A Inverted-Wishart Distribution Approach for the Noise Covariance Matrix Estimation Applied to Target Tracking Problems [★]

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Abstract: Improving the precision of an instrument is a hard task. Generally, this can be accomplished using stochastic filters, e.g., Kalman Filter (KF). Normally, these filters have to be tuned by an arbitrary definition of its noise covariance matrices. In this context, the objective of this paper is estimate dynamically the measurement noise covariance matrix, by means of the Bayes Theorem (BT), in a Kalman Filter algorithm, using the Inverted-Wishart distribution (IW-R Kalman Filter). The proposed method is compared with three other classic stochastic filters. In this comparison the IW-R Kalman Filter has achieved the best RMSE (Root Mean Square Error), outperforming the other filters. Therefore, one can conclude that dynamic estimation of noise covariances could improve the performance of the traditional stochastic filters.

Keywords: Target tracking, Kalman filter, Adaptive Kalman filter, Inverted-Wishart distribution.

1. INTRODUCTION

In target tracking problems, one of the most important steps is the estimation of some kinematic values, as position, velocity and acceleration. This estimation is commonly done by stochastic filters as the Kalman Filter (KF) (Kalman, 1960), the Extended Kalman Filter (EKF) (Bar-Shalom et al., 2001), the IMM Filter (Rong-Li and Jilkov, 2005), the Particle Filter (PF) (Ristic et al., 2004) and others.

These stochastic filters are defined from recursive equations and they are usually based on discrete state space models, in which the dynamic and measurement models are described. Perturbations, that are inaccuracies in these models when compared to the real systems, are almost always present, and can affect the modelled results as a form of process noise. The measurements may also be immersed in noise, related not only to the sensor accuracy but also to external factors such as heat, moisture, and wind.

It is necessary to consider these noise effects on the mathematical model. In general, the noise is represented as a stochastic process with a known probabilistic distribution. Therefore, these models can be defined as stochastic models. The process noise, related to the dynamics, and the measurement noise, related to the measurements from the

sensor/radar, are commonly defined as Gaussian noises with zero mean and with its respective covariance matrix.

The noise covariance matrices are defined as time-varying matrices, but it is difficult to define how these matrices evolve over time. Some stochastic filters define mathematical expressions for these two noise covariance matrices, obtaining new matrices for each discrete time instant $k \in \mathbb{N}$. They are the Adaptive Kalman Filter (AKF) (Brown and Rutan, 1985), the Adaptive Extended Kalman Filter (AEKF) (Nayak et al., 2016) and the Optimal Adaptive Kalman Filter (OAKF) (Yang and Gao, 2006). Some of these filters are applied to target tracking problems as in Kumar et al. (2017); Tripathi et al. (2016); Dang (2008).

In this paper, it is proposed a new method to estimate one of these covariance matrices: the measurement noise covariance matrix. This method is based on a probability distribution named Inverted-Wishart distribution (IW) (Gamerman and Lopes, 2006). This distribution is defined for positive-definite matrices and it is used as a conjugate prior for the covariance matrix of a multivariate Gaussian distribution. Its characteristics are well suited to the target tracking problems. Some simulations were made in order to compare the performances of the proposed method and some adaptive stochastic filters found in the literature.

The paper is organized as follows. Section 2 presents the stochastic systems, including the stochastic model used in this paper and some stochastic filters from the literature. In Section 3, the proposed method is presented

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and detailed. Section 4 shows some simulations results and in Section 5 the conclusions are presented.

