Robust Current Estimation Applied to the Multiloop Resonant Control of Uninterruptible Power Supplies^{*}

V. C. da Cunha, J. V. Flores, L.F.A. Pereira, A.T. Salton,
J. M. Gomes da Silva Jr.

*School of Engineering, Universidade Federal do Rio Grande do Sul,
Brazil (email: {vicente.cunha, jeferson.flores, lfpereira, aurelio.salton,
jmgomes}@ufrgs.br).*

Abstract: This paper addresses the use of inductor current estimators in the multiloop control of uninterruptible power supplies (UPS), assuming that only the output voltage is available for feedback and taking into account the robustness against load variations. Resonant and proportional controllers are assumed to regulate the voltage and inductor current loops, respectively. Conditions to independently design estimator and controller gains are provided, allowing the use of the proposed estimator in preexisting control schemes. Closed-loop stability under estimated current feedback is assessed a posteriori. Simulation results for a single-phase 3.5 kVA UPS show compliance with the IEC 62040 standard.

Keywords: Uninterruptible Power Supply, Current Estimation, Robust Control.

1. INTRODUCTION

Uninterruptible power supplies (UPS) are power electronic devices with the primary purpose of maintaining the continuity of a power source, safeguarding its load against short-term outages. Depending on design, UPS may also be able to provide regulated power quality by keeping the supplied voltage characteristics (e.g. frequency, amplitude, harmonic distortion) within desirable ranges. UPS performance can be assessed in terms of industrial normative standards, such as the IEC 62040-3 (2011).

In terms of control design, the UPS output voltage should track a sinusoidal reference of specific frequency and amplitude, while rejecting disturbances with high harmonic content. From the theoretical point of view, compensators based on the internal model principle – IMP, are best suited to achieve this goal, such as resonant (Pereira et al., 2014) and repetitive (Escobar et al., 2007) controllers. Most control topologies reported in the literature (Lee et al., 2015; Xu et al., 2013; Ryan et al., 1997) make use of multiloop structures that include the feedback of inductor current to improve the overall transient performance.

The feedback of inductor current implies the need of a current sensor, which may be implemented in a variety of ways. Nevertheless, the most relevant requirement for high-power UPS is that the sensor should be electrically isolated from the measured conductor. Most off-the-shelf sensors are based on the Hall effect, usually mounted around the measured conductor. However, one may point

out a number of issues inherent to these sensors, e.g., its positioning dependence, temperature drift and measurement errors due to magnetization (Ziegler et al., 2009). Also, the use of analog to digital converters for data acquisition and control introduces quantization noise. Finally, UPS control is implemented through semiconductor switching, which may also induce high frequency noise (Sangswang and Nwankpa, 2004).

Besides having to account for the mentioned noise issues, another major design concern for UPS control is related to load uncertainty. From a designer perspective, the only available specification for load is its maximum power consumption, which translates into a range of possible load admittance values. The standard Kalman filter approach for systems with an uncertain parameter consists in assuming a stochastic process model for the uncertain parameter and taking it as an additional state to be estimated (Jazwinski, 1970). Another possible estimation strategy is similar to “high-gain” filtering (Prasov and Khalil, 2013): specify a nominal model for the system and assume the load variation as additional process noise, therefore increasing the Kalman filter gain and the estimation responsiveness to measurements. To keep computational requirements low, a non-adaptive formulation of the Kalman filter can be used at the expense of estimation performance degradation (Simon, 2006), using only the steady-state filter gain instead of periodically recalculating it. Besides that, one could consider an estimator design procedure that is robust to load uncertainty, so from these arguments, (Petersen and McFarlane, 1994; Xie and Soh, 1994) present a steady-state Kalman filter extension to systems with norm-bounded uncertainties. Both Petersen and McFarlane (1994) and Xie and Soh (1994), using slightly different approaches, obtained an estimator with an upper-bound on the estimation error covariance by the solution of two

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continuous-time algebraic Ricatti equations (CAREs) that are coupled by a scaling parameter.

