Use of Subspace Identification Techniques for Solar Fields Mathematical Modeling

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Abstract: Concentrated solar fields (CSP) are used for the generation of electric energy through the conversion of thermal energy into electricity. Due to the complexity of the dynamical system behavior, identification techniques are an alternative to the use of mathematical models based on phenomenological equations. The use of subspace identification technique for the deterministic-stochastic case in this paper has proved to be interesting from the point of view of the prediction of the behavior of the output temperature of the solar field. Moreover, the linear time-invariant state-space model can be used in designing model-based controllers.

Resumo: Campos solares concentrados são utilizados para a geração de energia elétrica por meio da conversão de energia térmica em elétrica. Devido à complexidade do comportamento dinâmico destes processos, as técnicas de identificação são uma alternativa ao uso do modelamento matemático baseado em equações fenomenológicas. A utilização de uma técnica de identificação por subespaço para o caso determinístico-estocástico neste artigo mostrou-se ser interessante do ponto de vista da predição do comportamento da temperatura de saída do campo solar. Ainda, o modelo de espaço de estados linear invariante no tempo obtido pode ser empregado no projeto de controladores baseados em modelos.

Keywords: Concentrated solar power; solar energy; system identification; subspace identification; mathematical modeling; Acurex; Almeria Solar Platform.

Palavras-chaves: Campos solares concentrados; energia solar; identificação de sistemas; identificação por subespaços; modelamento matemático; Acurex; Plataforma solar de Almería.

1. INTRODUCTION

Solar energy is used for the generation of electricity through photovoltaic (PV) plants, and indirectly, by concentrated solar power (CSP) plants to produce steam used to drive a turbine to generate electrical power. If compared to photovoltaic devices, which is a direct conversion process, a CSP plant has better conversion efficiency and the possibility of efficient storage of the thermal energy to be subsequently converted into electricity (Sioshansi and Denholm, 2010).

In a CSP plant, the solar collectors concentrate sunlight to heat a fluid (HTF - Heat Transfer Fluid) at a high temperature. The hot working fluid is used directly for different applications or for generating steam that drives a power conversion system for producing electricity (Camacho et al, 2012).

In solar power systems, the main source of energy cannot be manipulated as in other power generating processes. Moreover, a solar plant needs to deal with problems of seasonal and daily variations that are not found in other thermal power plants: the solar radiation depends on atmospheric conditions such as cloud cover, humidity, and air transparency (Camacho et al, 2012).

CSP plants are represented as a complex grid where each element has stationary and transient losses with changing dynamics, nonlinearities, and uncertainties. To avoid the use of complex mathematical models based on phenomenological equations that govern the dynamics of the system, an alternative is the use of system identification. According to the input and output data of a given dynamic system, the mathematical model can be represented by transfer functions, difference equations and/or state-space, for example.

In system identification, prediction error methods (PEM) and subspace identification are highlighted. Van Overschee and De Moor (1995), Viberg (2002) and Qin (2006) describe the advantages of subspace identification methods over traditional PEM, mainly applied to multivariable systems.

In terms of CSP plants, based on the mathematical model through the phenomenological equations for the Acurex plant at the Solar Platform of Almería, Spain, Camacho et al (2007a, 2007b) present a survey about the use of models based on system identification by PEM for application in the design of basic and advanced controllers. Alsharkawi and Rossiter (2016, 2017) apply PRBS signals generating output data, with such models identified by the subspace identification method N4SID (Numerical algorithms for

Subspace State-Space System Identification). These identified models were used with dual MPC (Model-based Predictive Control) and GS (Gain Scheduling) predictive control strategies, in order to improve the solar fields closed-loop performance.

