MULTIPLE CRITERIA DECISION ANALYSIS AND COMPLEX NETWORKS APPLIED TO NBA SEASON LEADERS STUDY

Vanessa Helena Pereira-Ferrero
School of Applied Sciences (FCA), University of Campinas (UNICAMP)
Pedro Zaccaria, 1300, 13484-350, Limeira, São Paulo, Brazil
nessahelena@gmail.com

Theodore Gyle Lewis
Center for Homeland Defense and Security (CHDS), Naval Postgraduate School (NPS)
1 University Circle, 93943, Monterey, California, United States
tlewis@nps.edu

Luciane Graziele Pereira Ferrero
School of Applied Sciences (FCA), University of Campinas (UNICAMP)
Pedro Zaccaria, 1300, 13484-350, Limeira, São Paulo, Brazil
lucianegraziele@gmail.com

Leonardo Tomazeli Duarte
School of Applied Sciences (FCA), University of Campinas (UNICAMP)
Pedro Zaccaria, 1300, 13484-350, Limeira, São Paulo, Brazil
leonardo.duarte@fca.unicamp.br

ABSTRACT

Collective and competitive games such as basketball are a kind of assisted and practiced category by millions of people around the world. The Complex Network theory associated with MCDA is a novel method for weight attribution and players ranking study, which involves important tools of both statistics and mathematics. The main goal is to analyze the NBA leading players' variables like nodes and their level of interactions as links, for calculation of networks metrics which can be used for ranking athletes. This approach made possible to rank players at 2 distinct scenarios for comparative analysis. Our study introduces a new way of interpreting the ranking of leaders' players, focusing on which variables the player can prioritize for improve performance.

KEYWORDS. Multiple Criteria Decision Making, Computational Science, Complex Networks, Basketball, NBA, Performance.

Paper topics:
TAG – Graph Theory and Algorithms
OA – Other Operations Research Applications

1. Introduction

The Multiple Criteria Decision Making (MCDM) also known as Multiple Criteria Decision Analysis (MCDA) is an important scientific tool used to support decision making that is capable of involving both quantitative as qualitative factors [Mardani et al., 2015].

As a discipline, multi-criteria decision making has a relatively short history. Since the 1950s and 1960s, when the foundations of modern multi-criteria decision-making methodologies were laid, much research devoted it's time to the development of new models and decision-making techniques [Zavadskas, Turskis, & Kildiené, 2014].

The applications of MCDA can be related to: electricity, environment, sustainability, supply chain management, quality management, GIS-Geographic Information System, construction and project management, safety and risk management, manufacturing systems, technology, operational research and soft computing, strategic management, knowledge management, production management, tourism management, among other areas [Mardani et al., 2015, Oztaysi, 2014].

Throughout the world, methodologies have been developed, although not concentrated on systematically presenting the most remote theoretical basis. Especially because the focus of research involving decision making is on the methodological choice and the structure for evaluation of decisions, which will impact those involved more directly. Such research is still under discussion and evolution. In addition, there is a growing need for research that shows not only strengths and weaknesses of different decision-making methodologies, but also develops strategies for how to associate it with another tools and their various scenarios and applications [Zavadskas, Turskis, & Kildiené, 2014].

MCDA models may present an ideal choice and even the best course of action. The recommendations may include rankings and may present decision support at different stages of the overall decision-making process [Bouyssou et al., 2006]. Decision making is a process that often involve different actors: people, groups of people, institutions, etc. In the last decades, research and development in the field has accelerated and seems to continue to grow. However, given the diversity of needs, there are still many gaps related to ideal models in each decision scenario [Oztaysi, 2014].

Mardani et al. [2015] point out that different modeling techniques are presented in the literature, and a number of approaches are provided for modeling decision support. However, there is a growing need to develop alternatives, considering that the complexity of decision-making processes has increased.

The choice of a more appropriate approach to problem solving and a model also depends on the actors involved in the process, the desired goals, information and time available, among other issues. The most important advantage of MCDAs is the ability to address the problems of conflicting interests. In some MCDA applications, actors can solve problems that are not solvable using common optimization models. MCDA techniques and approaches are increasingly being used for the evaluation of alternatives and comparative analyzes [Mardani et al., 2015].

This methodology is part of the area of operational research and has its focus on mathematical and computational modeling as a tool to support decision makers in various areas of knowledge, but there are several gaps to be addressed with new approaches [Zavadskas, Turskis, & Kildiené, 2014].

