

ALLOCATIONS FOR A BITCOIN MINING POOL USING RISK MEASURES

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ABSTRACT

Mining Bitcoins is an uncertain activity. By nature of the process, participants take part in the Proof of Work process, where they try to decrypt the hash that the new block added to the Blockchain contains. One may spend months, or even years, without positive cash flows, while still having associated costs. This can often drive participants away from the technology affecting the network itself, as it cannot survive without their contributions. This study proposes to analyze what has already naturally begun: miners joining forces in mining pools and dividing revenues and costs, thereby reducing risks and generating more positive cash flows. The revenues and costs are modeled and a stochastic optimization problem is proposed to find the optimal allocations that guarantee that all members are still willing to stay within the pool. This group of miners is characterized by a coalition, a sub theory of Game Theory. A monetary risk measure, in the form of CVaR (Conditional Value at Risk) is used to represent the miner's risk profile and consequences to the optimal allocations.

KEYWORDS. Blockchain, Bitcoin, Game Theory, Stochastic Optimization, CVaR.

1. Introduction

With every newly introduced technology comes an inevitable period of adaptation. What we use as money (or monetary value for trading) has had many representations throughout history, be it gold, silver, coins, paper currency, debit cards etc [Eichengreenm 2019]. The system that backs up the monetary circulation today is the fiduciary one, in which there is no financial coverage (as there used to be with gold or dollar). A system based on the trust that those who own the money will pay their debts. Also, money exchanges are the responsibility of governments and banks, the first being able to print new money at will.

These aspects generate criticism. One being that this very centralized model, together with the fiduciary aspect, benefits inflation and at long-term, creates unemployment [Friedman, 1977]. Furthermore, individuals have no real knowledge of the amount of money available and circulating through the system. With the increasing scandals arising from the lack of digital privacy and leaking of personal data, more people want the control to be, at least partially, backed at their hands.

One technology that proposes a solution to this problem is the Bitcoin, supported by the Blockchain network. This cryptocurrency does not have physical financial coverage and is not under the current fiduciary system. It runs on a Blockchain network, where each transaction needs to be verified and registered (within chained blocks) by the miners, with the Blockchain consequently working as a “ledger”. The network is maintained, secured and expanded by the process known as Proof of Work, in which all miners take part. Using computational power, every miner participates in a “guessing game” of a hash in the new block's header. The network is not hosted at any specific server and rather is available at every computer that uses it, making less prone to be attacked. In a seminal article on this topic, Satoshi Nakamoto [Nakamoto, 2008] indicated that the maximum amount of Bitcoins on the system would be 21 Million, thus creating the noninflationary aspect of the cryptocurrency.

Each miner in the network has costs every time he takes part in the Proof of Work, but not always has revenues, only when he is the winner of the process. This creates an uncertain environment, that can soften by miners taking part in a mining pool, where they share profits and costs, thus having more predictable positive cash flows.

Studying player interactions in such a competitive environment is of interest so that there are further and continued incentives for the players to remain in the network, thus keeping it alive and functional ([Kiayias et al, 2016], [Eyal et al, 2013]). Game theory has been at the core of Bitcoin ever since its birth when Nakamoto analyzed incentives in a simple revenue distribution model. Those interactions can be modeled by Game Theory, and this is a Cooperative game scenario.

An optimization model is hereby proposed in this work to find possible allocations for each player in the mining pool, so that all players are satisfied and remain in the network. A Risk Measure Tool, in the form off CVaR (Conditional Value at Risk) is implemented to capture the risk profile of the players into result. This model is tested in an illustrative case study that aims to validate and highlight the benefits of the methodology.

This work is structured as follows; section 2 contains the necessary Blockchain background to understand this work, also as the idea behind the mining pools. Section 3 describes the mathematical models for revenue, cost and profit, as well as the understanding of the CVaR. Later it dives into the mining with a game theory perspective, including the notions of coalition and nucleolus, and finishes by grouping all up then into the optimization model. Finally, section 4 describes the tests and the results, and section 5 concludes with the findings.