2. STOCHASTIC SYSTEMS

2.1 Stochastic Models

A discrete-time system can be described by the following state space equations:

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}_k) + \mathbf{w}_k, \quad (1)$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \quad (2)$$

in which $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\mathbf{g} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\mathbf{h} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ are functions, linear or nonlinear, of the state vector, $\mathbf{w}_k \sim \mathcal{N}(0, Q_k)$ is the zero-mean Gaussian process noise vector with covariance matrix $Q_k \in \mathbb{R}^{n \times n}$ and $\mathbf{v}_k \sim \mathcal{N}(0, R_k)$ is the zero-mean Gaussian measurement noise vector with covariance matrix $R_k \in \mathbb{R}^{m \times m}$, $\mathbf{z}_k \in \mathbb{R}^m$ is the measurement vector. The system in equations (1) and (2) can be defined as a stochastic model due to the random noise terms $\mathbf{w}_k$ and $\mathbf{v}_k$.

2.2 Stochastic Filtering

The Kalman filter is based on a linear time invariant discrete system with additive Gaussian noise, that can be described by the following state space equations:

$$\mathbf{x}_{k+1} = G\mathbf{x}_k + \mathbf{w}_k, \quad (3)$$

$$\mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k, \quad (4)$$

in which $G \in \mathbb{R}^{n \times n}$ is the dynamic matrix and $H \in \mathbb{R}^{m \times n}$ is the measurement matrix. Equations (3) and (4) can be seen as a linearized version of (1) and (2), respectively.

2.3 Kalman Filter

The Kalman filter equations are divided into two main stages: prediction and update stages. Each stage has a set of equations, as follows:

- Prediction:

$$\hat{\mathbf{x}}_{k+1|k} = G\hat{\mathbf{x}}_{k|k}, \quad (5)$$

$$P_{k+1|k} = GP_{k|k}G^T + Q_k. \quad (6)$$

- Update:

$$\hat{\mathbf{z}}_{k+1|k} = H\hat{\mathbf{x}}_{k+1|k}, \quad (7)$$

$$\bar{\mathbf{z}}_{k+1} = \mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}, \quad (8)$$

$$S_{k+1} = HP_{k+1|k}H^T + R_k, \quad (9)$$

$$K_{k+1} = P_{k+1|k}H^TS_{k+1}^{-1}, \quad (10)$$

$$\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + K_{k+1}\bar{\mathbf{z}}_{k+1}, \quad (11)$$

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k}. \quad (12)$$

in which $\hat{\mathbf{x}}_{k+1|k}$ is the predicted state vector, $P_{k+1|k}$ is the state prediction error covariance matrix, $\bar{\mathbf{z}}_{k+1}$ is the innovation, $\hat{\mathbf{z}}_{k+1|k}$ is the predicted measurement, S_{k+1} is the innovation covariance matrix, K_{k+1} is the Kalman gain, $\hat{\mathbf{x}}_{k+1|k+1}$ is the estimated state vector, $P_{k+1|k+1}$ is the state estimation error covariance matrix, and I is the identity matrix with appropriate dimensions.

2.4 Adaptive Kalman Filter

In the literature, there are many approaches of the AKF, as presented in Section 1. All of them are based on

the Kalman filter equations in (5)–(12). However, the main difference between them is the way that the noise covariance matrices, Q_k and R_k , are obtained: recursively, based on closed-forms, etc.

For instance, the AKF using the Sage-Husa method (Wan-Xin, 2014) calculates both matrices recursively, as follows:

$$\begin{aligned} \hat{Q}_{k+1} = & (1 - d_k)\hat{Q}_k + d_k [K_{k+1}\bar{\mathbf{z}}_{k+1}\bar{\mathbf{z}}_{k+1}^TK_{k+1}^T + \\ & + P_{k+1|k+1} - GP_{k|k}G^T], \end{aligned} \quad (13)$$

$$\hat{R}_{k+1} = (1 - d_k)\hat{R}_k + d_k [\bar{\mathbf{z}}_{k+1}\bar{\mathbf{z}}_{k+1}^T - HP_{k+1|k}H^T], \quad (14)$$

in which

$$d_k = \frac{1 - b}{1 - b^{k+1}}, \quad (15)$$

and $b \in [0.95, 0.99]$ is a forgetting factor.