In this paper, a robust subspace identification algorithm for the combined deterministic-stochastic case proposed by Van Overschee and De Moor (1996) was used to represent a mathematical model which describes the dynamical behaviour of the ACUREX distributed solar collector fields, located in Almería. In spite of having presented it in Van Overschee and De Moor (1996), the methodology has not received so many practical contributions for its use. The data set used in this paper is adequate for the combined deterministic-stochastic case, in which field inlet temperature, volumetric flow rate of the HTF, and solar radiation are treated as inputs and the field output temperature is treated as an output, representing a MISO (Multiple Input Single Output) case.

The paper is organized as follows: the CSP plant ACUREX in Almería, Spain is presented in section 2. The robust subspace identification algorithm used for obtain the mathematical model for ACUREX is introduced in section 3. In section 4 are presented the results, and in section 4 are given the conclusions.

2. CSP PLANT ACUREX IN ALMERIA

The CSP plant used to investigate the mathematical models by subspace identification was the ACUREX field prototype of the Plataforma Solar de Almería (PSA) located in the Tabernas Desert (Southern Spain). ACUREX field is a parabolic trough-based solar thermal power plant. The ACUREX field consists of 480 east-west aligned single axis tracking concentrating solar collectors organized in 10 parallel loops with a mirror area of 2672 m². The loop is 172 m long formed by 4 modules of 12 collectors connected in series. The operation limits for HTF volumetric rate are 2,0 to 12,0×10⁻³ m³s⁻¹. The maximum value is for avoiding the HTF decomposing, which happens when the temperature is up to 305°C. For avoiding stress in the absorber pipe material, due to high HTF pressure in the pipe system, the difference between the inlet and outlet HTF temperatures must be less than 80°C.

In Fig. 1 is shown a scheme of a solar collectors field, similar to the ACUREX field, and an overview of it can be found in Camacho et al (2012).

In terms of ACUREX field mathematical modelling, Camacho et al (2012) describe the dynamic behaviour by the set of energy balance partial differential equations (PDEs):

Fig. 1 - Solar collector field scheme. Field inlet temperature (T_{in}), volumetric flow rate of the HTF (v), field output temperature (T_{out}), control signal (u) (Elias, 2018).

$$\rho_m C_m A_m \frac{\partial T_m}{\partial t} = n_0 G I - D_0 \pi H_l (T_m - T_a) - D_i \pi H_l (T_m - T_f) \quad (1a)$$

$$\rho_f C_f A_f \frac{\partial T_f}{\partial t} + \rho_f C_f q \frac{\partial T_f}{\partial x} = D_i \pi H_l (T_m - T_f) \quad (1b)$$

where the sub-index m refers to the metal of the receiver tube and f to the HTF. In Table 1 there is a description of the parameters and variables and their SI units (Alsharkawi and Rossiter, 2016, 2017).

Table 1. Parameters and variables of the ACUREX PDEs.

Symbol	Description	SI unit
ρ	Density	Kg/m ³
C	Specific heat capacity	J/kg°C
A	Cross-sectional area	m ²
T	Temperature	°C
t	Time	s
n_0	Mirror optical efficiency	-
G	Mirror optical efficiency	m
D_0	Outer diameter of the receiver tube	m
H_l	Global coefficient of thermal losses	W/m°C
T_a	Ambient temperature	°C
D_i	Inner diameter of the receiver tube	m
H_t	Coefficient of metal-fluid heat transfer	W/m ² °C
q	HTF volumetric flow rate	m ³ /s
x	Space	m

3. ROBUST SUBSPACE IDENTIFICATION ALGORITHM

In this section, the robust subspace identification algorithm for the deterministic-stochastic case is described. There are several forms of state-space representation. The innovation form is the one most commonly used in system identification, given as (Qin, 2006):

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (2a)$$

$$y_k = Cx_k + Du_k + e_k \quad (2b)$$

where $x_k \in \mathbb{R}^n$ (internal state vector), $y_k \in \mathbb{R}^l$ (output vector), $u_k \in \mathbb{R}^m$ (input vector), and $e_k \in \mathbb{R}^m$ is white noise. In these vectors, n is the dimension of the state vector and represents the system order too, m is the input dimension, and l is the output dimension. In equations (2a)-(2b), $A \in \mathbb{R}^{n \times n}$ is the system matrix, $B \in \mathbb{R}^{n \times m}$ is the input

matrix, $C \in \mathbb{R}^{l \times n}$ is the output matrix, $D \in \mathbb{R}^{l \times m}$ is the direct feedthrough matrix, and $K \in \mathbb{R}^{l \times m}$ is the Kalman gain.