2. Blockchain and mining

Bitcoin run on its own Blockchain network. Each block registers and saves all transactions, fees and public information from the participants. There is no specific server hosting the

information, the network is available for any user that wishes to download it and verify the registered information. This is a solid security aspect that prevents hacking of the network.

Each block contains a hash [Nakamoto, 2008], a class of functions with specific encryption properties. These are functions composed of a specific mathematical algorithm (the Mathematical Trap-door [Chalkias et al, 2008] where, in theory, the encrypted information cannot be reversed). Bitcoin utilizes SHA256 encryption [Courtois et al, 2014]. The hash of each block is contained within each of the following blocks, consequently, an attack to the network would have to be directed at all blocks and a new hacked fork would need to ensure that the network would follow it. However, from a financial point-of-view, for an attacker with such power, it would be more interesting to actually mine the Bitcoin.

The Bitcoin network controls the difficulty of the new header according to the computational power from the miners, so that each new block takes about ten minutes to be successfully mined. The continuous verifying of the hash is called mining and its difficulty is exponentially related to the number of zeros in each hash. The mining process is what ensures that anyone can add a new block, while guaranteeing that posterior block revisions are not possible. As a reward, miners obtain a fixed plus a variable Bitcoin amount.

Bitcoin first appearance was in an article written by Satoshi Nakamoto [Nakamoto, 2008]. Following the ideas presented by the author, intermediaries such as banks and governments would not be necessary, as each transaction would be verified by computer algorithms in a peer-to-peer network. Although up until 2012 Bitcoin was more commonly used in virtual black markets, such as the Silk Road [Christin, 2013], but has potential for many lawful applications, such as p2p (Peer-to-Peer) transactions, digital sales, securities trading, energy trading, network infrastructure etc [PwC, 2015].

2.1. Mining Pools

The act of mining is one of uncertainty and competitiveness. It entails guessing the right hash at the right time against many other participants while constantly having costs, but profiting only every now and then.

The miners are the ones that maintain the network operational with their computational power. Without them, people would retain their Bitcoins in their private wallets, and would not be able to negotiate them. Miners have two sources of income: a fixed and a variable revenue. The first is halved at every 210.000 new blocks created and the latter is composed by the sum of the taxes from all the transactions of a given newly mined block. Once the halving process reduces the fixed income to zero, only the variable revenue will be available.

Taking part in the Proof of Work demands great computational power and thus, miners with higher computational power (known as “hashpower” or the computational ability to generate new hashes) are more likely to succeed. Not having an assured positive cash flow for each block makes for great uncertainty and risk, with the participant spending months, or even years, without successfully mining a block.

One possible solution that has already been achieved [Kroll et al, 2013] is to participate in a mining pool, where a group of miners share their revenues and costs, increasing their chances of positive cash flows, even if they are lower in value than the revenue of mining alone. It's a trade-off where one receives less more often rather than receiving more at fewer times. A mining pool constitutes a coalition [Lewenberg et al, 2015], a particular case of cooperative mining game.

3. Mathematical Models

To obtain the results desired from the work, the models for revenue/cost/profit need first to be obtained, as well the theory of CVaR.

3.1. Revenue, costs and profit

The revenue of player j from mining a block can be written as:

$$R_j = R_j^f + R_j^v \quad (1)$$

Where R_j^f is the total fixed income from mining a block, and R_j^v is the total variable income from mining a block.

Mining costs include fixed costs (such as Capex) and marginal costs (mainly energy costs). Because miners can be anywhere in the world, both fixed and marginal costs may vary significantly from player to player. In this study, as a simplification, we assume C_j^e as the fixed cost, in BTC, per block mined.

With the modeled revenues and costs, the profit of player j , considering all n blocks he tries to mine, becomes:

$$L_j = \sum_{i=1}^n (R_j^f + R_j^v - C_j^e) \quad (2)$$

3.2. Conditional Value at Risk

Monetary risk measures treat undesirable results in a more pragmatic manner, where investors try to design portfolios on the basis of a comfortable trade-off between the risk of loss and the possibility of profit. This can be done by the use of a quantitative risk measure, where the risk of loss is quantified.