Other example is the residual-based estimation of R_k and the adaptive estimation of Q_k presented in Wang (1999) and Paul et al. (2013). The R_k estimation is obtained in a closed-form way as follows:

$$\bar{\mathbf{v}}_k = \mathbf{z}_k - H\hat{\mathbf{x}}_{k|k}, \quad (16)$$

$$\hat{C}_{\bar{\mathbf{v}}} = \frac{1}{m} \sum_{i=1}^m \bar{\mathbf{v}}_{k-i}\bar{\mathbf{v}}_{k-i}^T, \quad (17)$$

$$\hat{R}_k = \hat{C}_{\bar{\mathbf{v}}} + HP_{k|k}H^T, \quad (18)$$

and the Q_k adaptive estimation is based on the relation:

$$\hat{Q}_{k+1} = \hat{Q}_k\sqrt{\alpha}, \quad (19)$$

$$\alpha = \frac{\text{trace}(\hat{C}_{\bar{\mathbf{v}}} - \hat{R}_k)}{\text{trace}(HP_{k+1|k}H^T)}, \quad (20)$$

in which $\bar{\mathbf{v}}_k$ is the residue, $\hat{C}_{\bar{\mathbf{v}}}$ is the covariance matrix of the residue and m is the window size. if $k < m$, then Q_k and R_k are constant matrices; else ($k \geq m$), R_k and Q_k are obtained from (18) and (19), respectively.

These two methods presented here were chosen to have their performances compared to the proposed method.

3. INVERTED-WISHART DISTRIBUTION BASED ESTIMATION METHOD

In problems that are based on multivariate scenarios, such as target tracking scenarios, the related random variables are also considered multivariate. In order to treat these variables properly, it is necessary to use an suitable probability distribution. In this case, one of the possible choices is the Inverted-Wishart (IW) distribution.

The estimation method proposed in this paper for the measurement noise covariance matrix R_k is based on the Bayes Inference Theory. Thus, R_k will not be a fixed value but a random variable with an IW *a priori* probability distribution.

This choice is justified by the fact that the measurement probability distribution follows a Gaussian distribution and the IW distribution is a conjugate distribution of the Gaussian distribution. Conjugate distributions have similar core in their probability distributions (Gamerman and Lopes, 2006).

Then, from the Bayes' Theorem, the *a posteriori* distribution will be an IW distribution due to the conjugation

of the distributions. This approach will produce different estimates R_k at each instant k of the stochastic filter.

The IW distribution is not as common as uniform and Gaussian distributions. For this reason, it is important to briefly describe the related probability density function and its parameters.

3.1 Inverted-Wishart Distribution

A random variable (r.v.) $\mathbf{X} \in \mathbb{R}^{p \times p}$, following IW distribution, with parameters Ψ and ν , denoted by $\mathbf{X} \sim \mathcal{W}^{-1}(\Psi, \nu)$, can be define by a probability density function (pdf) given by:

$$f(\mathbf{X}) = \frac{\det(\Psi)^{\nu/2}}{2^{\nu p/2} \Gamma_p(\nu/2)} \det(\mathbf{X})^{-\frac{\nu+p+1}{2}} e^{-\frac{\text{trace}(\Psi \mathbf{X}^{-1})}{2}}, \quad (21)$$

in which $\Psi \in \mathbb{R}^{p \times p}$ is a positive definite scale matrix, $\nu > p - 1$ is the number of degrees of freedom and $\Gamma_p(\cdot)$ is the multivariate gamma function.

In order to obtain the noise covariance matrix estimator in the next section, it is important to define the mean of a IW-distributed r.v. $\mathbf{X}$, which is given by:

$$E[\mathbf{X}] = \frac{\Psi}{\nu - p - 1}. \quad (22)$$

Based on (21) and (22), the following section presents the proposed method for the noise covariance matrix estimation.