This paper proposes a robust design procedure for the multiloop resonant control of a single-phase UPS assuming that only the UPS output voltage is measured. The proposed controller structure is based on a resonant controller to regulate the voltage loop and a proportional inductor current feedback to improve transient performance. Assuming a norm-bounded modeling of the uncertain load connected to the UPS output, the robust steady-state Kalman filter presented in Petersen and McFarlane (1994) is employed for inductor current estimation. A three-step design procedure is considered: i) controller design is carried out in terms of an optimization problem under linear matrix inequality (LMI) constraints that guarantee closed-loop performance in transient and steady-state; ii) an inductor current estimator is computed by the solution of an optimization problem with CARE constraints that minimize an upper bound for the estimation error covariance; iii) closed-loop stability under the estimated inductor current is verified *a posteriori*. The proposed method is then validated in PSIM for a 3.5 kVa single phase commercial UPS in terms of the IEC 62040 performance requirements.

Notation: $\text{tr}\{X\}$ denotes the trace of matrix X and $E\{x\}$ the expected value of x ; $\text{diag}\{X, Y\}$ is a block diagonal matrix with X and Y in the main diagonal; $\text{He}\{X\} \triangleq X + X^T$. $\mathcal{R}\{s\}$ denotes the real part of s .

2. AVERAGE MODEL OF UPS OUTPUT STAGE

In most applications, online double-conversion UPS are composed of an input rectifier, a DC energy storage device and an output inverter. Only the output inverter will be considered in this paper, while the rectifier and energy storage stages are assumed to provide a DC voltage referred to as "DC link". Fig. 1 illustrates the output stage as comprising a half-bridge inverter and a LC low-pass filter (Pereira et al., 2014).

A common approach for obtaining a linear model of switched converters is to use an averaged state-space model for the circuit over a switching period, also known as state-space averaging Middlebrook and Cuk (1976). From Fig. 1 and assuming a PWM high frequency switching, then the control input $u(t)$ can be defined as the averaged input voltage of the low-pass filter, which is proportional to the switching duty-cycle. With the connection of a time-varying load with admittance $Y_{\text{load}}(t)$ and a current source $i_d(t)$, the equivalent circuit for the averaged model is presented in Fig. 2. Disturbances in the DC link (such as ripple voltage) are modeled by $v_d(t)$. The output current

Figure 1. The output stage of UPS.

Figure 2. Average model of UPS output stage with load.

disturbance $i_d(t)$ mainly represents the harmonic content of nonlinear loads, but both $v_d(t)$ and $i_d(t)$ may include other unmodeled disturbances as well (Pereira et al., 2014).

As UPS are not designed for specific loads, but rather for any load up to a maximum power, the load admittance is uncertain but restricted to the interval $Y_{\min} \leq Y_{\text{load}}(t) \leq Y_{\max}$. This range can be rewritten in terms of a time-varying parameter $\delta(t)$, as follows:

$$Y_{\text{load}}(t) = Y_0 + \delta(t) \frac{Y_{\min} - Y_{\max}}{2} \quad (1)$$

$$Y_0 := \frac{Y_{\max} + Y_{\min}}{2}, \quad \|\delta(t)\| \leq 1 \quad (2)$$

The state-space representation of the UPS output stage average model is then given by

$$\begin{cases} \dot{x}(t) = [A + B\delta(t)C_\delta]x(t) + Bu(t) + B_w w(t) \\ y(t) = Cx(t) \quad e(t) = r(t) - y(t) \end{cases} \quad (3)$$

$$\begin{aligned} A &= \begin{bmatrix} -R_{L_f}/L_f & -1/L_f \\ 1/C_f & -Y_0/C_f \end{bmatrix} & B_\delta &= \begin{bmatrix} 0 \\ \frac{Y_{\max} - Y_{\min}}{2C_f} \end{bmatrix} \\ C_\delta &= [0 \ 1] & B &= \begin{bmatrix} 1/L_f \\ 0 \end{bmatrix} & B_w &= \begin{bmatrix} 1/L_f & 0 \\ 0 & -1/C_f \end{bmatrix} \\ C &= [0 \ 1] & w(t) &:= [v_d(t) \ i_d(t)]^T \end{aligned} \quad (4)$$

where $x(t) := [x_1(t) \ x_2(t)]^T$, with $x_1(t)$ and $x_2(t)$ being, respectively, the averaged inductor current and capacitor voltage.