Conditional Value at Risk (CVaR) is a coherent risk measure (a risk measure that satisfies four axioms; translation invariance, subadditivity, positive homogeneity and monotonicity) that assesses what is the expected loss in the worst α -fraction of cases. Essentially, this is the expected value off the area off the tail off the density up to the α -quantile.

Monetary risk measures are important tools to simulate the game with the risk profile of each player. If only the expected value is used, the results might show that all players tend to go with the coalition.

3.3. Game Theory

Taking part in a mining pool means participating in a cooperative game and the Cooperative game theory studies such interactions, benefits and costs of said players [Manea, 2017].

A cooperative game is an interactive opinion making tool that focuses on the behavior of a group of participants. Each participant has a preference allocation in the outcome of the game. A coalition (C) is a group of players $S \subset N$, and we refer to N as the grand coalition. The coalition happens when these players join together towards a common goal.

Every coalition S has a set of available actions, and an outcome consists off a partition of N and one action associated with each coalition in the partition. Each agent j has preferences represented by an utility function u_j . A coalition action is stable if any subgroup of participants chooses an action that benefits all of these.

The profit of a coalition (C) that aims to mine Bitcoin becomes:

$$L^{pool} = \sum_{j \in N} L_j \quad (3)$$

Considering a coalition in which every player makes a contribution in the form of a hashpower hp_j , and where each player has a probability p_j to mine (related to its hashpower hp_j):

$$\sum_{j \in C} p_j < 1 \quad (4)$$

Equation (6) means that there are more participants trying to mine outside the pool. Each participant in the coalition model proposed will assist with a share of the pools' power.

This is when the concept of nucleolus of a cooperative game arises. Given a measure of value (such as expected value or expected utility). The nucleolus is the set of all allocations x_j in which the income of a player j (or subset of players) in the pool is higher than his income, if he was alone.. For example: If a coalition has three members, the nucleolus is stable (or exists), if there is an allocation between the members that will always benefit all its participants, bringing a higher value than if they worked alone (or in doubles or any combination different from the grand coalition). When a coalition includes an action that all its members prefer, other than the grand coalition, it is said that the former blocks the latter, breaking the concept of the nucleolus. In this work, the nucleolus of the game can be represented by:

$$\sum_{j \in C} x_j \cdot L^{pool} \geq \sum_{j \in C} L_j \quad (5)$$

This work aims to obtain the optimal allocations that guarantee the coalition of a group of miners working to conclude the next block in the Blockchain of Bitcoin, therefore having more predictable positive cash flows and more incentives to remain a player j in the mining game.

3.4. The optimization model

Equation (5), together with the knowledge of CVaR, leads the study of the optimization model used to find the optimal allocations. The variables of the problem are the allocations of each player, x_j . The measure of value used is a (convex) combination of the expected value and the CVaR. An error variable ϵ is used as a tool to help solve the model, that can be written as:

$$\begin{aligned} & \min_{x, \epsilon} \epsilon \\ \text{s.t.:} & \\ & x_j \geq 0, \forall j \in N \\ & \sum_{j \in N} x_j = 1 \\ & \lambda \cdot CVaR_{\alpha}^c \left(\sum_{j \in C} x_j \cdot L_j^{pool} \right) + (1 - \lambda) \cdot \mathbb{E} \left[\sum_{j \in C} x_j \cdot L_j^{pool} \right] \geq \\ & \lambda \cdot CVaR_{\alpha_i} \left(\sum_{j \in C} L_j \right) + (1 - \lambda) \cdot \mathbb{E} \left[\sum_{j \in C} L_j \right] - \epsilon, \quad \forall C \in \mathcal{P}(N) \end{aligned} \quad (6)$$

After all optimal allocations x_j have been proposed by the results of the optimization model, those can be used to obtain the incomes for every player in the pool:

$$L_j^{pool} = L^{pool} \cdot x_j, \forall j \in N \quad (7)$$

L_j^{pool} is different from L_j . The latter is the income that player j would have, had he mined the blocks alone. The former is the income the same player obtains by participating in the pool, with a quota of x_j .

