

DUAL FORMULATION FOR LI AND REEVES MODEL

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RESUMO

Em 2016, Chaves e outros apresentaram uma formulação dual para o modelo MCDEA de Li e Reeves (1999). No entanto, eles consideraram formulações parciais em duas funções bi-objetivo de cada vez usando o método lexicográfico para resolver o modelo tri-objetivo original de Li e Reeves (1999). O modelo de Chaves et al (2016) apresenta pouca liberdade para os decisores e não consideram todas as características do método de Li e Reeves (1999). Neste artigo tem como objetivo apresentar melhoria para formulação dual com uma abordagem que considera todas as características e aspectos do modelo tri-objetivo de Li e Reeves (1999). Combina-se, pelo método de soma ponderada, as três funções objetivo em uma função única. O modelo dual proposto oferece a busca de benchmarks. Com a função única pelo método da soma ponderada pode-se ainda analisar a eficiência para a DMU no método do Li e Reeves. Além disso, o modelo apresentado apresenta maior liberdade ao decisor nas análises.

PALAVRAS CHAVE. MCDEA, dual, multiobjetivo.

ABSTRACT

In 2016, Chaves et al presented a dual formulation for MCDEA from Li and Reeves (1999) model. Nevertheless, they considered just partials formulations for two objective functions each time, using a lexicography method to solve the tri-objective model from Li and Reeves (1999). Chaves et al (2016) model present lower freedom for decision-makers and did not consider whole aspects from Li and Reeves (1999) method. In this paper, we aim to present an improvement in the dual approach considering all aspects and characteristics from tri-objective Li and Reeves (1999) method. We combine, by weighted sum, the three objective functions into single one. We seek to provide a dual model, which is possible to check benchmarks. With the weighted sum method, we can also analyze the efficiency value for the DMU in Li and Reeves (1999) method. Moreover, our approach for formulation brings more freedom for the decision-maker analysis.

KEYWORDS. MCDEA, dual, multiobjective.

1. Introduction

Li and Reeves (1999) developed a model using Data Envelopment Analysis (DEA) associated with Multiobjective Linear Programming to solve recurrent problems in classic DEA models.

This intervention aims to increase the discriminatory power of the evaluated units (DMU – decision making unit) and also obtain a more homogeneous distribution in the weights.

To solve these problems, Li and Reeves (1999) model uses a multiobjective function that considers additional the criteria of minisum and minimax. In the end, after the resolution procedure, the problem gets few efficient DMUs which becomes valid when the sample of the problem is small (Cooper et al., 2001).

In this line, Chaves et al (2016) proposed a dual model for Li and Reeves (1999) formulation to provide benchmarks and targets for the DMU's. However, this approach considers in pairs the objective functions from Li and Reeves (1999) model by lexicographic method.

With this process, Chaves et al (2016) do not aggregate all aspects from the tri-objective Li and Reeves (1999) formulation. According to Li & Reeves (1999), all three objective functions are independent and equally important.

Herein, we bring the characteristics from Li and Reeves (1999) formulation of efficiency analysis, equality of the sample and overall optimization for a dual approach (Soares de Mello et al, 2009). We use a weighted sum methodology to linearize the tri-objective functions into a single one and then we formulate the dual model.

We show in the first part of the article theoretical bases regarding Multiple Criteria Data Envelopment Analysis, then a review for TRIMAP software and MCDEA, following with our dual formulation Li and Reeves (1999) model. Then we use this dual method in a numerical example, finalizing with some conclusions.

2. Theoretical bases

a. Multi Criteria Data Envelopment Analysis (MCDEA)

Multi Criteria Data Envelopment Analysis is a DEA model with additional objective functions (Clímaco et al, 2008), useful because classic DEA models are not suitable for small data sets (Cooper et al., 2001). MCDEA is also used to improve the discrimination and dispersion of weights (Bal et al, 2008, Chang et al 2007 and Ghamesi et al, 2014).

Li and Reeves (1999) presented a multiobjective model that seeks to improve the discrimination of DMUs in DEA and to promote better distribution of multipliers through the primal model. The model proposed by Li and Reeves (1999) was formulated restricted only by classic CCR model from DEA.

This model consists of three objective functions: (1) to determine the efficiency of each analyzed DMU, (2) to minimize the sum of the deviations (minisum) and (3) to minimize the maximum deviation M (minimax). These three objective functions are independent of each other.