3. MULTILOOP RESONANT CONTROL DESIGN

In the sequel, a design method for the state-feedback multiloop resonant controller is proposed, corresponding to step i) mentioned in section 1.

Consider a transfer function

$$G_{\omega_0}(s) = \frac{(\alpha_{\omega_0} + \beta_{\omega_0})s + (\alpha_{\omega_0} - \beta_{\omega_0})\omega_0}{s^2 + \omega_0^2} + \gamma_{\omega_0} \quad (5)$$

where ω_0 is a given resonance frequency and α_{ω_0} , β_{ω_0} , γ_{ω_0} are constants. From the IMP, the inclusion of $G_{\omega_0}(s)$ in the direct path between tracking error $e(t)$ and plant input $u(t)$ guarantees asymptotic reference tracking for a sinusoidal reference signal $r(t)$ with frequency ω_0 , as long as α_{ω_0} , β_{ω_0} , γ_{ω_0} are tuned to ensure the closed-loop stability. In control literature this structure is usually referred as a resonant controller (Pereira et al., 2014).

The IMP also guarantees asymptotic disturbance rejection if plant disturbances $w(t)$ are sinusoids with the same frequency as $r(t)$. However, this is not the case when dealing with nonlinear loads connected to the UPS output, since the output current disturbance $i_d(t)$ will contain harmonic components which are odd-multiples of ω_0 . To

achieve the disturbance rejection of N odd harmonics, (5) can be expanded to include N resonance modes, as follows:

$$G_c(s) = G_{\omega_0}(s) + G_{3\omega_0}(s) + \dots + G_{(2N-1)\omega_0}(s), \quad (6)$$

which can be written in state-space by:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c e(t) \\ y_c(t) = C_c x_c(t) + D_c e(t) \end{cases} \quad (7)$$

$$\begin{aligned} A_c &= \text{diag}\{A_{\omega_0}, A_{3\omega_0}, \dots, A_{(2N-1)\omega_0}\} \\ B_c &= \begin{bmatrix} B_{\omega_0}^T & B_{3\omega_0}^T & \dots & B_{(2N-1)\omega_0}^T \end{bmatrix}^T \\ C_c &= \begin{bmatrix} C_{\omega_0} & C_{3\omega_0} & \dots & C_{(2N-1)\omega_0} \end{bmatrix} \end{aligned} \quad (8)$$

$$\begin{aligned} D_c &= \gamma_{\omega_0} + \gamma_{3\omega_0} + \dots + \gamma_{(2N-1)\omega_0} \\ A_{n\omega_0} &:= \begin{bmatrix} 0 & n\omega_0 \\ -n\omega_0 & 0 \end{bmatrix} & B_{n\omega_0} &:= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ C_{n\omega_0} &:= [\alpha_{n\omega_0} \quad \beta_{n\omega_0}]. \end{aligned} \quad (9)$$

Assuming now a multiloop control structure with (7) regulating the voltage loop and a proportional controller with gain K_i in the current loop, then the control signal $u(t)$ is defined as

$$u(t) := y_c(t) + K_i x_1(t). \quad (10)$$

This particular control structure results in the closed-loop block diagram of Fig. 3, where the block named UPS refers to the output stage depicted in Fig. 2.

The closed-loop interconnection between the plant and controller can be represented as follows:

$$\begin{cases} \dot{x}_a(t) = [A_a + B_\Delta \delta(t) C_\Delta] x_a(t) + B_a u(t) + B_W w(t) + B_r r(t) \\ y(t) = C_a x_a(t) \end{cases} \quad (11)$$

$$\begin{aligned} x_a(t) &:= \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} & A_a &= \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix} \\ B_\Delta &= \begin{bmatrix} B_\delta \\ 0 \end{bmatrix} & C_\Delta &= [C_\delta \quad 0] & B_a &= \begin{bmatrix} B \\ 0 \end{bmatrix} \\ B_W &= \begin{bmatrix} B_w \\ 0 \end{bmatrix} & B_r &= \begin{bmatrix} 0 \\ B_c \end{bmatrix} & C_a &= [C \quad 0] \end{aligned} \quad (12)$$

with

$$u(t) = K x_a(t) + D_c r(t) \quad (13)$$

and $K = [K_i \quad -D_c \quad C_c]$.