The formal and final decision, as well as the most appropriate evaluation models, should, in most cases, be implemented through complex decision-making processes. Hence the idea of association of decision-making methodologies with other topics such as complex networks, little explored for this purpose. The best decision depends not only on the qualitative but also quantitative aspects of the tool itself that was implemented in the structuring of the problem [Bouyssou et al., 2006]. In this way, complex networks and their tools can be associated to decision support methodologies, given this identified gap and aiming to be applied in real problems and improvement of the processes involved.

An emergent discipline in scientific field in general and, particularly, in Sports Science is the Complex systems or Complex networks. Ecological dynamics are understood as the study of complex systems that try to explain the coordinated sport behaviors (patterns of variability, stability and symmetry-breaking) that emerges from interactions of performers with environment [Vilar et al. 2012]. Any complex analysis of sports performance makes it possible to identify collective variables that describe, characterize and explain game restrictions. The main observation of research involving complex mechanisms was the identification of the spatial-temporal patterns, highlighting their importance [Duarte et al. 2012].

Complex networks for games analysis are example of decision making processes and were applied using professional tennis players, and showed complex characteristics, typical of many real networks of contacts studied in the literature. A diffusion algorithm applied to the network of tennis contacts to classify professional players, identified Jimmy Connors as the best player in tennis history according to a classification procedure. This work shows the importance of complex network analysis in sports and in identifying the best players. They observed that the rank complex method had a higher predictive power and did not require the arbitrary introduction of external criteria for the correct assessment of player quality. These new evidences of the usefulness of tools and methods of complex network theory in real applications shows the need for the development of such methods applied in other sports [Radicchi 2011].

The usefulness of network methods to understand human interpersonal interactions within neurobiological social systems as sports teams has been the subject of studies for bringing benefits and advances. The networks of collective systems are supported by the sum of the interpersonal interactions that emerge from the activity of the agents of the system (like the players of a sports team). To test this idea, the intra-team collective behaviors in the collective sport of the water polo were analyzed. Future research on small-scale methodologies needs to formalize node-fitting measures in analyzes of collective behaviors in sports teams to see if a high frequency of interactions between players is required to achieve competitive performance results under the optics and complex networks [Passos et al. 2011].

Authors have argued that the self-organization of a sports competition emerges from the relationships between teams and their exchanges of information. Playing behaviors in team sports may be a consequence of the interaction of players interacting, but at the highest level, the team of the pair, that is, the pair of both teams in a match. It is important to note that there is a relative phase to evaluate the relationships between teams. The results of such analyzes showed a strong phase-in relationship in the longitudinal direction that reflects the nature of basketball teams moving at the same time from basket to basket, which has direct consequences on scores [Bourbousson et al. 2010].

There is a growing need for rigorous data collection in the field of collective sport sciences that provide empirical evidence on the complex reality to which they refer, given the nature of the items analyzed. Sports psychology in quality and quantity has advanced in recent years, but a research area that remains relatively unexplored is the dynamics of the sports group and the influence of the group on its members as individuals. The present study is part of a larger research project on the teams of the Spanish Basketball First Division (ACB) and considers the interaction context before teams try to score (where it is understood how teams create scoring opportunities). A model of analysis to study the effectiveness and construction of offensive basketball games to identify their results provides coaches with an important device to enhance or consolidate those [Fernandez et al. 2009].

Moreira et al. [2015] show in their study on injuries in Brazilian men's basketball the need to analyze different components but in a co-related form, such as: Functional Capacity, Mental Domain, Pain, etc. Listening to different components in a comparative way is very interesting and growing, hence the need for multi-analyzes such as those proposed in the approach to complex networks.

The team offensive network model to identify players' abilities to team success in players is also analyzed [Skinner & Guy 2015]. Studying the games of the 2011 playoff series between the Memphis Grizzlies and Oklahoma City Thunder, even with a very limited data set,

the authors show that a complex network can describe a player's interactions related to their performance, and even may focused on tracking algorithms utilizing public real available data, it is highlighted the importance of complex analysis on more players and practical data [Shitrit et al. 2014].

With the focus more on group behavior and the impact of social networks on team players, it is important to note that status is derived from both their individual attributes and the groups with which they are inserted. The status of individuals can benefit or suffer from the group. When the group stands out, high status members potentially receive a lot of credit and increase status. This result is in line with the research on status inconsistency, suggesting that individuals do not emphasize their group affiliation when it compromises their individual status. An additional contribution is the advancement of the Social Relations Model for the analysis of binary links in social networks [Koster & Aven 2018].