The objective function is a direct representation of the deviation sum. A DMU is classified as efficient in the minimax function if, and only if, the value corresponding to the solution that minimizes the third objective function of the MCDEA model is zero (Chaves et al, 2016).

In the proposed model, Li and Reeves (1999) developed only their primal formulation by Equation (1):

$$\begin{aligned}
& \text{Min } d_o \\
& \text{Min } \sum_{k=1}^n d_k \\
& \text{Min } M \\
& \text{Subject to} \\
& \sum_{i=1}^r v_i x_{io} = 1 \\
& \sum_{j=1}^s u_j y_{jk} - \sum_{i=1}^r v_i x_{ik} + d_k = 0, \quad k = 1, \dots, n \\
& M - d_k \geq 0, \quad k = 1, \dots, n \\
& u_j, v_i \geq 0, \quad \forall j, i
\end{aligned} \tag{1}$$

Chaves et al (2016) highlighted Li and Reeves model does not provide efficiency index for all DMU's, nor targets or benchmarks. Soares de Mello et al (2009) propose a model for a MCDEA index to efficient DMU's, while Chaves et al (2016) suggest a dual model to provide benchmarks.

Classic DEA model is widely used to analyze the benchmarks of inefficient units, from association between the similar units, in order to obtain the best practices adopted by the sample. Application of DEA modeling has been used in several areas (Kao, 2014), as well as has presented mathematical evolutions (Cook et al., 2014).

In a classic DEA formulation there are n DMU's to be evaluated, x_{ij} is the value of the input i to the DMU j , and y_{rj} is the value of the output r for the DMU j . With the decision of the weights of the output variables $u_1, u_2, u_3, \dots, u_s$ (one for each output item) and weights of the input variables $v_1, v_2, v_3, \dots, v_m$ (one for each input item) the mathematical formulation of the method is systematized below, where the relative efficiency of the DMU k will be determined by the classical CCR model represented by Equation (2).

$$\begin{aligned}
& \text{Min } \theta \\
& \text{Subject to} \\
& \sum_{j=1}^n y_{rj} \lambda_j \geq y_{rk} \quad r = 1, \dots, s \\
& \sum_{j=1}^n x_{ij} \lambda_j - \theta x_{ik} \leq 0 \quad i = 1, \dots, m \\
& \lambda_j \geq 0, \quad \forall j = 1, \dots, n
\end{aligned} \tag{2}$$

From the definition of the variable λ_k , which can also be defined as the participation of the efficient DMU j in the target goal DMU k under analysis, the benchmarks are defined from the values found in λ_k , if they are greater than zero ranging up to one, the value will be the ratio that the DMU j is a reference for the DMU k . We highlight that a DMU is efficient if, and only if, the efficiency equals one ($\theta = 1$) and all slacks from the constraints are zero.

3. MCDEA-TRIMAP

The TRIMAP software, developed by Clímaco & Antunes (1987, 1989) is a free search method that provides the set of non-dominated solutions in tri-criteria linear programming problems, through a learning process. Despite its interactive environment, TRIMAP could be used for MCDEA problems, as a generator of solutions and a tool for weights space analysis (Soares de Mello et al., 2009).

Besides computing all optimal solutions for MCDEA's three objective functions, TRIMAP also presents graphical representations, one of which is very useful for MCDEA, namely the weights space decomposition. This representation shows the indifference regions of the weights space that correspond to the non-dominated basic solutions. See Figure 1 with an example from TRIMAP triangle solutions result.

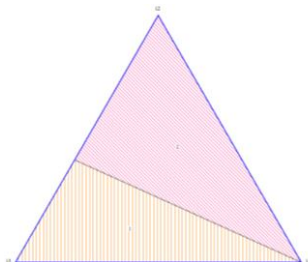


Figure 1 – Regions with set of solutions from TRIMAP

We should highlight that such weights refer to the multipliers of the objective functions, and that, in indifference regions, these weights could vary without altering the solution. The weights space is represented in a triangle, whose corners correspond to the optimization of each objective function.

With such tool, it is possible to evaluate whether the DMUs' optimal evaluations have stable solutions or if they depend on specific multipliers. Large indifference regions indicate that the solution remains the same even with moderate changes of the objective function's weights. For instance, if the same indifference region contains two triangle corners, one associated with the classic objective function and the other associated with the minimax or minisum objective functions, the observed DMU is therefore minimax or minisum efficient, respectively. Moreover, it is also possible to identify potentially good solutions, which improve results for the second and third objective functions, even if they do not confer the DMU's maximum efficiency, for instance.