4. Results and discussion

For the results in this section, a game with 3 players in a pool is proposed and analyzed in 4 different scenarios. On the first scenario simulated, the 3 members of the pool are the only ones trying to mine in the entire Bitcoin network and with no costs of doing so. It is assumed that player 1 has 15% of the hashpower, player 2 has 25%, and player 3 has 60%. On the second scenario, they are still the only ones trying to mine, but player 1 has 0.2 BTC of cost to mine, player 2 has 1.5 BTC of cost to mine, and player 3 has 2.5 BTC as cost to mine. Those are related to the computational power of each player, as it is expected that a player with higher hashpower spends more energy to mine. Finally, third scenario shows the introduction of all the other players (in the form of a player 4), with the same costs associated for player 1 to 3. Data of Player 4 is not important, since blocks mined by this Player will not be considered, nor the revenue or costs.. Now, player 1 has 15% of the hashpower, player 2 has 25%, and player 3 has 35%. This makes the pool own 75% of the bitcoin network, leaving 25% for player 4 (or, all the other miners). There is also no presence of a fee to participate in the pool. The fourth scenario is the same as the third, but now $\lambda=0.5$, introducing CVaR α .

α is considered as 0.25 and $\lambda=0.5$, unless expressed differently in the results.

The fixed income from mining is currently at 12.5 BTC (Bitcoins) and is expected to be maintained as such until mid-2020.

The variable income is more complicated and was modeled considering data from January/2018 to July/2018 [Blockchain info, 2019], where its presented a behavior that could be characterized by a log-normal distribution, with parameters: μ : - 0.048 and λ : 0.274 (obtained by maximum likelihood method).

100,000,000.000 blocks are simulated. For each, a result of the log-normal distribution is drawn, so as the result from a uniform distribution from 0 to 1.

The cumulative sum of all the hashpowers in the Bitcoin Network sum 1, so to find the miner for a given block, the result from the uniform distribution must be lower or equal to the given player's hashpower in the cumulative sum. For instance, if the result is 0.12, the successful miner is player 1, if it is 17%, it is player 2, since it is higher than player's 1 contribution, but lower than the sum of player 1 and player 2 (in the cumulative distribution).

4.1. Results of the game

Tables 1 to 4 show the expected revenue from mining alone and in the pool, obtained from the drawn of the 100,000,000.000 scenarios, and can be seen under $\mathbb{E}$. Value, in both cases, is the result of the measure of value used in this work, or in other words, is the result of the right-hand side of the main restriction in equation (6) for a player mining alone, and the result from the left-hand side of the main restriction in equation (6) when mining in a coalition.

The results of the first scenario can be seen in table 1, with the setup previously exposed, the first result is obtained from setting λ to zero, or making the CVaR α irrelevant:

Table 1: Results for the simulated case where there are no costs, $\lambda=0$ and there are only 3 players

	Out of the pool			x_i	In the pool		
	E	CVaR α	Value		E	CVaR α	Value
Player 1	2.30	0.0	2.30	0.15	2.30	0.0	2.30
Player 2	3.83	0.0	3.83	0.25	3.83	0.0	3.83
Player 3	9.18	0.0	9.18	0.60	9.18	0.0	9.18

The expected optimal allocations were the hashpower from each player, as ignoring CVaR α would lead to the optimization using Expected Values. Here, the measure of value returns the same result for both mining alone, and in a coalition. For the second scenario:

Table 2: Results for the simulated case where there are costs, $\lambda=0$ and there are only 3 players

	Out of the pool			x_i	In the pool		
	E	CVaR α	Value		E	CVaR α	Value
Player 1	2.10	-0.2	2.10	0.189	2.10	1.94	2.10
Player 2	2.33	-1.5	2.33	0.209	2.33	2.15	2.33
Player 3	6.68	-2.5	6.68	0.602	6.68	6.19	6.68

Introducing costs to the game alters the optimal solution in terms of allocations, but the revenues are still the same for both cases, as the measure of value. Introducing costs alters the revenue per block, so the new allocations guarantee the existence of the core, for the same value. Now, for the scenario where the pool is not alone in the network:

Table 3: Results for the simulated case where there are costs, $\lambda=0$ and there are more than 3 players