To address the performance constraints imposed by UPS standards, the design of gain K must consider the following control objectives:

- the closed-loop system should be robustly $\mathcal{D}$ -stable (i.e. eigenvalues of $A_a + B_\Delta \delta(t) C_\Delta + B_a K$ should lie in a $\mathcal{D}$ -region of the complex plane for all $\|\delta(t)\| \leq 1$);
- the RMS gain from disturbance $w(t)$ to the regulated output $y(t)$ should be minimized.

The $\mathcal{D}$ -region under consideration is the same one as in Pereira et al. (2014), resulting from the intersection of

Figure 3. UPS control loop using sensor measurements.

a left half-plane ($\mathcal{R}\{s\} \leq -\sigma$) with an origin-centered disk ($\|s\| \leq \rho$). The choice of $\mathcal{D}$ -region with real part boundary $-\sigma$ imposes a minimum exponential decay rate to the closed-loop state trajectories, which can be tuned to achieve the desirable transient performance. The disk boundary ρ limits the maximum eigenvalues of $A_a + B_\Delta \delta(t) C_\Delta + B_a K$, which helps to avoid an arbitrarily high control signal.

LMI constraints for the quadratic $\mathcal{D}$ -stability of the system are given in the following theorem:

Theorem 1. If there exists $R = R^T > 0$, $Y = KR$, $\mu_1 \geq 0$, $\mu_2 \geq 0$ that satisfy the following LMI constraints:

$$\begin{bmatrix} X_2 & RC_\Delta^T \\ C_\Delta R & -\mu_1 \end{bmatrix} < 0 \quad (14)$$

$$X_2 := \text{He}\{A_a R + B_a Y\} + 2\sigma R + \mu_1 B_\Delta B_\Delta^T \quad (15)$$

$$\begin{bmatrix} -\rho R + \mu_2 B_\Delta B_\Delta^T & A_a R + B_a Y & 0 \\ RA_a^T + Y^T B_a^T & -\rho R & RC_\Delta^T \\ 0 & C_\Delta R & -\mu_2 \end{bmatrix} < 0 \quad (16)$$

then system (11) is quadratically $\mathcal{D}$ -stable (in the intersection of a half plane to the left of $-\sigma$ and an origin-centered disk with radius ρ) for all $\|\delta(t)\| \leq 1$.

Proof. The proof of this theorem follows the same procedure presented in Chilali et al. (1999) and will be omitted due to space constraints.

From Boyd et al. (1994), if there exists $R = R^T > 0$, $Y = KR$, $\mu_3 \geq 0$ and $\gamma \geq 0$ that satisfy the following LMI constraint:

$$\begin{bmatrix} X_3 & RC_\Delta^T & RC_a^T \\ C_\Delta R & -\mu_3 & 0 \\ C_a R & 0 & -\gamma^2 \end{bmatrix} \leq 0 \quad (17)$$

$$X_3 := \text{He}\{A_a R + B_a Y\} + B_W B_W^T + \mu_3 B_\Delta B_\Delta^T \quad (18)$$

then the RMS gain of system (11) from $w(t)$ to $y(t)$ will be less than γ for all $\|\delta(t)\| \leq 1$. The design of feedback gains $K = YR^{-1}$ that result in an optimal RMS gain bound γ while accounting for quadratic $\mathcal{D}$ -stability can then be obtained by solving the optimization problem:

$$\min_{R, Y, \mu_1, \mu_2, \mu_3} \gamma \text{ subject to: } \begin{cases} (14) \text{ to } (18) \\ R = R^T > 0 \\ \mu_1, \mu_2, \mu_3, \gamma \geq 0 \end{cases} \quad (19)$$

4. STATE ESTIMATOR DESIGN

Based on the assumption that the UPS output voltage is the only state measurement available, the idea is to design a robust state estimator to implement the inductor current feedback loop. With this aim, consider now that the UPS average model (3) presents both measurement and process noise given by $\psi(t)$ and $\phi(t)$, respectively. Hence, the open-loop plant is now represented by

$$\begin{cases} \dot{x}(t) = [A + B_\delta \delta(t) C_\delta] x(t) + Bu(t) + \phi(t) \\ y(t) = Cx(t) + \psi(t) \end{cases} \quad (20)$$

where $\phi(t)$ encompasses both the disturbance $w(t)$ and other effects such as modeling errors due to the state-space averaging technique.