The basic idea of subspace identification methods is to estimate the dynamical system order n , and the matrices A, B, C, D , and K of the linear invariant-time (LIT) discrete state-space model (2a)-(2b), from the availability of the dynamical system acquired data set.

In representation (2a)-(2b), e_k can be eliminated from the (2a) to yield a system description in one-step-ahead predictor form:

$$x_{k+1} = \tilde{A}x_k + \tilde{B}u_k + Ky_k \quad (3a)$$

$$y_k = Cx_k + Du_k + e_k \quad (3b)$$

where $\tilde{A} = A - KC$ and $\tilde{B} = B - KD$. It is assumed the eigenvalues of $A - KC$ are strictly inside the unit circle.

After some algebraic manipulations, the subspace matrix equations are established for combined deterministic-stochastic identification (Van Overschee and De Moor, 1996; Katayama, 2005):

$$Y_p = \Gamma_i X_p^d + H_i^d U_p + Y_p^s \quad (4a)$$

$$Y_f = \Gamma_i X_f^d + H_i^d U_f + Y_f^s \quad (4b)$$

$$X_f^d = A^i X_p^d + \Delta_i^d U_p \quad (4c)$$

with: Y_p and Y_f , output vectors of the "past - p " and the "future - f ", respectively; U_p and U_f , input vectors of "past" and "future", respectively. The index d and s indicate the use of subspace techniques for deterministic and stochastic systems, respectively. The matrices Γ_i , Δ_i^d and H_i^d contained in (4a)-(4c) are referred as the extended observability matrix, the extended controllability matrix, and the Toeplitz matrix, respectively (Van Overschee and De Moor, 1996; Katayama, 2005). The extend observability (Γ_i) and Toeplitz (H_i^d) matrices, effectively used in the robust subspace identification algorithm, are written respectively as:

$$\Gamma_i \triangleq (C \quad CA_d \quad CA_d^2 \quad \dots \quad CA_d^{i-1})^T \in \mathbb{R}^{l \times n} \quad (5a)$$

$$H_i^d = \begin{bmatrix} D & 0 & 0 & \dots & 0 \\ CB & D & 0 & \dots & 0 \\ CAB & CB & D & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \dots & D \end{bmatrix} \in \mathbb{R}^{li \times mi} \quad (5b)$$

The input (U_p and U_f) and output (Y_p and Y_f) Hankel matrices are formed through input and output data set. Hankel's matrices of past " p " and future " f " input are defined, respectively (Van Overschee and De Moor, 1996):

$$U_p \triangleq U_{0|i-1} \triangleq \begin{pmatrix} u_0 & u_1 & \dots & u_{j-1} \\ u_1 & u_2 & \dots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_N & \dots & u_{i+j-2} \end{pmatrix} \quad (6a)$$

$$U_f \triangleq U_{i|2i-1} \triangleq \begin{pmatrix} u_i & u_{i+1} & \dots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \dots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \dots & u_{2i+j-2} \end{pmatrix} \quad (6b)$$

where $U_p, U_f \in \mathbb{R}^{mi \times j}$ with the subscript $k_1|k_2$ indicating the first column of the subspace matrix/vector begins at the discrete time k_1 and ends at k_2 . The index j (number of columns) is typically equal to $s - 2i + 1$, implying that all given data samples are used. The output block Hankel matrices, $Y_p, Y_f \in \mathbb{R}^{li \times j}$ are defined in a similar way to the matrices (6a) and (6b). The matrices Y_f^-, U_f^- on the other hand, are defined by shifting the border between past and future one block row down.