These studies are examples of how complex a game or an individual performance can be, once we find studies from Psychology and Social Sciences until specific biological conditions of athletes, representing the growing necessity of join in complex analysis different aspects for understanding the ideal performance in athletes during competitions.

The dynamics of the team and their role in the team's overall role considering games in the first round of the 2010 NBA play-offs were studied as complex networks. The analyzes demonstrate the usefulness of network approaches in quantifying the team strategy and show that testable hypotheses can be evaluated using this approach. These analyzes also highlight the richness of basketball networks as a set of data to explore the relationships between the structure of the network and the dynamics with the organization and effectiveness of the team [Fewell et al. 2012].

Considering the MCDA contextualization here explained and the cited research involving techniques of performance analysis and basketball results, the performance determinants of this complex sport are not completely clear in the literature. Besides, the majority of the research is focused on social networks construction, this is, the nodes are the athletes and the links are some level of interaction between them. A multifactorial analysis of athletes considering data available from the most recent season could help in the development of study approaches that point to modeling the specifics parameters involved on the games and may help to understand the role of each variable analyzed, and determine its weights.

The idea is to elaborate a complex model that, through complex analysis and its metrics, identifies which variable presents a point of convergence and helps in the understanding of why the performance of the individual was superior considering the 19 different factors gathered in the games that the all 259 athletes participated, analyzed together at one great complex network model. The complex models represent a dynamic system of interactions, which may help in determining the strategy adopted by individuals and that are decisive in the team's victory.

2. Materials and Methods

2.1 NBA athletes and variables selection

The National Basketball Association - NBA - is the leading professional basketball league not only North America but due to its large number of franchises is considered the main basketball league in the world. We consider in our analysis the NBA official data in a newsworthy complex and multifactorial manner. It was possible to select 19 parameters: GP: Games Played; MIN: Minutes Played; PTS: Points; FGM: Field Goals Made; FGA: Field Goals Attempted; FG%: Field Goal Percentage; 3PM: 3 Point Field Goals Made; 3PA: 3 Point Field Goals Attempted; 3P%: 3 Point Field Goals Percentage; FTM: Free Throws Made; FTA: Free Throws Attempted; FT%: Free Throw Percentage; OREB: Offensive Rebounds; DREB: Defensive Rebounds; REB: Rebounds; AST: Assists; STL: Steals; BLK: Blocks and TOV: Turnovers, as shown in the Table 1.

Table 1. The variables (nodes) analyzed including each technical description and abbreviation, officially used on NBA games.

Variables	Abbreviation	Technical description*
Points	**PTS**	Points can be accumulated by making field goals or free throws. The winner team scored the most points at the end of the game.
Field Goals Made	**FGM**	Field goal is the official terminology used by NBA. It is a basket scored on any shot or tap other than a free throw, worth two or three points depending on the distance of the attempt from the basket. If a player makes a field goal from within the three-point line, the player scores two points.
Field Goals Attempted	**FGA**	Number of attempts to score points by Field Goals.
Minutes Played	**MIN**	It is the total in minutes that player has already played and accumulated in his career in NBA official games. Currently, each game has 4 quarters of 12 minutes each, totalizing 48 minutes by game. However, each player has different game time.
Free Throws Made	**FTM**	Free throws are unopposed attempts to score points by shooting from behind the free throw line, a line situated at the end of the restricted area. Each successful free throw is worth one point.
Free Throws Attempted	**FTA**	Number of attempts to score points by Free Throws.
Turnovers	**TOV**	A turnover occurs when a team loses possession of the ball to the opposing team before a player takes a shot at their team's basket.
Defensive Rebounds	**DREB**	The defending team gains possession.
Rebounds	**REB**	A Rebound refers to a player who retrieves the ball after a missed field goal or free throw.
Assists	**AST**	An assist is attributed to a player who passes the ball to a teammate in a way that leads to a score by field goal, meaning that he or she was "assisting" in the basket.
3 Point Field Goals Attempted	**3PA**	It is a field goal attempted from beyond the three-point line.
Steals	**STL**	A defensive player may cause a turnover by his action. It is when occurring a steal.
3 Point Field Goals Made	**3PM**	If the player makes a field goal beyond the three-point line, the player scores three points.
Offensive Rebounds	**OREB**	The ball is recovered by the offensive side and does not change possession.
Blocks	**BLK**	A block or blocked shot occurs when a defensive player legally deflects a field goal attempt from an offensive player to prevent a score.
Free Throw Percentage	**FT%**	It is the percentage result of successful free throw made by each player.
Field Goal Percentage	**FG%**	It is the percentage result of field goal made by each player.
3 Point Field Goals Percentage	**3P%**	It is the percentage result of 3 point field goal made by each player.
Games Played	**GP**	Number of official NBA games played by each athlete.