In view of all these possible analyses, Soares de Mello et al. (2009) developed the MCDEA-TRIMAP efficiency index, which takes into account all possible infinite combinations of the objective function weights.

4. Li and Reeves dual model proposal

The Chaves et al (2016) model split the Li and Reeves multiobjective method in two bi-objective model considering first the efficient function with minsum and then the efficient with minmax. Further to solve the linear programming Chaves et al (2016) use the multiobjective method considering one main function to solve and the others as restrictions for both models. In the end Chaves et al (2016) consider a benchmark for inefficient DMU when they find benchmark in both partial models minisum and minimax.

However, with Chaves et al (2016) we hide the main characteristics from Li and Reeves both additional objective functions limit the possibilities of multipliers for efficient DMUs. These intermediate functions (minisum and minimax) have no meaning in the DEA context, as they only seek to evaluate the efficiency in reference to the analyzed DMU (Pereira et al, 2018).

Clímaco et al (2010) verify Li and Reeves model main characteristics with the three objective functions consist in DMU efficiency by the first objective. By minisum, the model optimizes the overall evaluation of the DMU and by minimax function we measure an equity for all DMU analysed.

The proposed Li and Reeves dual model of this article seeks to determinate benchmarks considering all characteristics from Li and Reeves (1999). However, as the primal Li and Reeves (1999) model is originally a multi-objective method, to set a dual formulation we shall adjust primal model in a mono objective linear programming model.

Among different methods to linearize a multi-objective into a mono objective model, we used the weighted sum method to transform the Li and Reeves (1999) formulation. In this

line, from Li and Reeves (1999) model (3), we generate a set of solution with TRIMAP software.

For each objective function we verify at TRIMAP a set of weights α_1, α_2 and α_3 which provides an optimized solution for the multi-objective programming. Then, using one multi-objective solution we could find a linear programming with a mono objective function (3) considering weights α_1, α_2 and α_3 provided by TRIMAP:

$$\begin{aligned}
 & \text{Min } (\alpha_1 d_o + \alpha_2 M + \alpha_3 \sum_{k=1}^n d_k) \\
 & \text{st } \sum_{i=1}^r v_i x_{io} = 1 \\
 & \sum_{j=1}^s u_j y_{jk} - \sum_{i=1}^r v_i x_{ik} + d_k = 0, \quad k = 1, \dots, n \\
 & M - d_k \geq 0, \quad k = 1, \dots, n \\
 & u_j, v_i \geq 0, \quad \forall j, i
 \end{aligned} \tag{3}$$

We assessed the (4) linear programming to transform into a canonic form, which it is possible to reach duality model. In this consideration, as the (4) is a minimize function, all restrictions must be equal or more form. As Li and Reeves (1999) model the restriction equal form, we created one opposite additional restriction for each original one. In the end we have one positive restriction and another negative (4).

$$\begin{aligned}
 & \text{Min } (\alpha_1 d_o + \alpha_2 M + \alpha_3 \sum_{k=1}^n d_k) \\
 & \text{st} \\
 & \sum_{i=1}^r v_i x_{io} \geq 1 \tag{4.1} \\
 & - \sum_{i=1}^r v_i x_{io} \geq -1 \tag{4.2} \\
 & \sum_{j=1}^s u_j y_{jk} - \sum_{i=1}^r v_i x_{ik} + d_k \geq 0, \quad k = 1, \dots, n \tag{4.3} \\
 & - \sum_{j=1}^s u_j y_{jk} + \sum_{i=1}^r v_i x_{ik} - d_k \geq 0, \quad k = 1, \dots, n \tag{4.4} \\
 & M - d_k \geq 0, \quad k = 1, \dots, n \tag{4.5} \\
 & u_j, v_i \geq 0, \quad \forall j, i
 \end{aligned} \tag{4}$$

Additionally, another modification is needed to provide the dual formulation. We shall relate each restriction into a variable for dual model. In this line, we set the following relationship restriction from primal into variable for dual:

$$\begin{aligned}
 4.1 &= b_o^+ \\
 4.2 &= b_o^- \\
 4.3 &= w_k^+ \\
 4.4 &= w_k^- \\
 4.5 &= m_k
 \end{aligned}$$