In the subspace identification literature, the notation of (7) is often used (Van Overschee and De Moor, 1996):

$$W_p = \begin{pmatrix} Y_p \\ U_p \end{pmatrix} \quad (7)$$

where $W_p \in \mathbb{R}^{(mi \times li) \times j}$ is the matrix of past input and output data.

3.1 The Algorithm

The algorithm is started by calculating the oblique (O_i) and orthogonal (Z_i) projections, respectively by:

$$O_i = Y_f / U_f W_p, \quad Z_i = Y_f / \begin{bmatrix} W_p \\ U_f \end{bmatrix}, \quad (8)$$

$$Z_{i+1} = Y_f^- / \begin{bmatrix} W_p^+ \\ U_f^- \end{bmatrix}$$

In (7), the projection Z_i could be considered an optimal prediction of the future output Y_f , given the past input and output data W_p and the future input data U_f . The interpretation for Z_{i+1} is that the offset projection of the edge between past and future one down (Van Overschee and De Moor, 1996). It can be noticed the oblique and orthogonal projections do not depend on the calculations of the state-space matrices but only the input and output data.

The SVD (Singular Value Decomposition) calculation is done by the weighted oblique projection:

$$W_1 O_i W_2 = USV^T \quad (9)$$

with W_1 and W_2 weighting matrices chosen as I (identity matrix) and $\Pi_{U_f^\perp}$, respectively. $\Pi_{U_f^\perp}$ denotes the operator that projects the row space of a matrix onto the row space of $U_f^\perp$ and $U_f^\perp$ denotes the subspace perpendicular to the row space of U_f (Van Overschee and De Moor, 1995).

The system order is determined by inspecting the singular values in S and partition the SVD accordingly to obtain U_1 and S_1 . The matrices Γ_i and Γ_{i-1} are determined as:

$$\Gamma_i = U_1 S_1^{1/2}, \quad \Gamma_{i-1} = \underline{\Gamma}_i \quad (10)$$

where $\underline{\Gamma}_i$ denotes Γ_i without the last l rows.

To calculate the matrices A (with stability guarantee) and C can be extracted from Γ_i . The stable matrix can be calculated by following procedure indicated in (Van Overschee and De Moor, 1996):

$$A = \Gamma_i^+ \cdot \begin{bmatrix} \bar{\Gamma}_i \\ 0 \end{bmatrix} \quad (11)$$

where 0 represent l rows of zeros, and $\bar{\Gamma}_i$ denotes Γ_i without the first l rows. The matrix C can be determined from the first l rows of Γ_i . Γ_i^+ denotes the Moore-Penrose inverse of Γ_i .

From A and C one must recompute Γ_i and Γ_{i-1} since original Γ_i and Γ_{i-1} determined from (10) are only approximations of the exact Γ_i and Γ_{i-1} .

The B and D matrices accuracy tends to be poor in case of badly conditioned input Hankel matrices. As A , C , Γ_i , and Γ_{i-1} are known, B and D are computed as:

$$B, D = \arg \min_{B, D} \left\| \underbrace{\begin{bmatrix} \Gamma_i^+ Z_{i+1} \\ Y_{i|i} \end{bmatrix}}_{\text{known}} - \underbrace{\begin{bmatrix} A \\ C \end{bmatrix}}_{\text{known}} \cdot \Gamma_i^+ \cdot Z_i - \kappa(B, D) \cdot \underbrace{U_f}_{\text{known}} \right\|_F^2 \quad (12)$$

with $\|\cdot\|_F^2$ refers to the Frobenius norm squared, and $\kappa(B, D)$ refers to the linear matrix function defined by

$$\kappa \triangleq \begin{bmatrix} (B | \Gamma_{i-1}^+ \cdot H_{i-1}^d) - A \cdot \Gamma_i^+ \cdot H_i^d \\ (D | 0) - C \cdot \Gamma_i^+ \cdot H_i^d \end{bmatrix} \quad (13)$$

Since $\kappa(B, D)$ is linear in B and D , the overall optimization problem is convex, and thus has a unique minimum. The problem can be solved by either using a optimization algorithm, or by rewriting the function between the norm signs in (12) as an explicit linear combination of B and D and solving this set of equation in a least squares sense, this second way is used in this paper (Van Overschee and De Moor, 1996).

