On the use of Frequency-Domain Subspace-based System Identification for Estimating Resonant Systems

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Abstract: In this paper, a parametric identification of high order resonant systems from frequency response data is analyzed. Among many techniques for obtaining black-box models for such systems, subspace-based algorithms stand out. Subspace methods uses geometric projections to obtain specific information necessary to compute the system extended observability matrix and, eventually, to obtain a state-space model representation. Throughout the paper, the method is also shown to use only well conditioned matrices to avoid numerical problems with high order models. The validity of the algorithm is firstly demonstrated on a simulated data of a synthetic 15th order system. Then, in order to state the systems effectiveness, the method is applied to actual data extracted from a resonant beam.

Keywords: Black-Box Modeling, System Identification, Subspace-Based Algorithms, Resonant System, High voltage equipment

1. INTRODUCTION

Resonant systems and vibration analysis are object of many study areas in engineering and widely applied in structural analysis, civil construction, aeronautics, electrical engineering and others. Several recent scientific research about resonant systems are addressed willing to obtain detailed description of its models, to pattern its resonant poles and behavior (Jeong et al., 2013; Noshadi et al., 2016; Kumon et al., 2000). Indeed, due to the oscillatory nature of resonant systems, it is highly recommended to investigate their operational limits via reliable simulations in order to avoid disruptions and injuries to equipment. Therefore, procedures to obtain an accurate description of the system behavior is of great interest.

A considerable number of studies has focused in computing resonant systems models in the field of structural analysis, such as in El-Gazzar (2017); Dou and Jensen (2015); Ruzziconi et al. (2013). An important challenge facing researchers is that to investigate and monitor the effect of oscillations in the system's stability (Zhao et al., 2018; Yao and Li, 2015). Likewise, in the area of energy supply, given the growing complexity in operating power grids, to supervise the stability of the system became increasingly important. In a electrical power system, resonant equipment are interconnected in the network, and there is an ongoing power exchange among the components. In such

cases, even small perturbations added to contingencies, can cause large power variations and lead to instability.

An equipment with resonant behavior that requires special attention are high voltage power transformers, due to its high cost equipment and importance to connect different parts of the grid (Cigrè-JWG-A2/C4.52, 2020; Cigrè-JWG-A2/C4.39, 2014). Brazilian study shows that for the 20 step-up transformers inspected due to failure, 6 of the failures were associated with resonant transients generated on the system (Bechara, 2010; Shipp et al., 2011). Moreover, literature shows that the effect of switching in Gas Insulated Substations (GIS) can damage connected power and instrument transformers due to the occurrence of Very Fast Transient Over-voltages (VFTO) (Gorayan and Chandrakar, 2013; Shu et al., 2013).

In this context, black-box models based on exogenous experimental data is an efficient procedure for describing dynamic behavior resonant system and requires little or no prior knowledge of the system to be modeled (Deschrijver et al., 2008; Grivet-Talocia and Gustavsen, 2016). However, obtaining a black-box model can be a complex and error-prone task at various stages of the process. Initially, a set of measurement data should be collected. Due to the properties of resonant systems, it is common in the electrical engineering to perform the data acquisition and system identification from frequency-domain data samples. Moreover, literature shows a frequent use of data analyzers to obtain information in frequency domain by collecting samples non-uniformly spaced (McKelvey, 1995; Ljung, 2007). Indeed, due to wide band nature of the measure-

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ment spam, the number of samples would be huge if the data was linearly spaced. Considering that, this paper is interested in system identification techniques based on non-uniformly spaced frequency-domain wide-band data.

Among noniterative methods to system identification, subspace-based algorithms have been the object of many research. It is a widely explored group whose applications are found in different fields as in Rongshan et al. (2014); Bakir (2011); Park and Noh (2011). This approach does not involve parametric optimization and is based on concepts from system theory, algebra and statistics (McKelvey et al., 1996).

When compared to traditional identification methodologies, this approach is advantageous because it requires no initial parametrization, but also due to its effortlessness on computing the state sequence directly from input-output data. Another point to highlight is the fact that there is no difference between SISO and MIMO identification for subspace-based algorithms. Furthermore, the idea of estimating one single state-space model in a minimal realization makes the technique attractive.

