \forall i \geq 0, \alpha_{i+1} \in Act \text{ and } s_i \xrightarrow{\alpha_{i+1}} s_{i+1} \in \rightarrow \text{ and } \exists i > 0, \alpha_{i+1} \in Act_f\}$;

- $\rightarrow'$ is the smallest relation defined by the rule:
· If $s \xrightarrow{\alpha} p \in \rightarrow$ and $s, p \in S'$ then: $s \xrightarrow{\alpha} p \in \rightarrow'$

By definition, TS_F models all the possible faulty behaviors of TS . The idea behind the construction of TS_F , is to perform a reachability analysis in TS , recording all runs of TS that has at least one transition with an action $\alpha \in Act_f$.

We also define TS_{NObs}^{compl} , which is a transition system that models all the observable runs that goes out of the normal behavior of the system. We attribute an atomic proposition D to indicate that a run has escaped from the observable normal behavior of the system, in other words, it indicates that we identified that a fault has occurred.

Algorithm 2 is used to compute TS_F and TS_{NObs}^{compl} . At lines 2 and 3 we compute TS_N from TS and Act_f and TS_F from TS and Act_f . At line 4 we compute TS_{NObs} , the observer of TS_N . At line 5 we complete TS_{NObs} with a dummy state Al which is a state that indicates that we detected an occurrence of a failure. At line 6, we attribute an atomic proposition D to state Al . Finally, at line 7 we rename TS_{NObs} to TS_{NObs}^{compl} .