Finally, the covariance matrices Q , R and S can be approximated from the residuals ρ_w and ρ_v :

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = E \left(\begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix} \cdot \begin{bmatrix} \rho_w^T & \rho_v^T \end{bmatrix} \right) \quad (14)$$

with the covariance matrices and residuals are positive definite. These covariance matrices are used for calculation of Kalman gain in the forward innovation model by solving the Riccati equation in (15):

$$K = (G - APC^T)(\Lambda_0 - CPC^T)^{-1} \quad (15)$$

where

$$P = (APA^T) + (G - APC^T)(\Lambda_0 - CPC^T)^{-1}(G - APC^T)^T \quad (16a)$$

$$E[x_k^s (x_k^s)^T] = \Sigma^s = A \Sigma^s A^T + Q \quad (16b)$$

$$E[y_k y_k^T] = \Lambda_0 = C \Sigma^s C^T + R \quad (16c)$$

$$E[x_{k+1}^s (x_{k+1}^s)^T] = G = A \Sigma^s C^T + S \quad (16d)$$

with P as the forward state covariance matrix, Σ^s is the state covariance matrix, and Λ_0, G are the output covariance matrices also considered as Markov parameters of the deterministic linear time invariant system, as described in (Van Overschee and De Moor, 1996).

In addition, a detailed scheme of the robust combined deterministic-stochastic identification algorithm is illustrated in Fig. 2.

Fig. 2 - Scheme of the robust combined deterministic-stochastic identification algorithm.

4. RESULTS AND VALIDATION

The data set used in this paper was obtained from the Acurex field, installed in the Plataforma Solar de Almería (PSA) located in the Tabernas Desert (Southern Spain). The Acurex field input and output data set were acquired in February 1997, with 11 ms of interval, where the inputs are the field inlet temperature ($^{\circ}\text{C}$), volumetric flow rate of the HTF (l/s), and solar radiation (W/m^2), and the output is the field output temperature ($^{\circ}\text{C}$), with about 410 samples in each. In the data pre-treatment stage, these are normalized, outliers and offset values are removed, a data resampling by a factor of 2 is applied, without filter usage. The normalized data set is shown in Fig. 3.

Fig. 3 - The normalized data set of the ACUREX field identification and validation, after the data pre-processing step. (a) Volumetric flow rate, (b) field inlet temperature, (c) solar radiation and (d) field output temperature.

The SVD calculated by (9) - (10) are used to investigate the order of the dynamical system model. By inspecting the gap between the high values in Fig. 4, order 3 can be chosen. The Mean Relative Square Error (MRSE) index is performed to determine whether the immediately higher order 4 would be better than third order to represent the dynamical behavior of the solar field outlet temperature. The MRSE index was calculated for the robust combined deterministic-stochastic and MOESP (Multivariable Output Error State sPace) identification algorithms.

MRSE (Mean Relative Square Error) index is calculated as:

$$MRSE = \left[1 - \left(\frac{1}{l} \sum_{i=1}^l \sqrt{\frac{\sum_{k=1}^N ((y_k)_i - (\hat{y}_k)_i)^2}{\sum_{k=1}^N ((y_k)_i)^2}} \right) \right] \times 100\% \quad (17)$$

where l is the number of outputs of the dynamical system, y_k is the experimental output data used for the validation, and $\hat{y}_k$ is the estimated output data. The MRSE index varies between 0 (bad fit) to 100% (perfect fit).

Fig. 4 - SVD values using the robust combined deterministic-stochastic subspace identification algorithm.