Although subspace system identification methods based on time-domain data are widely used in the literature, much less works addressing subspace methods for non-uniformly spaced frequency domain wide-band data are found, specially for resonant system study. Here, it should be mentioned the works of McKelvey et al. (1996) and Overschee and Moor (1996a). In light of that, this paper discusses the differences between the algorithms and their performance for resonant system applications. Therefore, this paper presents a comparison between both algorithms and its performance for resonant system applications.

This paper is organized as follows: in Section 2 the problem of estimating models from frequency domain data is formulated. Section 3 discusses two subspace system identification algorithms based on frequency domain non uniformly spaced data. In Section 4, it is presented the application of the method in two case studies, the first one using data from a synthetic process and a second one considering an actual flexible beam.

2. PROBLEM STATEMENT

Consider a linear time-invariant, continuous-time system with p outputs and m inputs described by the following state-space equations:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (1a)$$

$$y(t) = Cx(t) + Du(t), \quad (1b)$$

with $u(t) \in R^m$ and $y(t) \in R^p$ input and output signals. Also assume that the system is of finite order n and consider it initially relaxed.

Using Laplace transform, the time-domain description in (1) turns to a frequency-domain representation as in (2).

$$sX(s) = AX(s) + BU(s), \quad (2a)$$

$$Y(s) = CX(s) + DU(s), \quad (2b)$$

where $s = j\omega$ is the Laplace complex variable. Eventually, the frequency response is given by

$$Y(s) = D + C(sI - A)^{-1}B. \quad (3)$$

When $U = (\mathbf{1} \ \mathbf{1} \ \dots \ \mathbf{1})$, a vector with ones, (2) is rewritten as (4). By making the input as mentioned we consider, at once, the information obtained from m different input vectors. Note that $\bar{X}(s) \in R^{n \times m}$ is the transformed state vector caused by an impulse applied to the input.

$$s\bar{X}(s) = A\bar{X}(s) + BI_1, \quad (4a)$$

$$Y(s) = C\bar{X}(s) + DI_1. \quad (4b)$$

Let $Y(j\omega_k)$ be the measured frequency response of an unknown device under test or equipment, where ω_k , $k = 1, \dots, N$, are the measured frequencies, which may not necessarily be equidistantly spaced and N is the number of samples. Then $\{Y(j\omega_1), \dots, Y(j\omega_N)\}$ is the set of samples.

Although equidistantly distributed data is widely applied in time-domain system identification, it is less common in practical applications for wide-band frequency-domain data. The use of equidistantly distributed data in logarithmic scale is a common tool to overcome the issues concerned to working with big sets of samples.

The problem addressed in this paper is the one to use subspace techniques to fit a state-space model in terms of the quadruple (A, B, C, D) , given a set of non-equidistantly spaced, wide-band, frequency domain samples $\{Y(j\omega_1), \dots, Y(j\omega_N)\}$.

3. SUBSPACE-BASED IDENTIFICATION

Subspace algorithms have been firstly formalized by Overschee and De Moor (1991) in which a novel algorithm to system identification by using stochastic models was proposed. Matter fact, this algorithm works well for systems of small order. Arguably, one may say that whereas the estimate order increases, the user faces some problems related to the conditioning number of some matrices, what may lead the procedure to undesired results. Latter, Overschee and Moor (1996b) have described more robust algorithms in their book, eliciting different approaches for time-domain system identification. After that, one can find many approaches of subspace methods in the literature. However, just a few authors have addressed their research to frequency-domain subspace-based identification using not equidistantly spaced data, as described in the previous section.

In this context, two works stand out: one, described in Overschee and Moor (1996a) whose objective is twofold: *i*) to present a simple frequency-domain system identification algorithm; *ii*) to overcome the problem of estimating models of high order systems with better conditioned matrices. The second work was latter described in McKelvey et al. (1996) where the authors present two subspace based algorithms, a first procedure based on equidistantly, in frequency, spaced data, and a second one on non equidistantly spaced data.