In addition, consider a state estimator in the form of

$$\begin{cases} \dot{\hat{x}}(t) = A_e \hat{x}(t) + B_e y(t) + Bu(t) \\ \hat{x}_1(t) = C_e \hat{x}(t) \quad C_e = [1 \quad 0] \end{cases} \quad (21)$$

where the state estimation error is given by $e_e(t) := x(t) - \hat{x}(t)$. Then, the goal is to determine estimator matrices A_e and B_e such that an upper bound for the estimation error covariance $E\{e_e(t)^T e_e(t)\}$ is minimized.

For the purposes of using the Kalman filter results, the following assumption holds.

Assumption 2. $\phi(t)$ and $\psi(t)$ are mutually uncorrelated zero-mean white gaussian noise processes. $\phi(t)$ has covariance matrix W and $\psi(t)$ has covariance matrix V .

The work of Petersen and McFarlane (1994) extended the steady-state Kalman filter results to systems with norm-bounded uncertainties, providing an upper-bound for the estimation error covariance. The main result of this reference is given under Lemma 3.

Lemma 3. Suppose that the uncertain system (20) is quadratically stable. Then there exists a constant $\bar{\epsilon} > 0$ such that for all $\epsilon \in (0, \bar{\epsilon})$ the Riccati equation

$$AS + SA^T + \epsilon SC_\delta^T C_\delta S + \frac{1}{\epsilon} B_\delta B_\delta^T + W = 0 \quad (22)$$

has a solution $S = S^T > 0$. For any $0 < \epsilon \leq \bar{\epsilon}$, the Riccati equation

$$AQ + QA^T + \epsilon QC_\delta^T C_\delta Q - QC^T V^{-1} CQ + \frac{1}{\epsilon} B_\delta B_\delta^T + W = 0 \quad (23)$$

also has a solution $Q = Q^T > 0$ and $Q \leq S$. Then, a state estimator (21) with matrices

$$A_e = A + \epsilon QC_\delta^T C_\delta - B_e C \quad (24)$$

$$B_e = QC^T V^{-1} \quad (25)$$

guarantees that $E\{e_e(t)^T e_e(t)\} \leq \text{tr}\{Q\}$, i.e. $\text{tr}\{Q\}$ is an upper bound of estimation error covariance. In addition, $\text{tr}\{Q\}$ is a convex function of ϵ over the range $(0, \bar{\epsilon})$.

It can be shown from (4) that all the eigenvalues of $[A + B_\delta \delta(t) C_\delta]$ have strictly negative real parts for any $Y_{\min} > 0$, i.e., the assumption of quadratic stability of (20) holds true. Firstly, in order to determine a suitable $\bar{\epsilon}$, we consider the following theorem.

Theorem 4. If λ^* is the solution to the following optimization problem

$$\lambda^* = \min_{\tilde{S}} \lambda \text{ subject to: } \begin{cases} (27) \\ \tilde{S} = \tilde{S}^T > 0 \\ \lambda > 0 \end{cases} \quad (26)$$

$$\begin{bmatrix} A\tilde{S} + \tilde{S}A^T + \lambda B_\delta B_\delta^T + W & \tilde{S}C_\delta^T \\ C_\delta \tilde{S} & -\lambda \end{bmatrix} < 0 \quad (27)$$

then $\bar{\epsilon} = 1/\lambda^*$ is such that, for all $\epsilon \in (0, \bar{\epsilon})$, the Riccati equation (22) has a solution $S = S^T > 0$.

Proof. From Petersen et al. (1991), if (22) is to have a solution S , then inequality

$$A\tilde{S} + \tilde{S}A^T + \epsilon \tilde{S}C_\delta^T C_\delta \tilde{S} + \frac{1}{\epsilon} B_\delta B_\delta^T + W < 0 \quad (28)$$

must also hold for $\tilde{S} > 0$. With the change of variable $\lambda = \epsilon^{-1}$ and through the Schur complement, it follows that the satisfaction of LMI (27) implies (28). Since λ and ϵ are inversely proportional, a maximum value for ϵ that satisfies these constraints can be obtained by finding a minimum for λ .