* Source of definitons: www.nba.com

The idea is to select data from the General List of Official Leading Players, totalizing a great number of athletes (259 players), from the Regular Season 2017-2018. Differently from the others studies above mentioned our model is focused on the specific parameters of each player, which will become nodes (vertices) in our complex network model. Then, using this considerable sample, the correlations between one data column and the other are calculated, using Pearson correlation method. It is worth mentioning that in the case of Turnovers, it is the only variable that behaves inversely proportional in performance. All the others the higher the value the greater the contribution in the overall results, that is, the better the contribution of the player to the team.

2.2 MCDA and Complex networks

In a first moment, the data of the 259 athletes were gathered to allow the calculation of the correlations between each set of data divided into different columns. Correlations between one column and another were calculated. For example, the correlation between Rebounds and Blocks was 0.67. The total correlations calculated for the 19 variables listed were 171. These 171 correlations were used within the proposal to construct a complex model that represents the dynamics of interactions between the analyzed variables, involving 19 different attributes of each 259 athletes (4921 values). The results of the correlations were then later transformed into links in the complex model.

A complex network is a mathematical graph that consists of nodes (vertices) and two connections or links (edges). A network is a graph $G = \{N, M, f\}$, where $N = \{n_1, n_2, \ldots n_k\}$ nodes; $M = \{m_1, m_2, \ldots m_l\}$ links, and the mapping function is $f: N \times N$.

Here, f is expressed as a connection matrix C, which defines the network topology. In this paper, nodes represent factors relating to fatigue, and links represent interactions among these factors [Pereira et al. 2015]. Generally, the elements under analysis are determined as nodes and their relationships, links. In this case, the variables that turned into nodes were mentioned above one NBA athletes and variables selection. The results of the correlations between these variables were transformed into weighted links in the complex system.

When working with complex networks, the concern is the structural organization of data and its relations dynamically. As a network is a mathematical graph composed of vertices and edges, it is possible to determine other measures arising from its structure. The degree of a node is the total number of connections incident to it. This measure may reflect this node's importance to the rest of this network. Another widely used measure is the eigenvalue of the node. The value of the node with the largest eigenvalue is also called the Spectral Radius. The Spectral Radius reflects global information about a complex network. Thus, the Spectral Radius considers the largest non-trivial value of the adjacency matrix of the graph, showing a value and pointing to a determined node that besides the individual importance also connects to nodes that have a greater number of connections that also connect to nodes influences in the complex context. Therefore, it is possible to visualize the importance of this measure. This measure is the eigenvalue, which is the largest eigenvalue of the matrix that represents the complex modeled system. The function for eigenvalues calculation follows the pseudo-code, according to complex network analysis [Pereira et al. 2018; Lewis 2009; Ray 2016].

Choose a random vector $v^{(0)} \in R^n$
for $k = 1, 2, \ldots$ while
$\left\| v^{(k-1)} - v^{(k-2)} \right\| > \epsilon,$
$z^{(k)} = A\, v^{(k-1)}$,
$v^{(k)} = z^{(k)} / \left\| z^{(k)} \right\|,$
$\lambda^{(k)} = \left[v^{(k)} \right]^T A v^{(k)},$
end

The MCDA usually deals with complex decision problems, involving a hierarchical of criteria or alternatives that aid in the final decision making. One of the most widely used methods in MCDA is weighted aggregation.

Considering n alternatives and m criteria, the aggregate preference P_i of alternative A_i $(I = 1, \ldots, n)$ is calculated according to:

$$P_i = \sum_{j=}^{m} a_{ij}\, W^j, (i = 1, \ldots, n)$$

Where W^j is the weight of the j^{th} criteria. In this work, we use this method to show how weights can be assigned and aggregated uniformly for athletes' classification, besides comparing with weight assignment through a complex metric, applying the aggregation next [Zahir 2016].

3. Results

The variables analyzed and the resulting complex models, involved the nodes displayed in the Figure 1. Following the analysis, the complex metric called eigenvalue of each variable (node) result, are shown in Figure 2. At Figures 3 and 4, it is shown the comparative analysis of equal weights and SR weights.