Algorithm 2: Construction of TS_F and TS_{NObs}^{compl}

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1 function Build – Verif( $TS, ACT_o, Act_f$ )
2   Compute  $TS_N$  from  $TS$  and  $Act_f$ 
3   Compute  $TS_F$  from  $TS$  and  $Act_f$ 
4   Compute  $TS_{NObs} = OBS(TS_N, Act_o)$ 
5   Complete  $TS_{NObs}$  with dummy state  $Al$ 
6   Define  $L(Al) = D$  and  $AP = \{D\}$  for  $TS_{NObs}$ 
7    $TS_{NObs}^{compl} \leftarrow TS_{NObs}$ 
8   return  $TS_F, TS_{NObs}^{compl}$ 

```

Example 1. In order to illustrate the application the method presented in this section, we present the example illustrated in figure 1.

Figure 1. Example of a non diagnosable system

In this example we consider the verification of the diagnosability of TS with the set of observations $Act_o = \{a, b\}$, to diagnose failure σ_f . TS_N , TS_{NObs} and the proposed diagnosis module TS_{NObs}^{compl} are illustrated in figure 2. TS_F is illustrated in figure 3.

Figure 2. Construction of the diagnosis module

Figure 3. TS that models the faulty behavior of the system.

The interaction between TS_F and TS_{NObs}^{compl} will be further discussed and used to derive our model checking approach to verify the diagnosability of TS in the next section.

4. DIAGNOSABILITY VERIFICATION VIA MODEL CHECKING

The diagnosability of a system is the necessary and sufficient condition to design a diagnosis module that complies to the following two properties:

- (1) The module should never issue false alarms;
- (2) If a fault happen in the system then the module should detect it in the future.

Which can be easily expressed as LTL formula as below:

- (1) $\Box(\neg F \rightarrow \neg D)$
- (2) $\Box(F \rightarrow \Diamond D)$

where, F and D are atomic propositions that indicate that a fault has occurred and that the system has diagnosed it, respectively.

For a given TS which has the normal and faulty behavior of the system, it's always possible to construct a diagnosis module that attends the first requirement by construction, since we consider that we know a priori all the possible normal and faulty runs. Informally, the module should issues an alarm if and only if it detects runs that are not feasible in normal conditions of the system.

In our approach like in standard ones for diagnosis of DES, the TS_F is responsible to generate actions (events), the TS_{NObs}^{compl} only observes the sequences generated by the TS_F without interfering with its behavior. In order to model this kind of interaction, we use the handshaking product as presented in section 2, computing $TS_V = TS_F \parallel_H Diag$, where $H = Act_o$. The idea is that observable actions of the plant are executed synchronously in both transition systems, while unobservable actions of occur freely in TS_F . Note that in TS_{NObs}^{compl} , observable actions are always enabled, thus, there are no blockage of TS_F actions.

The last step is to check if $TS_V \models \Box(F \rightarrow \Diamond D)$ using model checking, if positive the plant is diagnosable, otherwise the plant is not diagnosable, note that in our approach the diagnosability is tested in a indirect way. We apply the model checking technique presented in section 2 to try disprove $TS_V \models \Box(F \rightarrow \Diamond D)$, for this we compute the Buchi automaton for the negation of $\varphi = \Box(F \rightarrow \Diamond D)$, as shown in figure 4.

Algorithm 3 is used to perform our model checking approach. At line 2 we compute $TS_V = TS_F \parallel_{Act_o} TS_{NObs}^{compl}$ which models the interaction of the observed normal behavior with the faulty behavior of the plant. At line

Figure 4. Buchi Automaton for $\Diamond(F \wedge \Box \neg D)$

3 to 8 we apply the model checking technique presented in section 2 to try disprove $TS_V \models \Box(F \rightarrow \Diamond(D))$, if the LTL formula is disproved we output a counter example and inform that the system is not diagnosable, otherwise the system is diagnosable.

Algorithm 3: Diagnosability Model-Checking

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1 function DIAG( $TS_F, DM$ )
2   Compute  $TS_V = TS_F \parallel_{Act_o} TS_{N_{Obs}}^{compl}$ 
3   Construct NBA  $\mathcal{A}$  for  $\neg \Box(F \Rightarrow \Diamond D)$ 
4   Compute the product  $TS_V \otimes \mathcal{A}$ 
5   if exists a infinitely long path  $\pi$  accepted in  $TS_V \otimes \mathcal{A}$  then
6     return "The system is not diagnosable" and provide
       counter example  $\pi$ 
7   else
8     return "The system is diagnosable"

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The following theorem states the correctness of our method.

Theorem 1. Let TS_V be the transition system that model the interaction of the faulty behavior of the plant TS with the proposed diagnosis module $TS_{N_{Obs}}^{compl}$ and $\varphi = \Box(F \rightarrow \Diamond D)$. Then, TS is diagnosable w.r.t to Act_o and Act_f if and only if $TS_V \models \varphi$

