

MATHEMATICAL MODELS FOR COUPLED INTEGRATION OF PORT PROBLEMS WITH DIFFERENT PERFORMANCE MEASURES

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RESUMO

Este trabalho apresenta um conjunto de modelagens matemáticas para a resolução de problemas portuários. Os problemas considerados são de natureza operacional e pertencem à área marítima de um porto. O problema principal consiste na alocação de navios em berços de atendimento, sendo os problemas auxiliares a designação de guindastes a estes berços, e o posterior posicionamento e movimentação dos guindastes ao longo do horizonte de planejamento. É apenas com a resolução de tais problemas auxiliares que se obtém uma solução detalhada e de boa qualidade para o problema principal. Uma resolução integrada destes problemas promove considerável dimensionalidade para as variáveis de decisão, tal que a busca por soluções unificadas se torna um desafio. A pesquisa traz uma proposta de resolução sequencial de um modelo acoplado dos problemas citados e uma breve análise sobre o impacto que variadas funções objetivo causam no modelo e na qualidade da solução.

PALAVRAS CHAVE. Problema de Alocação de Berços. Problema de Designação de Guindastes. Problema de Planejamento de Guindastes.

PM – Programação Matemática, OC – Otimização Combinatória, LT – Logística e Transportes

ABSTRACT

This paper introduces a series of mathematical models for the resolution of port problems. The problems are from the seaside area of a port in operational level. The main problem consists in the allocation of vessels in berths. Auxiliary problems are the assignment of quay crane cranes to berths and the subsequent scheduling of quay cranes into the planning horizon. Solving these auxiliary problems is the only method to get a detailed and good quality solution for the main problem. An integrated model greatly increases dimensionality to the decision variables in such a way that the search for unified solutions becomes a challenge. This research proposes a sequential resolution for a coupled model of the mentioned problems and a brief analysis regarding the impact of several objective functions on the dimensionality of the model and quality of the solution.

KEYWORDS. Berth Allocation Problem. Quay Crane Assignment Problem. Quay Crane Scheduling Problem.

MP – Mathematical Programming, CO – Combinatorial Optimization, LT – Logistics and Transportation)

1. Introduction

In Brazil, expressiveness of maritime transport has great proportions. Exports by sea correspond to 83.5% of total products exported by the country, about US \$ 153.2 billion from January to October 2017. This amount has a total of 521 million tons exported in the period. In imports, the relevance is similar. Until October, the trading made by sea totaled US \$ 113.3 billion which corresponds to 73.6 % of the total purchases by the country. Brazilian ports export the main products of our commercial goods: soy, soybean meal, corn, mineral products, meats, sugar, automobiles, coffee and other. Currently, Brazil has 34 ports and more than 100 port facilities covering 8,500 kilometers of navigable coast [Brasil, 2017].

Besides the importance of maritime transport in the country, it was also noted the deepness as port decisions are studied in the scientific community. Numerous bibliographic reviews have been made over the years to compile and summarize the researches done in the area. The first bibliographic survey was conducted based about 71 papers in [Meersmans e Dekker, 2001]. Few years later in [Steenken et al., 2004] a new survey was made about 212 papers. In [Vacca et al., 2007] were reviewed about 61 papers involving studies since 2004. In the following year [Stahlbock e Voß, 2008] made a continuation of the 2004 bibliographic survey ([Steenken et al., 2004]), now about 254 papers. In [Bierwirth e Meisel, 2010] a bibliographic database was analyzed about 114 papers of topics purely focused on operational problems that occur on the seaside of the port. A few years later in [Rashidi e Tsang, 2013], a brief review of the literature on all port problems was made, including 118 papers. In [Carlo et al., 2015], a bibliographic database was studied between 2004 and 2012 about 90 papers. In the same year, [Bierwirth e Meisel, 2015] made a follow-up of the paper [Bierwirth e Meisel, 2010], showing a brief classification of the problems and the current guidelines for future research. The volume of global studies on the problem shows not only the importance of the subject but the complexity of resolution and analysis provided. We are still looking for a way to effectively represent the problem with a complete and detailed resolution.