3.2 Measurement Noise Covariance Matrix Estimator

First, it is considered that the probability distribution for the model residue is a Gaussian given by

$$f(\mathbf{z}_{k+1} | \hat{\mathbf{x}}_{k+1|k+1}, \mathbf{S}_1) \sim \mathcal{N}(H \hat{\mathbf{x}}_{k+1|k+1}, \mathbf{S}_1). \quad (23)$$

Assuming that $\mathbf{S}_1$ has a IW distribution, this assumption is usually applied *a priori* for covariance matrices when the data follow Gaussian multivariate distributions (Gaman and Lopes, 2006), as in this case the $\mathbf{z}_{k+1}$. $\mathbf{S}_1$ has parameters Ψ_0 and ν_0 , i.e., $\mathbf{S}_1 \sim \mathcal{W}^{-1}(\Psi_0, \nu_0)$. The Bayes theorem is applied, resulting in the following probabilistic proportionality, represented by the “ $\propto$ ” symbol:

$$f(\mathbf{S}_1 | \mathbf{z}_{k+1}, \hat{\mathbf{x}}_{k+1|k+1}) \propto \prod_{i=1}^{n_k} f(\mathbf{z}_{k+1} | \hat{\mathbf{x}}_{k+1|k+1}, \mathbf{S}_1) f(\mathbf{S}_1), \quad (24)$$

$$f(\mathbf{S}_1 | \mathbf{z}_{k+1}, \hat{\mathbf{x}}_{k+1|k+1}) \propto \det(\mathbf{S}_1)^{-\frac{n_k}{2}} e^{-\frac{\sum_{i=1}^{n_k} \mathbf{e}^T \mathbf{S}_1^{-1} \mathbf{e}}{2}} \times \det(\mathbf{S}_1)^{-\frac{\nu_0+p+1}{2}} e^{-\frac{\text{trace}(\Psi_0 \mathbf{S}_1^{-1})}{2}}, \quad (25)$$

$$f(\mathbf{S}_1 | \mathbf{z}_{k+1}, \hat{\mathbf{x}}_{k+1|k+1}) \propto \det(\mathbf{S}_1)^{-\frac{\nu_0+p+1+n_k}{2}} \times e^{-\frac{\text{trace}((\mathbf{S}^* + \Psi_0) \mathbf{S}_1^{-1})}{2}}. \quad (26)$$

in which n_k is number of observations until the time instant k and

$$\mathbf{e} = \mathbf{z}_{k+1} - H \hat{\mathbf{x}}_{k+1|k+1}. \quad (27)$$

Observing the result in (26), it is possible to verify that $\mathbf{S}_1 \sim \mathcal{W}^{-1}(\Psi_n, \nu_n)$, in which $\Psi_n = \mathbf{S}^* + \Psi_0$, $\nu_n = \nu_0 + n_k$ and

$$\mathbf{S}^* = \sum_{k=1}^{n_k} (\mathbf{z}_{k+1} - H \hat{\mathbf{x}}_{k+1|k+1})(\mathbf{z}_{k+1} - H \hat{\mathbf{x}}_{k+1|k+1})^T. \quad (28)$$

Therefore, an estimator for $\hat{\mathbf{S}}_1$ can be chosen as the mean of the IW distribution in (22), as follows:

$$E[\mathbf{S}_1] = \hat{\mathbf{S}}_1 = \frac{\mathbf{S}^* + \Psi_0}{\nu_n - p - 1}. \quad (29)$$

Now, the residue covariance matrix S_k^* can be defined based on the formulation of (9), resulting in (30):

$$S_k^* = H P_{k|k} H^T + R_k. \quad (30)$$

The proposal here is to define the estimator for S_k^* as $\hat{\mathbf{S}}_1$ in (29). Thus, the estimator for R_k , $\hat{R}_k$, can be obtained as follows:

$$\hat{\mathbf{S}}_1 = H P_{k|k} H^T + \hat{R}_k, \quad (31)$$

$$\Rightarrow \hat{R}_k = \hat{\mathbf{S}}_1 - H P_{k|k} H^T. \quad (32)$$

3.3 Process Noise Covariance Matrix Estimator

In this paper, the matrix Q_k is not estimated and it does not appear as an additive factor as in (6). It was chosen to define a forgetting factor $\lambda \in [0, 1]$ and modify (6) as follows:

$$P_{k+1|k} = \frac{G P_{k|k} G^T}{\lambda}. \quad (33)$$

Future research might apply a new estimator for Q_k also based on IW distributions.