At this point, we could formulate the dual for Li and Reeves (1999) as described in (5):

$$\begin{aligned}
& \text{Max } b_o^+ - b_o^- \\
& \text{Subject to} \\
& b_o^+ x_{io} - b_o^- x_{io} - \sum_{k=1}^n w_k^+ x_{ik} + \sum_{k=1}^n w_k^- x_{ik} \leq 0, \quad \forall i = 1, \dots, r \\
& \sum_{k=1}^n w_k^+ y_{jk} - \sum_{k=1}^m w_k^- y_{ik} \leq 0, \quad \forall j = 1, \dots, s \\
& w_o^+ - w_o^- - m_0 \leq \alpha_1 + \alpha_3 \\
& w_k^+ - w_k^- - m_k \leq \alpha_3, \quad \forall k = 1, \dots, n \neq o \\
& \sum_{k=1}^n m_k \leq \alpha_2 \\
& m_k, w_o^+, w_o^- \geq 0
\end{aligned} \tag{5}$$

However, to simplify our formulation we shall consider $w_o^+ - w_o^- = w_k$ and $b_o^+ - b_o^- = b_o$. Then we afford the following Equation (6), which provide the dual linear programming for Li and Reeves (1999) model.

$$\begin{aligned}
& \text{Max } b_o \\
& \text{Subject to} \\
& b_o x_{io} - \sum_{k=1}^n w_k x_{ik} \leq 0, \quad \forall i = 1, \dots, r \\
& \sum_{k=1}^n w_k y_{jk} \leq 0, \quad \forall j = 1, \dots, s \\
& w_o - m_0 \leq \alpha_1 + \alpha_3 \\
& w_k - m_k \leq \alpha_3, \quad \forall k = 1, \dots, n \neq o \\
& \sum_{k=1}^n m_k \leq \alpha_2 \\
& m_k \geq 0, \quad \forall k \\
& b_o, w_k, \text{ free } \forall k
\end{aligned} \tag{6}$$

Analyzing (6) and comparing with dual for CCR classical model (1), we verify the variables w_k provides the λ_j numbers. However, as w_k is as free variable we could find negative numbers. In this case, when λ_j is positive at CCR classical dual we find the respective w_k negative.

The variable b_o means an efficiency for the DMU in analysis considering all aspects of the objective functions aggregated from Li and Reeves (1999) model. This efficiency is influenced by region of solution chose from Trimap. However, among from same region of solution is indifference the set of weights α_k chosen.

5. Numerical Example

We use herein the Brazilian main airports data to evaluate the proposal dual model for Li and Reeves (1999) method. Where we considered data from June 2015 of regular commercial flights analyzed by Pereira (2015). The main airports verified in this study consist in a sample of 6 central airports generated by h-index method (Pereira, 2015): Guarulhos (GRU), Congonhas (CGH), Brasília (BSB), Galeão (GIG), Confins (CNF) and Viracopos (VCP). These airports represent more than 50% of the flights approved by the Brazilian air fleet.

The data from each airport are: airport site, considering the total area, as input. As output, we consider four variables: the number of companies that operate in the airport, the number of certificated departures, quantity of passengers departed, and cargo load departed. Table 1 presents data on the inputs and outputs used to analyze the air transport system of Brazilian airports for the case of this article.

Airports	Airport Site ($\times 10^3$)/m ² (AR)	Airliners (EM)	Takeoffs (DE)	Cargo Load ($\times 10^3$)/t (CL)	Passengers ($\times 10^3$) (PX)
GRU	11905	50	11595	10.25888	985.946
CGH	1647	5	7526	2.208907	749.621
BSB	13774	18	6852	3.695521	739.21
GIG	17881	35	5425	2.164595	493.447
CNF	15120	15	4663	1.089886	417.231
VCP	17659	22	5566	1.01053	370.127

Table 1 - Data from main airports

The objective here is to determine benchmarks from proposed dual model for the main Brazilian airports selected by the h-index (Pereira, 2015). We shall analyze the airports that have a greater demand for the number of flights and passengers given the infrastructure of the analyzed network, in the case the Brazilian sector.

a. Results

Moreover, we compare the classic CCR model Equation (2) with proposed MCDEA model Equation (6) to understand the variables meaning. In this line, we use at Equation (6) $\alpha_1 = 1$, $\alpha_2 = 0$ and $\alpha_3 = 0$, then we could check the efficiency function in parallel with CCR model (2). For an numerical example we use the data from GIG (Galeão Airport) to perform the evaluation. Tables 2 and 3 provide results from classic CCR model Equation (2), while Table 4 and 5 provide results for Equation (3):