We briefly discuss the complexity of our algorithm, the complexity to compute TS_N and TS_F is linear in the size of TS , the complexity to compute $OBS(TS_N, Act_o)$ is $2^{|S|}$, it occurs due to the subset construction involved in the agregation of states in the unobservable reach of states of TS_N , the complexity to compute $Diag$ is linear in the size of $OBS(TS_N, Act_o)$. The handshaking product TS_V is linear in the size of TS_F , note that the handshaking only aggregates an atomic proposition D to each state of TS_F . The complexity of construction of NBA $\mathcal{A}$ is fixed since the number of until operators is fixed. The cost to perform model checking then is linear in the size of TS_V . Then the worst case complexity of our proposed model checking is $2^{|S|}$, which is the complexity of computation of the observer. Note however that, in practical situations the size of the observer is quite smaller than the worst case scenario.

Example 2. In order to illustrate the application of our model checking approach to verify the diagnosability of systems, we apply algorithm 3 in Example 1 presented before.

The Handshaking $TS_V = TS_F \parallel_H DM$, with $H = Act_o$, that models the interaction between the faulty runs of the plant TS_F and the observed normal behavior of the plant $TS_{N_{Obs}}^{compl}$ is illustrated in figure 5.

The next step is the verification if $TS_V \models \Box(F \rightarrow \Diamond D)$, the construction of the Buchi automata $\mathcal{A}$ for $\neg \varphi$ is shown in figure 4 and the product $TS_V \otimes \mathcal{A}$ is shown in figure 6. It is easy to check that the infinitely long path $\pi = (1, 1, 1) (3, \{3, 10\}, \{1, 2\}) (3, \{3, 10\}, \{1, 2\}) (5, 10, \{1, 2\}) (6, 10, \{1, 2\}) (8, 10, \{1, 2\}) (8, 10, \{1, 2\}) \dots$ is accepted in

Figure 5. Handshaking interaction between the plant and the diagnosis module

$TS_V \otimes \mathcal{A}$, with the respecting string $a\sigma_f b c a \sigma_u^\omega$, thus, the system is not diagnosable w.r.t Act_f and Act_o .

Figure 6. Product of TS_V and Buchi automaton $\mathcal{A}$

In the next section we will illustrate a method to implement algorithm 3 in the SPIN model checking tool.

5. SPIN IMPLEMENTATION

In this section we present our implementation method of our algorithm in SPIN to check if $TS_V \models \Box(F \rightarrow \Diamond D)$ by means of an application example as the one presented in section 4. SPIN uses a proprietary programming language called Promela (Process meta language) that allows the creation of models of concurrent processes, for more information regarding this programming language, the reader should refer to the Promela reference manual at <http://spinroot.com/spin/Man/Intro.html>.

It's important to note that in SPIN the interaction between processes can be defined trough channels in a synchronous or asynchronous way. In our approach we define the communications in a synchronous way, it means that once the plant issues an action, the diagnosis module will listen to it without delay. In this context we define that TS_F is responsible to generate and transmits actions trough a channel and $TS_{N_{Obs}}^{compl}$ listen and reacts to the actions received via the same channel.

In our implementation framework, we define Actions as an *mttype* declaration which allows the introduction of symbolic names for constant values, atomic propositions F and D are declared as Boolean variables. TS will be declared as *proctype* constructs with the prefix *active* that indicates that this process is automatically started at run time. The synchronous communication channel between TS_F and $TS_{N_{Obs}}^{compl}$ is declared with the command `chan F2D = [0] of mtype`. The LTL formula $\varphi = \Box(F \rightarrow \Diamond D)$ is expressed with the command `ltl L1 \Box(F -> <> D)`;

The idea of our implementation is, given TS_F , $TS_{N_{Obs}}^{compl}$, and the LTL formula $\varphi = \Box(F \rightarrow \Diamond D)$, the SPIN internally computes TS_V and verify if $TS_V \models \Box(F \rightarrow \Diamond D)$, if $TS_V \not\models \Box(F \rightarrow \Diamond D)$ then SPIN generates

an counterexample for verification purposes. The Buchi automaton $\mathcal{A}$ and the product $TS_V \otimes \mathcal{A}$ are also computed internally on SPIN.

One of the advantages of this approach is that SPIN does not explicitly constructs the entire state space of TS_V , it construct it *on the fly* during verification. If a counterexample is found the verification is stopped, another advantage is that as mentioned before, SPIN can handle system with up to 10^{476} states. Part of the Promela code used for declaration of the system presented in example 1 is illustrated below, showing the definition of the actions, the atomic propositions, the synchronous communication channel, the LTL formula and part of TS_F . We define two processes, active proctype TSF and active proctype DM, and we define LTL formula L1 for verification of diagnosability. For illustration purposes we assume that the code is saved in *Exemplo1.pml* file. The SPIN files for this example can be found at https://github.com/TuxiThiago/SPIN_FDIAGNOSIS.