After finding $\bar{\epsilon}$, the state estimator (21) can then be designed by solving the optimization problem

$$\min_{\epsilon, Q} \text{tr}\{Q\} \text{ subject to: } \begin{cases} (23) \\ Q = Q^T > 0 \\ 0 < \epsilon \leq \bar{\epsilon} \end{cases} \quad (29)$$

5. CLOSED-LOOP QUADRATIC STABILITY

The block diagram in Fig. 4 represents the closed-loop system when the inductor current estimate from (21) is considered in the current loop. The state-space representation for this closed-loop system is then given by:

$$\begin{cases} \dot{\bar{x}}(t) = [\bar{A} + \bar{B}_\Delta \delta(t) \bar{C}_\Delta] \bar{x}(t) + \bar{B}u(t) + \bar{B}_W w(t) + \bar{B}_r r(t) \\ y(t) = \bar{C} \bar{x}(t) \end{cases} \quad (30)$$

$$\bar{x}(t) := \begin{bmatrix} x(t) \\ x_c(t) \\ e_e(t) \end{bmatrix} \quad \bar{A} = \begin{bmatrix} A & 0 & 0 \\ -B_e C & A_c & 0 \\ A - A_e - B_e C & 0 & A_e \end{bmatrix} \quad (31)$$

$$\bar{B}_\Delta = \begin{bmatrix} B_\delta \\ 0 \\ B_\delta \end{bmatrix} \quad \bar{C}_\Delta = [C_\delta \ 0 \ 0] \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{B}_W = \begin{bmatrix} B_w \\ 0 \\ B_w \end{bmatrix} \quad \bar{B}_r = \begin{bmatrix} 0 \\ B_c \\ 0 \end{bmatrix} \quad \bar{C} = [C \ 0 \ 0]$$

with control signal $u(t)$:

$$u(t) = [(K_i C_e - D_c C) \ C_c \ -K_i C_e] \bar{x}(t) + D_c r(t) = \bar{K} \bar{x}(t) + D_c r(t) \quad (32)$$

In order to assess the internal stability of the final design, the exogenous signals $w(t)$ and $r(t)$ are disregarded since the internal stability of (30) also implies its BIBO stability (Dorf and Bishop, 2000), resulting in

$$\dot{\bar{x}}(t) = [\bar{A} + \bar{B}_\Delta \delta(t) \bar{C}_\Delta + \bar{B} \bar{K}] \bar{x}(t). \quad (33)$$

Closed-loop quadratic stability can then be verified if a solution $U = U^T > 0$ and $\mu \geq 0$ exists to the feasibility problem with the following LMI constraint (Boyd et al., 1994):

$$\begin{bmatrix} \text{He}\{\bar{A}U + \bar{B} \bar{K} U\} + \mu \bar{B}_\Delta \bar{B}_\Delta^T & U \bar{C}_\Delta^T \\ \bar{C}_\Delta U & -\mu \end{bmatrix} < 0. \quad (34)$$

Summarizing the method proposed in this paper, we present a step-by-step design procedure for the multiloop resonant control of UPS with inductor current estimation:

- i) Solve the optimization problem (19) to obtain the feedback gains C_c , D_c and K_i . In this step, the design

Figure 4. UPS control loop using state estimation.

- parameters are the number of resonance modes (N) and the $\mathcal{D}$ -region parameters (σ and ρ);
- ii) Solve the optimization problem (26) to obtain $\bar{\epsilon}$. With the value of $\bar{\epsilon}$, solve optimization problem (29) and construct estimator matrices A_e and B_e as in (24) and (25), respectively;
 - iii) Verify the resulting closed-loop stability by solving the feasibility problem with constraint (34). If stability is not verified, then repeat the previous steps relaxing the $\mathcal{D}$ -region or reducing the noise covariance matrices W and V .