4. SIMULATIONS AND RESULTS

In this section it is presented a performance comparison between the proposed method from Section 3, the linear Kalman filter from Section 2.3 and the two methods presented in Section 2.4. These comparisons are based on a target tracking situation where two 2D trajectories were simulated, creating two distinct scenarios. The simulated noisy measurements served as input to the stochastic filter, in order to provide position and velocity estimations.

The first scenario, shown in Fig. 1, is a rectilinear trajectory with constant velocity. The second scenario is a more complex trajectory, which is composed by nine paths, five rectilinear trajectories with constant velocity and four constant turn trajectories, forming a “zig-zag” trajectory. Fig. 2 shows the second scenario.

For both scenarios, it was used the constant velocity model with the following matrices and state vector, with $T = 4$ s, which is standard sweep time for aircraft control terminal area radars:

$$G = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (34)$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (35)$$

$$\mathbf{x} = \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}, \quad (36)$$

in which p_x and p_y are the target positions in x and y axis, respectively. v_x and v_y are the target speeds in x and y axis.

Figure 1. Scenario 1: Rectilinear trajectory.

Figure 2. Scenario 2: "Zigzag" trajectory.

4.1 Scenario 1

In Scenario 1, 20 Monte Carlo runs were simulated with the following parameters:

- Linear Kalman filter:
 - (a) $P_{0|0} = 1500^2 I$;
 - (b) $Q_k = Q = 100^2 I$;
 - (c) $R_k = R = 1000^2 I$;
 - (d) $\hat{\mathbf{x}}_{0|0} = [500 \text{ m} \quad -200 \text{ m/s} \quad 400 \text{ m} \quad -120 \text{ m/s}]^T$.
- AKF with Sage-Husa method:
 - (a) $b = 0.95$;

- (b) $Q_0 = 100^2 I$;
- (c) $R_0 = 1000^2 I$;
- (d) $P_{0|0} = 1500^2 I$;
- (e) $\hat{\mathbf{x}}_{0|0} = [500 \text{ m} \quad -200 \text{ m/s} \quad 400 \text{ m} \quad -120 \text{ m/s}]^T$.

- AKF with residual-based estimation of R_k and adaptive estimation of Q_k :
 - (a) $m = 10$;
 - (b) $Q_0 = 100^2 I$;
 - (c) $R_0 = 1000^2 I$;
 - (d) $P_{0|0} = 1500^2 I$;
 - (e) $\hat{\mathbf{x}}_{0|0} = [500 \text{ m} \quad -200 \text{ m/s} \quad 400 \text{ m} \quad -120 \text{ m/s}]^T$.
- Kalman Filter with proposed method (IW distribution):
 - (a) $\lambda = 0.8$;
 - (b) $Q_0 = 100^2 I$;
 - (c) $R_0 = 1000^2 I$;
 - (d) $P_{0|0} = 1500^2 I$;
 - (e) $\hat{\mathbf{x}}_{0|0} = [500 \text{ m} \quad -200 \text{ m/s} \quad 400 \text{ m} \quad -120 \text{ m/s}]^T$.

Figure 3 shows a visual comparison between the estimated trajectory, obtained from the proposed method, and the exact trajectory. Note that the position estimations are very close to the exact trajectory, showing a good performance of the proposed filter.

Figure 3. Scenario 1: Rectilinear trajectory with position estimation.