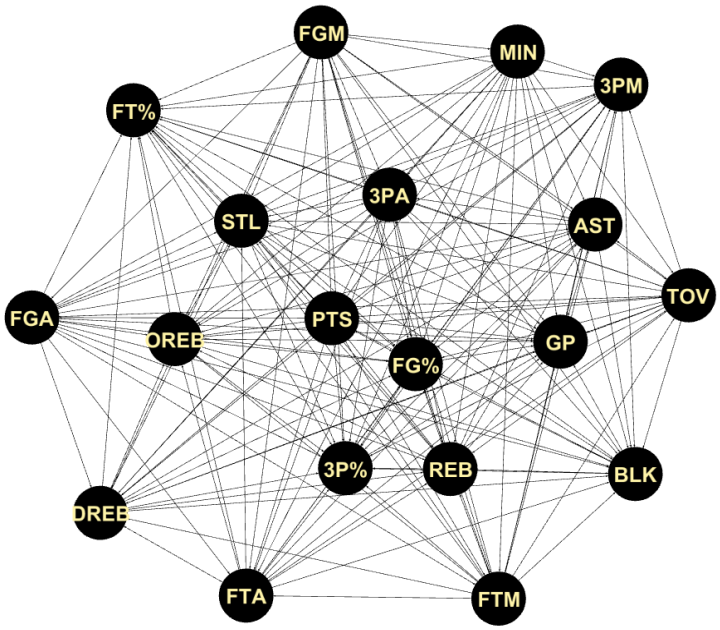


Figure 1. The variables (nodes) analyzed, officially used on NBA games to build the complex model proposed.

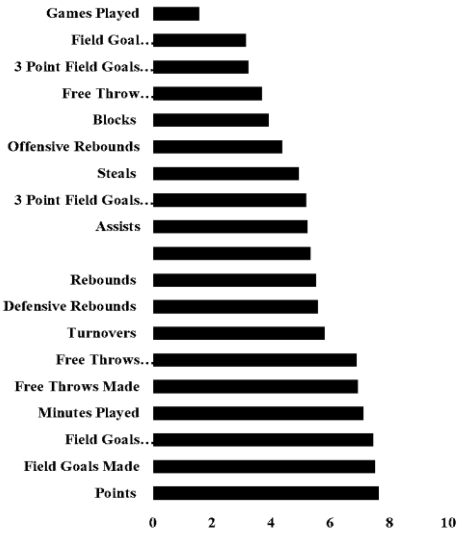


Figure 2. The eigenvalue results by variable (node) ordered by the highest to the lowest. The highest eigenvalue found is the variable Points, which may play a central role in athlete performance.

The eigenvalue is an important measure that showed the importance of the points variable compared to others analyzed together. This was possible through complex modeling, which unlike other works that mostly deals with the gains of complex networks in modeling players like nodes and their interactions as links, which shows the impact of the social network on team performance. Here it is evident that calculating the player's points can be crucial in his classification as an excellent player on the pitch.

After the analysis of the eigenvalues of each variable, and considering each importance of them, it was possible to rank the players through an aggregation method. If we consider each variable as the same contribution for the player, after an aggregation calculation, we have a comparing ranking with the players ordered by points, as shown at Figure 3.