The differences between the methods are to be highlighted in this paper. Lets begin stating that in McKelvey et al. (1996), the system identification methodology is designed for discrete-time systems, whereas in Overschee and Moor (1996a), a continuous-time frequency domain subspace system identification is developed. Because we are interested in continuous-time model estimation, this author

transformed McKelvey et al. (1996) to a continuous-time approach.

3.1 The Algorithm

In general, subspace-based algorithms follow a few steps to obtain the system matrices A, B, C and D . Initially, an output basis matrix is determined using the system response samples. Also, an input basis matrix is created usually from unit vectors somehow concatenated, depending on the approach. Then, an orthogonal projection is applied to the data matrices, which is latter decomposed via SVD. The SVD generates matrices (U, S, V^T) , where $(\cdot)^T$ denotes a transpose matrix, from which important information is obtained. At this point, it is either possible to estimate the order of the model, using the singular values information, and the extended observability matrix, what leads to compute the matrices A and C . Lastly, B and D is determined through a least squares solution of a linear set of equations.

Step 1- Computing the data matrices Because we want to estimate high order models, we need to compute well conditioned input/output matrices. An efficient tool to overcome problems related to ill conditioned matrices are the Forsythe recursions, applied to determined the input basis I_F and the output basis Y_F .

Let D_w be a complex matrix with the measured frequencies, as follows

$$D_w = \text{diag}[(jw_1)(jw_2)\dots(jw_N) \otimes (\mathbf{1} \ \mathbf{1} \ \dots \ \mathbf{1})] \quad (5)$$

where $\otimes$ denotes a Kronecker product. Also assume the size of $(\mathbf{1} \ \mathbf{1} \ \dots \ \mathbf{1})$ as $p \times m$.

To find Y_F and I_F , first initialize defining R_0, R_1 and Z_0, Z_1 as in (6) to (9). Lets denote the complex conjugated transpose as $(\cdot)^*$, and $\text{diag}(\cdot)$ returns a square diagonal matrix with the elements on the main diagonal.

$$R_0 = (Y(jw_1), Y(jw_2), \dots, Y(jw_N)), \quad (6)$$

$$Z_0 := \text{diag}(\text{diag}(R_0 R_0^*)), \quad (7)$$

$$R_1 := R_0 D_w, \quad (8)$$

$$Z_1 := \text{diag}(\text{diag}(R_1 R_1^*)). \quad (9)$$

Then, compute the recursion for $k = 2$ to $q - 1$, where q is a user defined parameter corresponding to an initial estimate order.

$$R_k := R_{k-1} D_w + \frac{Z_{k-1}}{Z_{k-2}} R_{k-2}. \quad (10)$$

$$Z_k := \text{diag}(\text{diag}(R_k R_k^*)). \quad (11)$$

Note that $R_k \in \mathbf{C}^{q \times N}$ and $Z_k \in \mathbf{C}^{q \times p}$. Now, defining D_1 and D_2 , diagonal matrices of order $q - 2$ as in (12) and (13), respectively.

$$D_1 = \begin{bmatrix} \frac{Z_1^{1/2}}{Z_2^{1/2}} & 0 & \dots & 0 \\ 0 & \frac{Z_2^{1/2}}{Z_3^{1/2}} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{Z_{q-2}^{1/2}}{Z_{q-1}^{1/2}} \end{bmatrix}, \quad (12)$$

$$D_2 = \begin{bmatrix} \frac{Z_1}{(Z_0 Z_2)^{1/2}} & 0 & \dots & 0 \\ 0 & \frac{Z_2}{(Z_1 Z_3)^{1/2}} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{Z_{q-2}}{(Z_{q-3} Z_{q-1})^{1/2}} \end{bmatrix}. \quad (13)$$

Finally, the output basis, with the superscript c standing for complex.

$$Y_F^c = \begin{bmatrix} R_0 \\ \frac{Z_0^{1/2}}{R_1} \\ \frac{Z_1^{1/2}}{R_2} \\ \dots \\ \frac{R_{q-1}}{Z_{q-1}^{1/2}} \end{bmatrix}. \quad (14)$$

Hence, $Y_F = (\Re[Y_F^c] \ \Im[Y_F^c])$, with $\Re(\cdot)$ and $\Im(\cdot)$ meaning the real and imaginary part of $(\cdot)$. Then, we have

$$Y_F Y_F^T = I_{q \times m}, \quad (15)$$

$$\text{cond}(Y_F) = 1. \quad (16)$$

where $\text{cond}(\cdot)$ returns the condition number of a given matrix.