VARIABLE	VALUE	REDUCED COST
EM	0.665791	0.000000
DE	0.000000	0.232126
CL	0.000000	0.489003
PX	0.000000	0.199519
AR	1.000.000	0.000000

Table 2 – Variable value from primal CCR for GIG

6)	0.533946	0.000000
7)	0.645853	0.000000
8)	0.694637	0.000000

Table 3 – Slack value for constrains from primal CCR for GIG

ROW	SLACK SURPLUS	OR	DUAL PRICES
2)	0.000000		0.466054
3)	0.000000		0.700000
4)	0.025530		0.000000
5)	0.530630		0.000000

VARIABLE	VALUE	REDUCED COST
D4	0.533946	0.000000
M	0.694637	0.000000
D1	0.000000	0.700000
D2	0.025530	0.000000
D3	0.530630	0.000000
D5	0.645853	0.000000
D6	0.694637	0.000000
AR	1.000000	0.000000
EM	0.665791	0.000000
DE	0.000000	0.232126
CL	0.000000	0.489003
PX	0.000000	0.199519

Table 4 – Variable value from primal MCDEA (3) for GIG

ROW	SLACK SURPLUS OR	DUAL PRICES
2)	0.000000	-0.533946
3)	0.000000	0.700000
4)	0.000000	0.000000
5)	0.000000	0.000000
6)	0.000000	-1.000000
7)	0.000000	0.000000
8)	0.000000	0.000000
9)	0.694637	0.000000
10)	0.669107	0.000000
11)	0.164007	0.000000
12)	0.160691	0.000000
13)	0.048784	0.000000
14)	0.000000	0.000000

Table 5 – Slacks value of constrains from primal MCDEA (6) for GIG

From Table 4 and 5 we could verify the CCR model main property if we consider just the efficiency as an objective function. The proposed formulation primal formulation (3) provides the results in line with CCR model with the case where we do not consider minisum and minimax objective function weights ($\alpha_2 = 0$ and $\alpha_3 = 0$).

As we interest in provide benchmarks, considering all characteristics from Li and Reeves (1999) model, we shall compare the proposed dual formulation (6) with the CCR dual model. Tables 6 and 7 provide results from dual CCR model, while Tables 8 and 9 bring results from dual proposed for MCDEA (6), considering GIG as numerical example.

VARIABLE	VALUE	REDUCED COST
H4	0.466054	0.000000
L1	0.700000	0.000000
L2	0.000000	0.025530
L3	0.000000	0.530630
L4	0.000000	0.533946
L5	0.000000	0.645853
L6	0.000000	0.694637

Table 6 – Variable value from dual CCR for GIG

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-1.000000
3)	0.000000	-0.665791
4)	0.232126	0.000000
5)	0.489003	0.000000
6)	0.199519	0.000000

Table 7 – Variable value from dual CCR for GIG

VARIABLE	VALUE	REDUCED COST
B	0.533946	0.000000
W1	-0.700000	0.000000
W2	0.000000	0.000000
W3	0.000000	0.000000
W4	1.000000	0.000000
W5	0.000000	0.000000
W6	0.000000	0.000000
M1	0.000000	0.000000
M2	0.000000	0.025530
M3	0.000000	0.530630
M4	0.000000	0.533946
M5	0.000000	0.645853
M6	0.000000	0.694637

Table 8 – Variable value from dual formulated (6) for GIG

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.665791
4)	0.232126	0.000000
5)	0.489003	0.000000
6)	0.199519	0.000000
7)	0.700000	0.000000
8)	0.000000	0.025530
9)	0.000000	0.530630
10)	0.000000	0.533946
11)	0.000000	0.645853
12)	0.000000	0.694637
13)	0.000000	0.000000

Table 9 – Slacks value from dual formulated (6) for GIG

From Tables 8 and 9, we verify the CCR dual model main property if we consider just the efficiency as an objective function. The proposed formulation dual formulation (6) provides the results in line with CCR model with the case where we do not consider minisum and minimax objective function weights ($\alpha_2 = 0$ and $\alpha_3 = 0$).