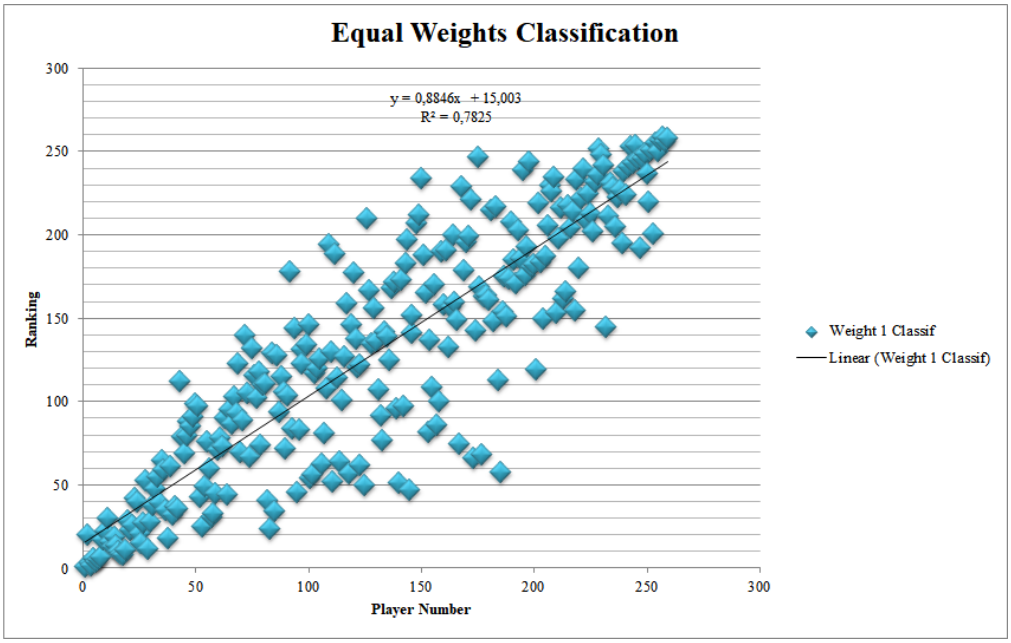


Figure 3. The Equal Weight Classification of athletes from NBA stats. We considered all the variables analyzed with the same weight (without pre-weight) and make a comparative aggregation.

On the other hand, ranking the athletes considering each variable analyzed with a different weight, i.e., the weight as the result of the complex networks eigenvalues, it is possible to calculate the aggregation. Comparing this new aggregation result for each player with the players ordered by points, it is possible to see that eigenvalues are a good measure once the players are closer to the ranking when ordered by points.

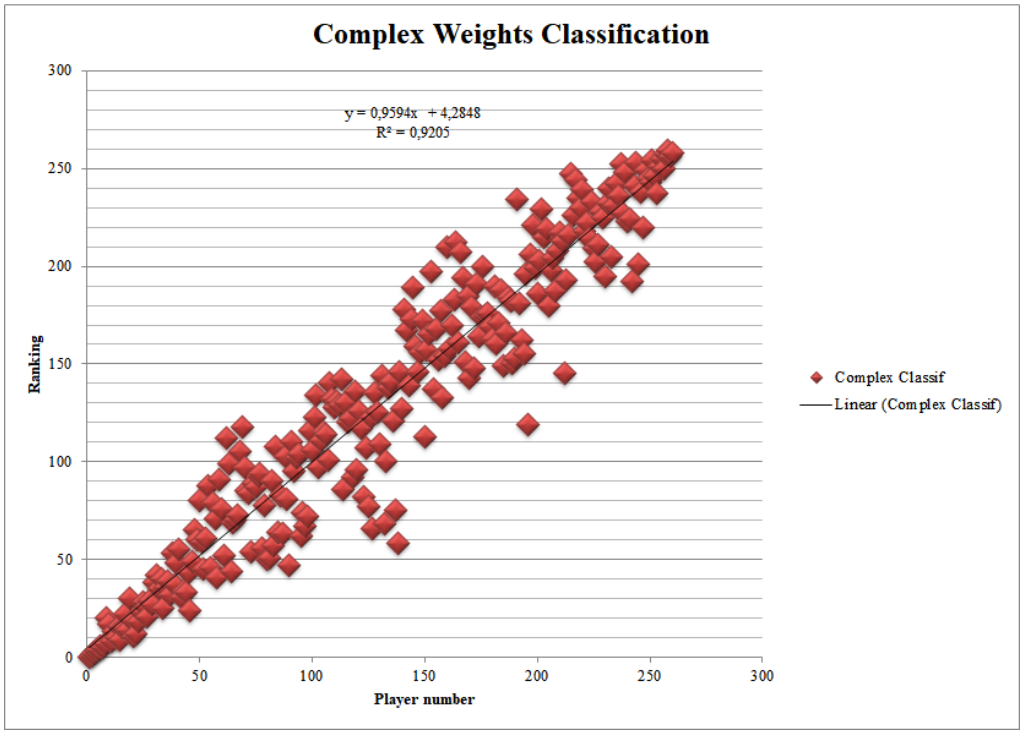


Figure 4. The Complex Weights Classification of athletes from NBA. We considered comparative aggregation with the attribution of weights via eigenvalues, an interesting alternative of analysis in the athletes ranking is observed.