To compute the input basis, make $R_0 = (I_m \ I_m \ \dots \ I_m)$ with I_m the identity matrix of order m , and the following steps are equivalent. Then, $I_F = (\Re[I_F^c] \ \Im[I_F^c])^1$.

Step 2 - Orthogonal projection Because we need to extract specific information from the ambient space defined by the response samples, one may be interested in the orthogonal projection.

Now that Y_F and I_F are computed, the orthogonal projection is defined as in (17) because $I_F I_F^T = I_{m \times q}$.

$$H_F / I_F^\perp = H_F - H_F I_F^T I_F. \quad (17)$$

Step 3- The SVD and its partitions

$$H_F / I_F^\perp = U S V^T = [U_1 \ U_2] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}. \quad (18)$$

Once the the decomposition is performed, it is possible to determine the order of the system (denoted by n) from the number of dominant singular values. An interesting decision is to make $n = \text{rank}(S)$.

Let $\gamma \in \mathbb{R}^{(p \times q) \times n}$ be:

$$\gamma = U_1 S_1^{1/2}. \quad (19)$$

where $\bar{\gamma}$ denotes γ without the first $2p$ rows; $\bar{\gamma}$ corresponds to γ without the first p rows and without the last p rows; $\underline{\gamma}$ denotes γ without the last $2p$ rows. Also γ_p as the first p rows of γ .

Step 4- Computing the system matrices A and C In this step, the aforementioned algorithms propose different approaches to compute A and C . In Overschee and Moor (1996a) the authors use the information from the SVD as follows

¹ Note that (15) and (16) holds for the input basis as well.

$$A = [D_1 \bar{\gamma}]^\dagger [\bar{\gamma} - D_2 \underline{\gamma}]. \quad (20)$$

$$C = Z_0^{1/2} \gamma_p. \quad (21)$$

where $(\cdot)^\dagger$ stands for the Moore-Penrose pseudo-inverse, a generalized inverse matrix.

In McKelvey et al. (1996), the authors replace the use of γ and its partitions by matrices J_1 , J_2 and J_3 , and compute A and C as follows. Differently from (20) and (21), this approach does not use information from matrix S , provided by the SVD.

$$A = (J_1 U_s)^\dagger J_2 U_s. \quad (22a)$$

$$C = J_3 U_s. \quad (22b)$$

where

$$J_1 = [I_{(q-1)p} \ 0_{(q-1)p \times p}], \quad (23a)$$

$$J_2 = [0_{(q-1)p \times p} \ I_{(q-1)p}], \quad (23b)$$

$$J_3 = [I_p \ 0_{p \times (q-1)p}]. \quad (23c)$$

where I_i denotes the i th order identity matrix and $0_{i \times j}$ denotes the $i \times j$ order zeros matrix.

Step 5- Computing the system matrices B and D Lastly, B and D is determined through a least squares solution of (24).

$$\min_{B,D} \left(L - M \begin{bmatrix} B \\ D \end{bmatrix} \right) \quad (24)$$

for L and M defined as follows

$$L = \begin{pmatrix} Y(jw_1) \\ Y(jw_2) \\ \vdots \\ Y(jw_N) \end{pmatrix} \quad (25)$$

$$M = \begin{pmatrix} C(jw_1 I_n - A)^{-1} I_p \\ C(jw_2 I_n - A)^{-1} I_p \\ \vdots \\ C(jw_N I_n - A)^{-1} I_p \end{pmatrix} \quad (26)$$

4. PRELIMINARY RESULTS

In this section, numerical examples are presented to corroborate and validate the previously exposed theory. We start this discussion with a comparison on the algorithms in Subsection 4.1, with regard to the fourth step of the algorithm. Then, to illustrate the performance of subspace techniques, we choose two highly resonant system of large order that span some decades of frequency. In both cases, the algorithm described in Subsection 3.1 was applied with number N of sample frequencies much bigger than q .