```
mtype = {a,b,c,sigma_f,sigma_u }
bool F,D;
chan F2D = [0] of { mtype }
ltl L1 { [] (F -> <> D) };

active proctype TSF()
{
S1: if
:: F2D!a -> goto S3
fi
S3:
if
:: F2D!sigma_f -> goto S4
fi
S4: F =true
if
:: F2D!b -> goto S5
fi
```

We use the following commands to compile the verifier for the system declared in *Exemplo1.pml*.

```
Console input: spin -a Exemplo1.pml
Console output: ltl L1: [] ((! (F)) || (<> (D)))
Console input: gcc -o pan pan.c
```

Then we run the verifier with the *./pan -a* command. For this example, the system is not diagnosable, the two first lines of the output indicates that the LTL formula L1 was violated, and that the counter example was saved at *Exemplo1.pml.trail*, the remaining lines of the output are omitted due to space limitations.

```
Console input: ./pan -a
Console output: pan:1: acceptance cycle (at depth 21)
pan: wrote Exemplo1.pml.trail
```

We then check the counter example with the *spin -t -c Exemplo1.pml* command. A run that violates the LTL formula will be printed, in this case the string $\alpha\sigma_f b c \alpha\sigma_u^\omega$ violates the LTL formula L1. This result matches the result obtained in example 1 in section 4. Part of the counterexample provided by SPIN is presented below.

```
Console input: spin -t -c Exemplo1.pml
Console output: proc 0 = TSF
proc 1 = TSN_OBS_COMPL
ltl L1: [] ((! (F)) || (<> (D)))
starting claim 2
```

```
using statement merging
Never claim moves to line 4 [(1)]
q\p 0 1
1 F2D!a
1 . F2D?a
1 F2D!sigma_f
1 . F2D?sigma_f
```

6. CONCLUSIONS AND FUTURE WORKS

In this paper we presented a model checking approach to deal with diagnosability of DES, our approach is quite different from the ones found in the literature where they direct test if a given DES is diagnosable with respect to a set of failures and observations. In our approach we propose an indirect test, first we construct a Transition System that indicates when an observed run goes out of the normal behavior of the system and then perform model checking in the model that represents the interaction between it with the faulty runs of the system, using LTL to express that whenever a fault occur in the system it should be always diagnosed in the future. One advantage of this approach is that by our examination and tests, we conjecture that it is quite general, and could be applied to check diagnosability of timed and hybrid systems. Another advantage of our approach is that it can be easily applied in state of the art model checkers such as SPIN with minors adaptations. SPIN is a quite powerful model checking tool and have been applied in several interesting study cases. In future works we intend to perform a benchmarking between our proposed approach and others found in the literature in order to better evaluate the potential of our. We also expect to extend this approach to deal with timed and hybrid system.

REFERENCES

- Baier, C. and Katoen, J.P. (2008). *Principles of Model Checking*. MIT Press.
- Cassandras, C.G. and Lafortune, S. (2008). *Introduction to Discrete Event Systems*. Springer, Massachussets, 2 edition.
- Cassez, F. (2012). The complexity of codiagnosability for discrete event and timed systems. *IEEE TRANSACTIONS ON AUTOMATIC CONTROL*, 57(7), 1752–1764.
- Clarke, E.M., Henzinger, T.A., Veith, H., and Bloem, R. (2018). *Handbook of Model Checking*, volume 1. Springer International Publishing.
- Jiang, S. and Kumar, R. (2004). Failure diagnosis of discrete-event systems with linear-time temporal logic specifications. *IEEE Transactions on Automatic Control*, 49(6), 934–945. doi:10.1109/TAC.2004.829616.
- Jiang, S. and Kumar, R. (2006). Diagnosis of repeated failures for discrete event systems with linear-time temporal-logic specifications. *IEEE Transactions on Automatic Control*, 3(1), 47–58.
- Sampath, M., Sengupta, R., Lafortune, S., Sinnamohideen, K., and Teneketzis, D. (1995). Diagnosability of discrete-event systems. *IEEE Transactions on Automatic Control*, 40(9), 1555–1575. doi:10.1109/9.412626.
- Vardi, M.Y. and Wolper, P. (1986). An automata-theoretic approach to automatic program verification (preliminary report). *1st Annual Symposium on Logic in Computer Science (LICS)*, 332–334.