4. Discussion and Conclusions

In the light of MCDA analysis, and from the idealization of such a complex model, construction, development and analysis of 19 measurable variables of each of the athletes, leaders of the NBA season 2017-2018, it is possible to observe the nodes weights (resulting eigenvalues) with each contribution correlated with the ranking of the players. Most decision methods involving multiple criteria need to be pre weighted for each of the analyzed questions or even pre-definitions of input and output variables [Masoumzadeh et al. 2016; Cooper et al. 2011]. One of the advantages of the Complex Networks lies in the fact that this pre-determination is not necessary and results in an order of weights for each item or attribute analyzed. Besides, the applied approach in this study in a different because it is not commonly used in athletes' analysis or competitive sports.

In addition, there is a novel construction of the complex network through a network creation method [Pereira et al. 2015; Pereira et al. 2018] to generate an important network metric: the eigenvalue. The complex metric shows a variable that has greater representativeness and importance in the complex network scenario. This is the case of the Points, a node that deeply relates to a central role in athletes and teams' final results. The analysis by eigenvalues, whose higher value determines the spectral radius, reflects a tendency of convergence of the model. The

complex model reinforces the idea of representing a dynamic system of interactions, which can help in determining the strategy adopted by individuals and that are decisive in the team's victory. The greater eigenvalue points to a kind of gravitational force exerted by that node on the network in which it is inserted. Considering the applicability of the results in training, aiming for a superior performance, efforts should focus on the Points: the bigger, better and more decisive this athlete will be.

The eigenvalue metric made possible to view each variable such as other MCDA methods, and prove to be an important tool for weight assignment, which makes it possible to establish rankings in athlete classification studies, as analyzed here.

Interestingly, if we consider all the variables analyzed with the same weight (without pre-weight) and make a comparative aggregation with the attribution of weights via eigenvalues, an interesting alternative of analysis in the athlete ranking is observed. The association of complex network tools as a new method of assigning weights to variables under analysis is close to a classification of better points, although there is no consensus as to which would be the best variable that more fairly orders the athletes. Future work may apply this association of complex network modeling as a MCDA tool and use to show which variables would be the most relevant in the training of the players, in addition to enabling them to be ranked in light of the new weights and aggregation of results, as presented here.

This way, a different method as proposed, is an alternative analysis with the goal of improve individual performance, which generates positive results for the entire team. Besides, such complex network model and metrics can be applied to other complex sports and decision making scenarios.

5. Acknowledgements

The authors would like to thank the National Post-Doctoral Program from the Coordination for the Improvement of Higher Education Personnel (PNPD - CAPES - grant 649962/2018), the São Paulo Research Foundation (FAPESP, grant #2015/16325-1), the National Council for Scientific and Technological Development (CNPq, grant 311357/2017-2) and the Graduate Program in Production and Manufacturing Engineering (EMP) at the School of Applied Sciences (FCA). This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001. All authors contributed to one or more of the following areas: proposing, leading and running the experiment; design, construction, optimization, data analysis, modelling, simulation and the writing of this article. The authors declare no competing interests.

References

Bourbousson, J., Seve, C. and McGarry, T. (2010). Space–time coordination dynamics in basketball: Part 2. The interaction between the two teams. Journal of Sports Sciences 28(3), 349-358.

Bouyssou, D., Marchant, T., Pirlot, M., Tsoukias, A., & Vincke, P. (2006). *Evaluation and decision models with multiple criteria: Stepping stones for the analyst* (Vol. 86). Springer Science & Business Media.

Cooper, W. W., Ramón, N., Ruiz, J. L., & Sirvent, I. (2011). Avoiding large differences in weights in cross-efficiency evaluations: application to the ranking of basketball players. *Journal of CENTRUM Cathedra: The Business and Economics Research Journal*, *4*(2), 197-215.

Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: the CRITIC method. *Computers & Operations Research*, *22*(7), 763-770.

Duarte, R., Araújo, D., Davids, K., Travassos, B., Gazimba, V. and Sampaio, J. (2012). Interpersonal coordination tendencies shape 1-vs-1 sub-phase performance outcomes in youth soccer. Journal of Sports Sciences 30(9), 871-877.

Fernandez, J., Camerino, O., Anguera, M. T., & Jonsson, G. K. (2009). Identifying and analyzing the construction and effectiveness of offensive plays in basketball by using systematic observation. *Behavior Research Methods*, *41*(3), 719-730.

Fewell, J. H., Armbruster, D., Ingraham, J., Petersen, A., & Waters, J. S. (2012). Basketball teams as strategic networks. *PloS one*, *7*(11), e47445.

Gaggioli, A., Morganti, L., Mondoni, M., & Antonietti, A. (2013). Benefits of combined mental and physical training in learning a complex motor skill in basketball. *Psychology*, *4*(9A), 1-6.