6. SIMULATION RESULTS

In order to illustrate the proposed methodology, consider the 3.5 kVA commercial UPS presented in Corleta et al. (2016) with parameters described in Table 1 and its corresponding nonlinear test loads under the specifications of IEC 62040-3 Commission (2011). With a resonant controller of $N = 3$ and a $\mathcal{D}$ -stability region with $\sigma = 50$ and $\rho = 5000$, optimization problem (19) results in $\gamma = 1.59$ and the following feedback gains:

$$K_i = -5.19, \quad D_c = 3.90, \\ C_c = [320.10 \quad 395.80 \quad 237.41 \quad 457.47 \quad 170.59 \quad 497.17].$$

The state estimator is then designed considering the measurement noise covariance matrix $V = 0.01$ and process noise covariance matrix $W = 1000I$. The optimization problem (26) yields $\bar{\epsilon} = 1.845$ and the solution of (29) results in $\text{tr}\{Q\} = 22.83$. The resulting closed-loop system is quadratically stable since (34) is feasible.

To validate the designed controller and estimator, we use a software environment comprising Matlab and PSIM. The controller, state estimator and feedback loop are simulated in Matlab Simulink, while the output stage of Fig. 1 (including the switching inverter) and the test loads are simulated in PSIM. Two simulation setups are considered. The first setup, as depicted in Fig. 3, does not use state estimation, but rather uses a feedback loop with an ideal current sensor. The second setup follows Fig. 4 and employs state estimation. Both setups consider an

Table 1. Parameter values used in simulation

Parameter	Symbol	Value
Filter inductance	L_f	1 mH
Filter inductor resistance	R_{L_f}	15 m Ω
Filter capacitor	C_f	300 μ F
Minimum load admittance	$Y_{\min}$	0.1 mS
Maximum load admittance	$Y_{\max}$	151.9 mS
Reference signal frequency	ω_0	$2\pi \times 60$ rad/s
Reference signal amplitude	-	$127\sqrt{2}$ V
DC Link Voltage	-	520 V
PWM switching frequency	-	21.6 kHz
Nonlinear load series resistor (25%)	-	0.73 Ω
Nonlinear load series resistor (75%)	-	0.25 Ω
Nonlinear load parallel resistor (25%)	-	41.57 Ω
Nonlinear load parallel resistor (75%)	-	13.86 Ω
Nonlinear load capacitor (25%)	-	3007 μ F
Nonlinear load capacitor (75%)	-	9021 μ F

Figure 5. UPS output voltage and current for 100% nonlinear test load employing the estimated current feedback.

output voltage sensor with zero-mean measurement noise of covariance $V = 0.01$.

From these setups, Table 2 presents a comparison of the output voltage total/individual harmonic distortion (THD/IHD). As it can be seen, the THD increases when the state estimator is used instead of an ideal sensor, nevertheless remaining within the margins imposed by IEC 62040. Fig. 5 shows a plot of output voltage and current for the 100% nonlinear test load using the simulation setup with state estimation.

The simulation of transient performance for nonlinear load steps from 25% to 100% nominal value is plotted in Fig. 6, showing compliance to the most restrictive overvoltage and undervoltage profile presented in the IEC62040. Fig. 7 presents a plot of the estimated inductor current for the different levels of nonlinear load. It can be seen that the overall behavior of the inductor current is correctly estimated by the proposed method.

7. CONCLUSIONS

In this work we have presented a systematic design procedure for inductor current estimation in the multiloop resonant control of single-phase UPS. A method based on three independent steps was proposed, where controller and estimator design was carried out by the solution of optimization problems under LMI/CARE constraints and

Table 2. Harmonic distortions of output (100% nonlinear load)

	max (IEC62040)	feedback with x_1 measurements	feedback with x_1 estimation
THD	8%	3.81%	4.14%
IHD ₃	5%	0.002%	0.001%
IHD ₅	6%	0.003%	0.0004%
IHD ₇	5%	3.46%	3.79%
IHD ₉	1.5%	0.63%	0.87%
IHD ₁₁	3.5%	1.22%	1.23%
IHD ₁₃	3%	0.34%	0.27%

Figure 6. Transient response to nonlinear load steps compared to overvoltage and undervoltage limits of IEC62040-3 with estimated current feedback.

Figure 7. Filter inductor current estimation under different nonlinear loads.

with closed-loop stability assessed a posteriori. Simulation results in PSIM showed compliance to IEC 62040 test standards and a small increase on the THD under current estimation. The experimental validation of the proposed method in an industrial UPS setup is an ongoing work.

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