The MRSE values index for one-step-ahead prediction of the dynamical system output (normalized field outlet temperature) for order 3 and order 4 are close and by parsimony analysis, the chosen model is of third order. The results in Table 2 are similar to those obtained by Alsharkwai and Rossiter (2017), who chose a 4th order model for the design of the developed controllers. In this paper, a 3rd order model is indicated so that model-based controller designs can be developed in the future.

Still with the data in Table 2, it can be seen that the MRSE index for the one-step-ahead prediction has a better percentage. This is explained by the inclusion of the Kalman gain (K) in the output predictions considering the existence of disturbances in the system input (solar radiation). In Table, The computation time recorded of each identification algorithm indicates the robust combined deterministic-stochastic is 4 times faster than MOESP algorithm, approximately (Intel Core i5 with 8 GB of RAM).

Table 2. MRSE index with variations in the order systems.

Order	Method	MRSE index (%)	Comput. time (s)
3	MOESP - deterministic case	92.34	0.37
	Robust combined deterministic-stochastic	95.04	0.13
4	MOESP - deterministic case	94.35	0.45
	Robust combined deterministic-stochastic	96.19	0.12

The data set used has 413 samples for identification, corresponding to half of the data, and the others for validation. In Fig. 6, the ACUREX plant field outlet temperature normalized graphs are shown with data used in the state-space model estimation step. The experimental data are compared with the data used in the free simulation

estimation (2a-2b), one-step-ahead prediction (3a-3b) both using the robust combined deterministic-stochastic algorithm and the free simulation prediction for MOESP. From the comparison with plant output, the effectiveness of robust combined deterministic-stochastic algorithm is clearly demonstrated. The identified system matrices and Kalman gain are given in the Appendix.

Fig. 6 - Estimated and real outputs of ACUREX field.

In the validation step, the output data is compared to the one-step-ahead prediction. From the results shows in Fig. 7, it can be observed that the employed algorithm was successful, with RMSE index of 98.56%. These results suggest that the estimated LIT discrete state-space model is satisfactory, showing the good properties of the robust combined deterministic-stochastic subspace identification algorithm.

Fig. 7 - Validation based on the robust combined deterministic-stochastic identification algorithm.

5. CONCLUSIONS

In this paper, an offline subspace identification algorithm was used to identify discrete LTI state-space model that represents the dynamic behavior of the ACUREX field in Almería, Spain. The technique that combines the stochastic and deterministic parts has an advantage over techniques that are based only on the deterministic part, such as MOESP, including a shorter computational time.

Moreover, the parameters of the phenomenological equation models do not need to be estimated, calculated or measured, since identification techniques use experimental data from the dynamical system.

In future works, the state-space model will be used in model-based controllers, and the connection of the ACUREX solar field in the context of smart grids to optimize the production of the active power.

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APPENDIX

Matrix	MOESP	Robust combined deterministic-stochastic
A	$\begin{bmatrix} 0.936 & -0.632 & -0.389 \\ 0.003 & 0.823 & 1.537 \\ -0.001 & -0.026 & 0.995 \end{bmatrix}$	$\begin{bmatrix} 0.935 & -0.073 & 0.033 \\ -0.013 & 0.958 & 0.190 \\ -0.036 & -0.049 & 0.901 \end{bmatrix}$
B	$\begin{bmatrix} 0.225 & -0.604 & -0.027 \\ -0.201 & -0.188 & -0.080 \\ -0.002 & -0.203 & -0.003 \end{bmatrix}$	$\begin{bmatrix} 1.101 & -0.382 & -0.291 \\ -0.467 & 0.542 & 0.222 \\ 0.852 & -0.281 & -0.248 \end{bmatrix}$
C	$[-0.504 \quad -0.657 \quad 0.467]$	$[-0.186 \quad -0.077 \quad 0.052]$
D	$[0 \quad 0 \quad 0]$	$[0.087 \quad 0.005 \quad 0.000]$
K	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -8.683 \\ 9.931 \\ -3.643 \end{bmatrix}$