The present research has as general objective to present a set of mathematical models for the coupled integrated resolution of seaside port problems. The problems considered are (i) the berth allocation problem (BAP), (ii) the quay crane assignment problem (QCAP) and (iii) the quay crane scheduling problem (QCSP).

The specific objectives for this research are guided by:

- A review on the scientific literature in order to find classifications and nomenclatures that helps to clarify the points not addressed in literature;
- A detailed description of mathematical models and its features found in literature;
- The proposition of an original approach that consists in solving a sequence of three mathematical models to find a solution for the integrated problem of BAP, QCAP, and QCSP;
- Brief analysis about the increase of dimensionality of the model according to the chosen objective function and;
- Brief analysis about the qualitative difference of the solutions found by the various objective functions.

The paper presents a literature review in the section 2, discussing some classifications and nomenclatures adopted by the scientific community about the problem. In the section 3 we have a discussion of research methodology. In the section 4 we present the models used and some discussions about dimensionality of the problem when choosing specific objective functions. In the section 5 we have a discussion about the objective functions that have found the same solution for a small and controlled toy problem. And, finally, the conclusion in the section 6.

2. Literature Review

In this section we will briefly present (i) the nature and classification of the cited problems ; (ii) a model classification and notation usually used by the scientific community in order to unify the notation and better understanding of the considered detail, and (iii) a presentation on how current researchers approach integrating similar problems.

2.1. Problem classification

There are many problems that can be optimized in a port, such that it is necessary to discriminate the sector and decision making level that will be considered in the proposed resolution. According to the Figure 1 there are three divisions of physical sectors of the port, about (i) seaside, (ii) yard and (iii) landside, subdividing yet another two levels of decision makers on (i) strategic and (ii) operational levels. Taking this

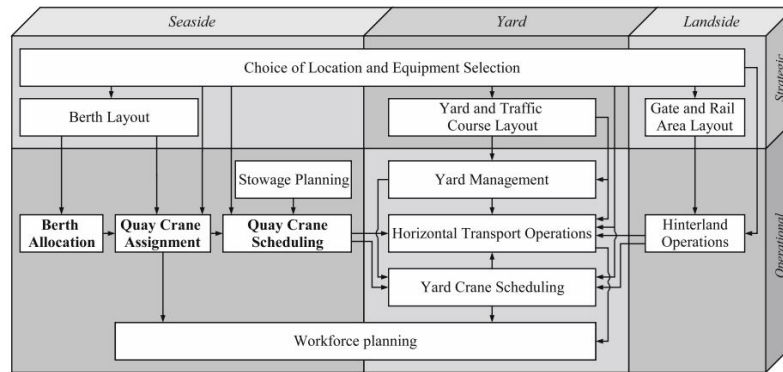


Figure 1: Scheme on classes of port problems. Source: [Bierwirth e Meisel, 2010].

nomenclature, the current research will be classified as operational problems of the seaside area of the port.

The Berth Allocation Problem (BAP) consists of deciding which vessels, within a predetermined set, will be allocated in which berths over a period of time. The problem has been proved to be NP-hard in [Lim, 1998] and has been studied extensively since then, gathering many detailing variations and approaches among those.

The Quay Crane Assignment Problem (QCAP) consists of determining the amount of quay cranes available for service in each berth. Such a determination greatly impacts the BAP, increasing its complexity.

The Quay Crane Scheduling Problem (QCSP) is the definition of how each crane will be allocated between the internal positions of a vessel over time. Usually the quay cranes are positioned queued along the quay, not having the possibility of crossing each other during their movement. The addition of this constraint to the mathematical model is proved NP-hard as seen in [Zhu e Lim, 2006].

2.2. Model classification and notation schemes

In the literature review paper by [Bierwirth e Meisel, 2010], a classification notation is proposed on strategies to solve different port problems in an integrated approach. The notations can be seen in the figure 2. There are three possibilities for an integrated model: (i) a monolithic model is commonly called deep integration, (ii) a sequential resolution of several models will be called a coupled model, and (iii) a sequential resolution with information exchange can be called a feedback resolution. More details on explanations and meanings of each notation can be seen in [Bierwirth e Meisel, 2010]. In the review of the updated literature by

Notation	Description
$\overline{A, B}$	Deep integration of problems A and B
$A \rightarrow B$	A is preprocessed to B
$A \rightleftarrows B$	Feedback loop integration of A and B

Figure 2: Scheme on integrated problem notation. Source: [Bierwirth e Meisel, 2010].