García-Rubio, J., Gómez, M. Á., Cañadas, M., & Ibáñez, J. S. (2015). Offensive Rating-Time coordination dynamics in basketball. Complex systems theory applied to Basketball. *International Journal of Performance Analysis in Sport*, *15*(2), 513-526.

Guerra, Y. D. S., González, J. M. M., Montesdeoca, S. S., Ruiz, D. R., López, N. A., & García-Manso, J. M. (2013). Basketball scoring in NBA games: An example of complexity. *Journal of Systems Science and Complexity*, *26*(1), 94-103.

Keidel, R. W. (1984). Baseball, football, and basketball: Models for business. *Organizational Dynamics*, *12*(3), 5-18.

Koster, J., & Aven, B. (2018). The effects of individual status and group performance on network ties among teammates in the National Basketball Association. *PloS one*,*13*(4), e0196013.

Lewis, T. G. (2009). *Network science: theory and practice*. Wiley, New Jersey.

Mardani, A., Jusoh, A., Nor, K., Khalifah, Z., Zakwan, N., & Valipour, A. (2015). Multiple criteria decision-making techniques and their applications–a review of the literature from 2000 to 2014. *Economic Research-Ekonomska Istraživanja*, *28*(1), 516-571.

Masoumzadeh, A., Toloo, M., & Amirteimoori, A. (2016). Performance assessment in production systems without explicit inputs: an application to basketball players. *IMA Journal of Management Mathematics*, *27*(2), 143-156.

Moreira, A., Aoki, M. S., Franchini, E., da Silva Machado, D. G., Paludo, A. C., & Okano, A. H. (2018). Mental fatigue impairs technical performance and alters neuroendocrine and autonomic responses in elite young basketball players. *Physiology & behavior*, *196*, 112-118.

Moreira, N. B., Mazzardo, O., Vagetti, G. C., De Oliveira, V., & De Campos, W. (2016). Quality of life perception of basketball master athletes: association with physical activity level and sports injuries. *Journal of sports sciences*, *34*(10), 988-996.

Oztaysi, B. (2014). A decision model for information technology selection using AHP integrated TOPSIS-Grey: The case of content management systems. *Knowledge-Based Systems*, *70*, 44-54.

Passos, P., Davids, K., Araújo, D., Paz, N., Minguéns, J., & Mendes, J. (2011). Networks as a novel tool for studying team ball sports as complex social systems. *Journal of Science and Medicine in Sport*, *14*(2), 170-176.

Pereira, V. H., Gobatto, C. A., Lewis, T. G., Ribeiro, L. F. P., Beck, W. R., Masselli Dos Reis, I. G., ... & Manchado-Gobatto, F. B. (2018). Computational and complex network modeling for analysis of sprinter athletes' performance in track field tests. *Frontiers in Physiology*, *9*, 843.

Pereira, V. H., Gama, M. C. T., Sousa, F. A. B., Lewis, T. G., Gobatto, C. A., & Manchado-Gobatto, F. B. (2015). Complex network models reveal correlations among network metrics, exercise intensity and role of body changes in the fatigue process. *Scientific reports*, *5*, 10489.

Radicchi, F. (2011). Who is the best player ever? A complex network analysis of the history of professional tennis. *PloS one*, *6*(2), e17249.

Ray, S. S. (2018). *Numerical analysis with algorithms and programming*. CRC Press, Boca Raton-FL.

Shitrit, H. B., Berclaz, J., Fleuret, F., & Fua, P. (2014). Multi-commodity network flow for tracking multiple people. *IEEE transactions on pattern analysis and machine intelligence*, *36*(8), 1614-1627.

Skinner, B., & Guy, S. J. (2015). A method for using player tracking data in basketball to learn player skills and predict team performance. *PloS one*, *10*(9), e0136393.

Vilar, L., Araújo, D., Davids, K. and Button, C. (2012). The role of ecological dynamics in analysing performance in team sports. Sports Medicine 42(1), 1-10.

Zahir, S. (2016). Aggregation of priorities in multi-criteria decision analysis (mcda): Connecting decision spaces in the cognitive space. *American Journal of Operations Research*, 6(04), 317-333.

Zavadskas, E. K., Turskis, Z., & Kildienė, S. (2014). State of art surveys of overviews on MCDM/MADM methods. *Technological and economic development of economy*, *20*(1), 165-179.