This good performance can be confirmed from the results presented in Table 1 containing the norm of the Root Mean Squared Error (RMSE) vector $\mathbf{RMSE} = [RMSE_x \quad RMSE_y]$, where $RMSE_x$ is the RMSE for position in x axis and $RMSE_y$ is the RMSE for position in y axis. Analyzing Table 1, the proposed method has achieved a better performance than the other three methods. Improvement of results indicates the adequate choice of a *a priori* distribution for R_k .

Table 1. RMSE values: Scenario 1

Filter	$\ \mathbf{RMSE}\ $
Linear Kalman filter	108.81
AKF (Sage-Husa method)	155.29
AKF (residue-based R_k , adaptive Q_k)	124.77
Kalman filter with proposed method	86.73

4.2 Scenario 2

For Scenario 2, 20 Monte Carlo runs were simulated with the following parameters:

- Linear Kalman filter:
 - (a) $Q_k = Q = 100^2 I$;
 - (b) $R_k = R = 1000^2 I$;
 - (c) $P_{0|0} = 1500^2 I$;
 - (d) $\hat{x}_{0|0} = [60 \text{ km } -130 \text{ m/s } 40 \text{ km } -190 \text{ m/s}]^T$.
- AKF with Sage-Husa method:
 - (a) $b = 0.99$;
 - (b) $Q_0 = 100^2 I$;
 - (c) $R_0 = 1000^2 I$;
 - (d) $P_{0|0} = 1500^2 I$;
 - (e) $\hat{x}_{0|0} = [60 \text{ km } -130 \text{ m/s } 40 \text{ km } -190 \text{ m/s}]^T$.
- AKF with residual-based estimation of R_k and adaptive estimation of Q_k :
 - (a) $m = 10$;
 - (b) $Q_0 = 100^2 I$;
 - (c) $R_0 = 1000^2 I$;
 - (d) $P_{0|0} = 1500^2 I$;
 - (e) $\hat{x}_{0|0} = [60 \text{ km } -130 \text{ m/s } 40 \text{ km } -190 \text{ m/s}]^T$.
- Kalman Filter with proposed method (IW distribution):
 - (a) $\lambda = 0.4$;
 - (b) $Q_0 = 100^2 I$;
 - (c) $R_0 = 1000^2 I$;
 - (d) $P_{0|0} = 1500^2 I$;
 - (e) $\hat{x}_{0|0} = [60 \text{ km } -130 \text{ m/s } 40 \text{ km } -190 \text{ m/s}]^T$.

Figure 4 shows a visual comparison between the estimated trajectory, obtained from the proposed method, and the exact trajectory. Note that, once again, the position estimations are very similar to the exact trajectory, even for the four curve sections, showing a good performance of the proposed technique.

Figure 4. Scenario 2: “Zigzag” trajectory with position estimation.

This good performance can also be confirmed from the RMSE values presented in Table 2 for positions in x and y . Once again, the proposed method has a better performance than the other three methods from the literature.

Table 2. RMSE values: Scenario 2

Filter	RMSE
Linear Kalman filter	331.94
AKF (Sage-Husa method)	451.25
AKF (residue-based R_k , adaptive Q_k)	205.32
Kalman Filter with proposed method	192.02

As in the previous result the uncertainty about R_k decreases by the appropriate combination of an *a priori* information and the data produces more accurate estimates.

5. CONCLUSION

In this work it was presented a technique to estimate dynamically, in a Kalman Filter algorithm, the measurement noise covariance matrix by means of the Bayes Theorem (BT), using the Inverted-Wishart distribution (IW-R Kalman Filter).

The method was compared with the Linear Kalman filter, the AKF with Sage-Husa method and AKF with residual-based estimation of R_k and adaptive estimation of Q_k . In all cases the proposed method was better at estimating trajectories.

The improved performance is due to the probability distribution, which enables the updating of the Noise Covariance Matrix, via Bayesian inference, reducing the matrix uncertainty and improving the prediction results of the filter. A new estimator for Q_k , also based on IW distributions, will be the next step in this research, to be published as a future work.

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