[Bierwirth e Meisel, 2015], a classification scheme was elaborated for (i) spatial, (ii) temporal, (iii) handling time and (iv) performance measure detailing on how BAP can be addressed. This scheme can be seen in Figure 3. More details on explanations and meanings of each nomenclature can be seen in [Bierwirth e Meisel, 2015]. In the same paper, as well as for the BAP, a classification scheme was elaborated for the detail of the attributes (i) task, (ii) crane, (iii) interference and (iv) performance measure; on how QCSP can be

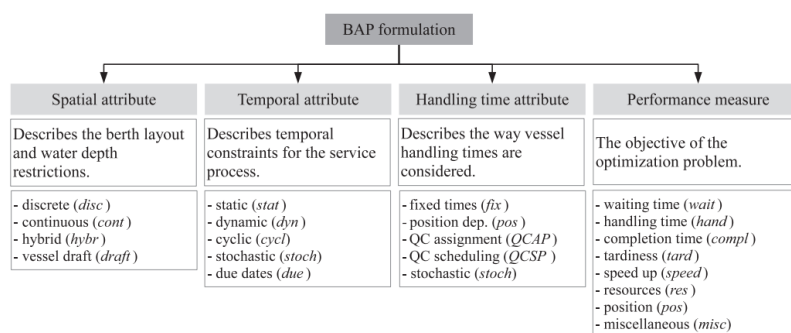


Figure 3: BAP classification scheme. Source: [Bierwirth e Meisel, 2015].

approached. This scheme can be seen in the Figure 4. More details on explanations and meanings of each nomenclature can be seen in [Bierwirth e Meisel, 2015].

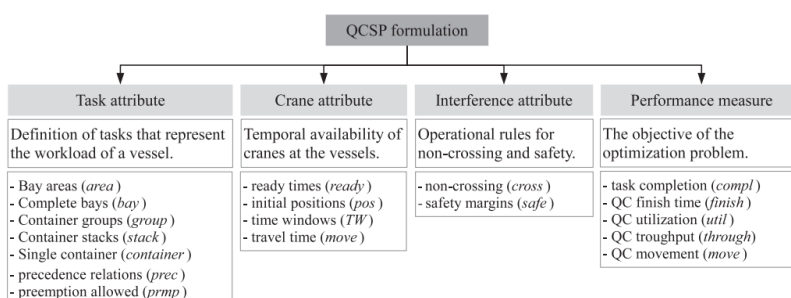


Figure 4: QCSP classification scheme. Source: [Bierwirth e Meisel, 2015].

2.3. Related literature

It was verified in [Bierwirth e Meisel, 2015] that the complete approach of the three problems was started in [BAP, QCAP, QCSP] with the detailing $cont \mid dyn \mid QCAP, QCSP \mid \sum w (wait + hand)$ in [Lu et al., 2011]; and detailing $cont \mid dyn \mid QCAP, QCSP \mid \sum w wait$ in [Rodriguez-molins et al., 2014]. It was also developed a model form $QCSP \rightarrow [BAP, QCAP] \Leftrightarrow QCSP$ with detailing $cont \mid dyn \mid pos, QCAP \mid (w1 speed + w2 tard + w3 res)$ for BAP detailing and $group, prec \mid TW, pos, move \mid cross, save \mid max (compl)$ for the QCSP detailing by [Meisel et al., 2013].

According to the details presented, it is noted that in the integrated approaches only "continuous" representations of the service berth were developed, unlike the current proposal with "discrete" representation. This representation makes the current modeling proposal a pioneer for the integration of the mentioned problems, which also leads to a difficult comparison between models and resolutions already presented.

3. Methodology

Looking at the current scenario of this area of research, the complexity of the problem can be noted, not only computationally as seen in the literature, but also mathematically (problems difficult to find an adequate modeling while integrating simpler models).

The current research has as general objective to present a set of mathematical models for the coupled integrated resolution of operational seaside port problems. The problems considered are (i) the berth allocation problem (BAP), (ii) the quay crane assignment problem (QCAP) and (iii) the quay crane scheduling problem (QCSP).