	Out of the pool			x_i	In the pool		
	E	CVaR α	Value		E	CVaR α	Value
Player 1	2.10	-0.2	2.10	0.288	2.10	-1.21	2.10
Player 2	2.33	-1.5	2.33	0.320	2.33	-1.34	2.33
Player 3	2.86	-2.5	2.86	0.393	2.86	-1.65	2.86

The introduction of the pool in the network of miners shifts the optimal allocations from the hashpower. The CVaR of the coalition now is negative, which is expected, since in scenarios 1 & 2, the pool would always mine, hence having positive cash flows every block. That does not happen when the 4th player is introduced, and having similar hashpower. Finally, the fourth scenario:

Table 4: Results for the simulated case where there are costs, $\lambda=0.5$ and there are more than 3 players

	Out of the pool			x_i	In the pool		
	E	CVaR α	Value		E	CVaR α	Value
Player 1	2.10	-0.2	0.948	0.616	4.48	-2.59	0.948
Player 2	2.33	-1.5	0.413	0.269	1.95	-1.13	0.413
Player 3	2.86	-2.5	0.178	0.116	0.84	-0.48	0.178

With the risk measure (CVaR α) being considered, there is a shift in the optimal allocations, but one interesting outcome is observed: for player 1 (the ones with the least computational power), the income from mining in the pool is now higher than mining alone, and player 2 & 3 see lower

incomes. With risk measures, it would be expected that players with more computational powers are being benefited from helping the pool, but Player 2 and Player 3 have both higher costs than Player 1, which can explain why, in the coalition, the outcome is the one observed. Since there is no creation of value by being in the pool, the measure of value stays the same for all the players.

4.2. Accumulated Probabilities results and the graphical representation of the core

Using the results of the last scenario from 4.1., the accumulated probability to mine for each player, as well as their accumulated gains from mining can be represented as:

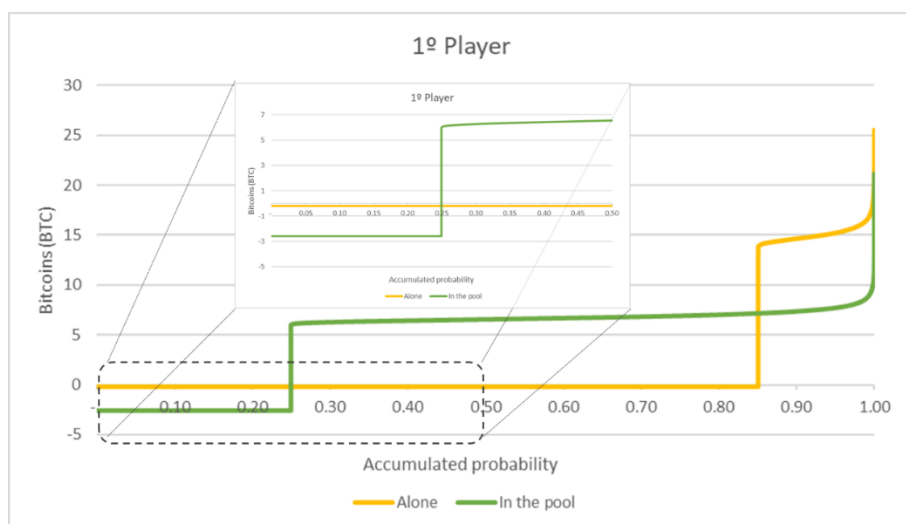


Figure 1: Accumulated probabilities and revenues for player 1

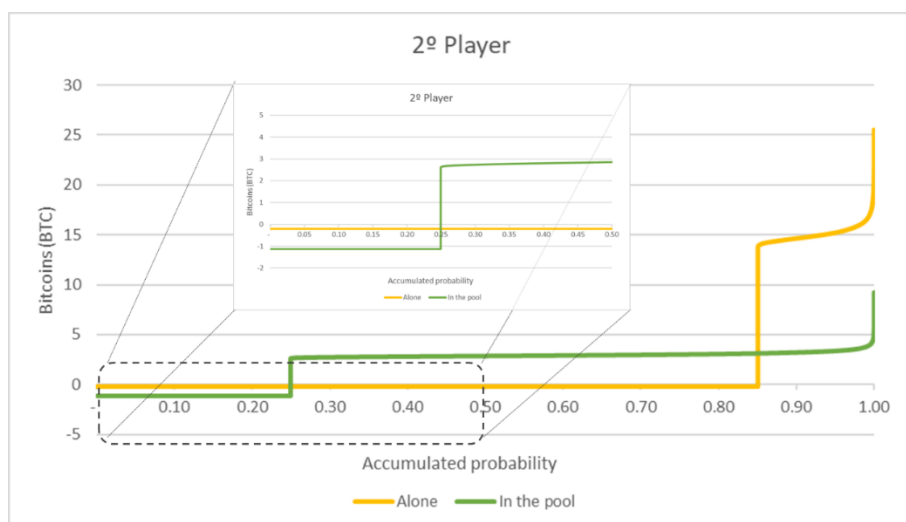


Figure 2: Accumulated probabilities and revenues for player 2

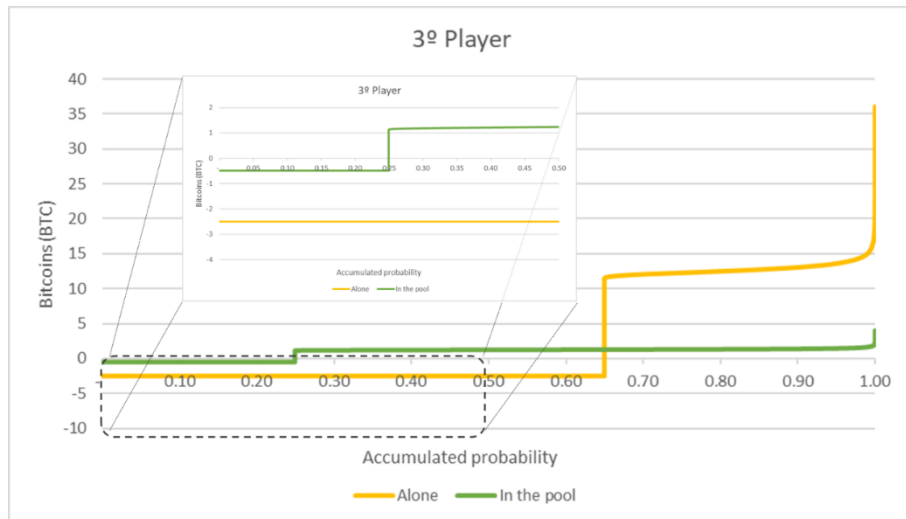


Figure 3: Accumulated probabilities and revenues for player 3

Player 1, when mining alone, spends approximately 0.85 percent of the time with negative cash flows, until his blocks are mined and he begins to have positive cash flows. This matches 1 minus his hashpower (0.15 %). On the other hand, Figure 1 shows that when mining in the coalition, player 1 spends approximately 0.25 percent of the time with negative cash flows, matching 1 minus the coalition's accumulated hashpower (0.75 percent). This result shows that being part of the coalition benefits player 1, as he obtains more constant positive incomes.

Players 2 & 3 exhibit similar behaviors, with player 2 spending approximately 75% of the time with negative cash flows, and player 3 approximately 65%. For player 3, here is where he has gains from taking part in the pool. In magnitude, the sum of revenues from all blocks for Player 1 is higher when he is in the coalition, but for Players 2 & 3, the sum when being alone is higher. Even though his average income is lower, he receives positive cash flows more often, giving more predictability and stability.