The first study case uses simulated data and the model approximation is shown in Subsection 4.2. The second one presents estimation results for real data collected from a resonant beam, in Subsection 4.3. The data set was shared by James S. Welsh from University of Newcastle, Australia.

4.1 Comparing the algorithms

At this point the reader may ask what entails the different approaches to compute A and C . To answer that, we

Figure 1. Magnitude of system and model frequency response.

use both methods to estimate a simple system, with the following parameters in regard to the representation in (4), and then check their eigenvalues.

$$A_1 = \begin{bmatrix} -0.1200 & -9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = [1 \ 0.1000 \ 7.5000],$$

$$D_1 = 0.$$

On one hand, the estimation of matrix C_1 returned exactly the same parameters for the two methods. About matrix A_1 , which eigenvalues are $[0; 0.0600 + 2.9993i; 0.0600 - 2.9993i]$, until the fourteenth decimal place, there was no difference in the estimated eigenvalues from both methods. Therefore, despite the different approaches, the final estimation may not be affected by choosing one method instead of one another. Taking that in consideration, from this time forth, we prefer to compute A and C via (20) and (21), respectively.

4.2 Illustrative example

Consider the noise-free case in which 350 samples were acquired from a synthetic continuous-time single-input single-output system of order 15, described by a set of state-space equations as in (2) with real A , B , C and D matrices.

The approximation between the model and the system frequency response is illustrated in Fig. 1 and Fig. 2, for magnitude and phase respectively. The Root Mean Square Error (RMSE) computed is approximately 10^{-7} . Moreover, to this illustrative example whose order was known a priori, the rank of S matrix from SVD provided the expected estimate order.

4.3 Real data from a resonant beam

In this topic, in addition to a higher level of resonance, real data from measurement is used to estimate a model.

Figure 2. Angle of system and model frequency response.

Figure 3. Resonant beam experimental setup.

The experimental setup and measurement samples have been presented in (Welsh and Goodwin, 2003; Gilson et al., 2018), the frequency axis is defined from 10 *rad/s* to 600 *rad/s* and 1600 points were used to estimate the model.

Fig. 3 illustrates the experimental setup described in Gilson et al. (2018). The bar consists of a 60cm long uniform aluminum beam and a pair of piezoelectric elements are attached symmetrically to either side of the beam. In this experiment, a vector analyzer is used to determine the frequency response of the piezoelectric laminate and the experimental data span approximately two decades.

The data frequency response is illustrated in full line and the approximation is illustrated in in dashed line in Fig. 4 for magnitude, and Fig. 5 for phase.

The estimate model is of 19th order, and the computed RMSE is approximately 0.4205, which is considerable. It is possible to note that the algorithm showed more difficulty to approximate a model from real data than simulated data.

Literature shows that in such cases, using real data to estimate parameters, the identification techniques fail in some peaks, specially the low frequency ones. Therefore, as a suggestion for future works, the authors point the necessity to weight the frequency response samples, so that a weighted subspace-based algorithm is developed. This procedure claims to improve efficiency and diminish the RMSE.

Figure 4. Magnitude of frequency response from resonant beam and model.

Figure 5. Angle of frequency response from resonant beam and model.

5. CONCLUSION

This paper discussed the application of subspace based algorithms to compute a state-space model of highly resonant systems. Two solid algorithms for frequency domain identification were discussed and they showed very similar estimation results. The discussed methodology deals with well conditioned matrices what is an advantage when comparing to other subspace-based procedures. To validate the technique, this paper have examined subspace approach efficiency using a simulated example and real data from a resonant beam. We showed that the research with subspace must forward toward a better approximation of real data, by means of improvements in the algorithm.

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