In additional, we could evaluate the benchmarks at proposed dual model are given by $w_k < 0$, from which is related to $\lambda_k > 0$. In our example, we check $w_1 < 0$ and $\lambda_1 > 0$ which provides GRU as a benchmark respectively in both formulations proposed dual and CCR dual model. We evaluate here the feasibility and the comparative implication of w_k and λ_k .

However, we are interest in the formulation considering the meaning of Li and Reeves (1999) model. Then we shall consider all weighted sum formulation with α_1, α_2 and $\alpha_3 \geq 0$. Then we use the TRIMAP software to find a set α_k from a region area of non-dominated solutions.

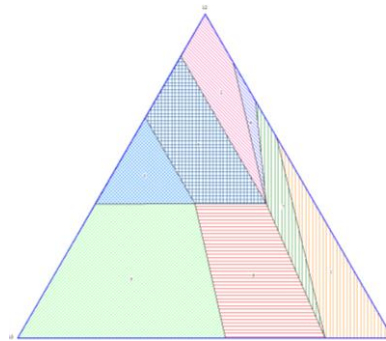


Figure 2 - GIG – Region of solutions for MCDEA

For the solution definition we used model similar to Rubens et al (2015), where arbitrarily, we chose the region 1 from Figure 2 to provide the weight from set α_k at a specific point considering that any solution at same region are non-dominated as discussed before. The set of α_k is following:

α_1	α_2	α_3
0,732237	0,205251	0,062512

Table 10 – Set of weights from region of solution 1 for GIG

Then we use formulation (6) to evaluate the dual proposed for GIG considering the Li and Reeves (1999) meaning. From Table 11 and 12, we analyze the results all aspects from objective function from Li and Reeves (1999).

VARIABLE	VALUE	REDUCED COST	ROW	SLACK SURPLUS	OR	DUAL PRICES
B	0,542917	0,000000	2)	0,0000000		1,0000000
W1	-0,631339	0,000000	3)	0,0000000		0,6657910
W2	0,062512	0,000000	4)	0,1268330		0,0000000
W3	0,062512	0,000000	5)	0,4148720		0,0000000
W4	0,794749	0,000000	6)	0,0892640		0,0000000
W5	0,062512	0,000000	7)	0,6938520		0,0000000
W6	0,062512	0,000000	8)	0,0000000		0,0255300
M1	0,000000	0,000000	9)	0,0000000		0,5306300
M2	0,000000	0,025530	10)	0,0000000		0,5339460
M3	0,000000	0,530630	11)	0,0000000		0,6458530
M4	0,000000	0,533946	12)	0,0000000		0,6946370
M5	0,000000	0,645853	13)	0,2052510		0,0000000
M6	0,000000	0,694637				

Table 11 – Variable value from dual formulated (6) for GIG with a set of α_k

Table 12 – Slacks value from dual formulated (6) for GIG with a set of α_k

Though, as Li and Reeves (1999) model is more conservative then classical CCR model, we find more w_k positives when the related λ_j is zero, but the for the benchmark DMU we still verify w_k negative. This conservative characteristic is because we included in the evaluation the minisum and minimax function to integrate de analysis.

In the numerical example for GIG we verify GRU as benchmark, which is in line with the characteristics from both airports. They are big hubs with international connection to many cities in world. Table 13 presents the benchmarks results from Formulation (6) for all DMU (airports).

Airports	Benchmarks
GRU	GRU ; CGH
CGH	GRU ; CGH
BSB	GRU ; CGH
GIG	GRU ; CGH
CNF	GRU ; CGH
VCP	GRU ; CGH

Table 13 – Slacks value from dual formulated

Results present as benchmark GRU – Guarulhos Airport and CGH – Congonhas Airport, both in São Paulo State. The first one as already described is the biggest international hub to Brazil and the second one is a principal domestic hub. Both airports operate in their full capacity.

GRU presents the highest numbers for all output variables, which is a benchmark variability and importance for the air network. While CGH is the second in passengers and takeoff even being the smallest airport analyzed.

6. Conclusions

In this paper, we presented a dual formulation in line with all characteristics from Li and Reeves (1999) model. We used a different approach of Chave et al (2016), we considered from weighted sum methodology to solve multiobjective functions in order to formulate the dual model.

In our analysis we also bring an efficiency number for the DMU considering the Li and Reeves (1999) model, and from our dual model we could provide the benchmarks for the DMU's.

Future studies shall be done to consider this dual model into an index of MCDEA-TRIMAP, so we could rank the DMU's comparing in a Li and Reeves (1999) methodology by a linearized objective function.

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