Now, considering that Player 2 has the same behavior that in figures 1 to 3 when mining alone, but testing sensibility for a low, medium and high λ , figure 4 shows the variation in the accumulated probabilities:

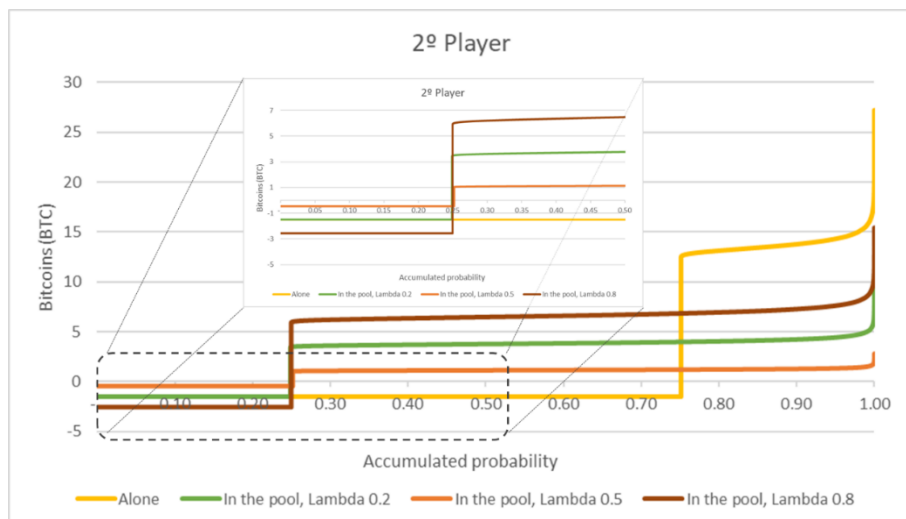


Figure 4: Accumulated probabilities and revenues for player 2, different λ

The higher λ is set, it was expected that the variance would be lower (since the risk is being considered), but $\lambda = 0.8$ shows the variance being higher.

For the next test conducted, the optimal allocations were "forced" into the model, with x_1 , x_2 and x_3 varying each from 0.0 to 1.0, with steps of 0.0001, and λ from 0.0 to 1.0, with the same steps of 0.1.

Figure 5 shows the obtained results. The colored dots show the results for ϵ with each λ for x_1 and x_3 (x_2 is perpendicular, outside the 2D realm), within a tolerance of 10^{-3} . The results show that the allocations within the nucleolus of the game are the optimal solutions found by the optimization model for each lambda, as will be seen in 4.3. Each of those results are, in fact, a visual representation of the nucleolus of the game. Values of λ for 0.6, 0.7, 0.8 and 0.9 do not produce any solution within the nucleolus of the game with a tolerance of 10^{-3} .



Figure 5: Representation of the allocations inside the core

4.3. Sensibility analysis for λ

It is of interest to investigate the results for different values of λ . This is the parameter that defines how much importance a player is giving to its risk profile and to expected value. The setup used was the same as for the last scenario in 4.1, and the results are:

Table 5: Sensibility test for λ

	λ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
x_1	0.288	0.304	0.328	0.367	0.438	0.616	1.00	0.00	0.00	0.00	0.047
x_2	0.320	0.317	0.313	0.307	0.296	0.268	0.00	0.465	0.386	0.366	0.357
x_3	0.392	0.379	0.358	0.326	0.266	0.116	0.00	0.535	0.614	0.634	0.595

It can be noted that when raising λ , the importance of the CVaR α raises, which shifts the allocations in an unpredicted behavior, producing degenerate solutions, such as for $\lambda=0.6$.

5. Conclusions

Models for the revenue, cost and profit of a miner of Bitcoin, whether alone or in a coalition have been obtained. In addition, an optimization model to find the allocations of the players in a nucleolus of a game theory approach has also been devised and implemented.

While mining alone can provide higher incomes for a player, accumulated probabilities result indicates that said player is likely to spend most of its mining work with negative cash-flows, since his costs are always present. While mining in a pool, on the other hand, nets him positive cash flows more often, offering a more “secure” way to operate.

The introduction of a risk measure is shown to change the optimal allocations of a pool, depending on the weight that it receives.

A visual representation of the core of the game was obtained, showing that inside all the possible allocations with the 3 players, the ones that visually make up the core of the game (the ones that are inside the nucleolus) with an accepted tolerance, are the ones obtained from the optimization model.

Further work aims to raise the number of players included in the pool, as well as obtaining more robust representations of the variable incomes and the costs associated with mining Bitcoin. A time-dependent representation of the game is also in the plans. Also important is to analyze the reason for the behavior of the allocations with different λ .

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