Using these mathematical models some analysis will be done about (i) the addition or choice of a certain objective function; and (ii) verification of a possible similarity among the final solutions according to the chosen objective function.

The analysis of choosing an specific objective function consists on verifying the need of creating continuous, integer or binary variables as much as the addition of constraints to the model. Similar final solutions found by different objective functions may indicate a positive correlation between objectives. This research has no purpose of identifying existence of correlation. Instead, it seeks to introduce a discussion about some possible complex objective functions that require the addition of several variables and constraints but find solutions of same quality of simpler objective functions.

4. Problem formulation

In this section we will discuss some original linear mathematical models specially made for the mentioned port problems. Initially the model for the problem of vessel allocation in service berths (BAP) is presented; followed by the model for the crane crane designation problem (QCAP); and, finally, the model of positioning and movement (scheduling) of quay cranes (QCSP).

Following the proposed notation by [Bierwirth e Meisel, 2010] and [Bierwirth e Meisel, 2015], the coupled resolution for this three problems is a model form called $BAP \rightarrow QCAP \rightarrow QCSP$ for integration approach and $disc | dyn, due | QCAP, QCSP | misc$ and $bay, prmp | ready | cross, safe | move$ for BAP and QCSP detailing approach.

It is important emphasizing that the individual resolutions of these problems assumes different classification and notation, and these information will be presented next. The variables and parameters will be presented in the discussion of each model. Another detail is the objective functions that are listed separately for better understanding but will be presented in a unified form by a summation. And finally the constraints that have the "subject to" index 1 are for the structural case of the problem and 2 are for the additional cases of a specific objective function.

4.1. BAP formulation

The BAP model presented was inspired by the classical modeling of parallel machine problems very studied by the scientific community. This inspiration is due to the fact that the mathematical representation of the service berths is discrete (berth 1, berth 2, ..., berth b , for b being the number of berths considered).

The parameters required to initiate the resolution are (i) the arrival time (Ch_n), (ii) the time required for attendance (TA_n), and (iii) the estimated time for departure (PE_n) of each vessel n in the considered horizon. The service time is made knowing initially the number of hours of service that the vessel will demand to be fully attended divided by the average number of service quay cranes available in a berth. The estimated time of departure shall be established by admitting, from the arrival of the vessel in question to the port, a sufficient time for the service, characterizing a time window. The unique resolution of this problem characterizes according to the notation proposed by [Bierwirth e Meisel, 2015] the BAP model detailing $disc | dyn, due | fix | misc$.

(BAP)

$$\text{Min} \quad \sum_{n \in N} HF_n + Mak + \quad (1)$$

$$\sum_{n \in N} (HF_n - Ch_n) + MTS + \quad (2)$$

$$\sum_{n \in N} (HI_n - Ch_n) + MAI + \sum_{n \in N} AIB_n + \quad (3)$$

$$\sum_{n \in N} AF_n + MAF + \sum_{n \in N} AFB_n; \quad (4)$$

$$\text{s.t.1} \quad Ch_n \leq HI_n; \quad n \in N \quad (5)$$

$$\sum_{b \in B} At_{n,b} = 1; \quad n \in N \quad (6)$$

$$HF_i \leq HI_j + M(1 - Or_{i,j}) + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (7)$$

$$HF_j \leq HI_i + M.Or_{i,j} + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (8)$$

$$TA_n = HF_n - HI_n; \quad n \in N \quad (9)$$

$$\text{s.t.2} \quad HF_n \leq Mak; \quad n \in N \quad (10)$$

$$HF_n - Ch_n \leq MTS; \quad n \in N \quad (11)$$

$$HI_n - Ch_n \leq MAI; \quad n \in N \quad (12)$$

$$HI_n - Ch_n \leq M.AIB_n; \quad n \in N \quad (13)$$

$$HF_n - PE_n \leq AF_n; \quad n \in N \quad (14)$$

$$AF_n \leq MAF; \quad n \in N \quad (15)$$

$$AF_n \leq M.AFB_n; \quad n \in N \quad (16)$$

4.1.1. BAP constraints

The constraint 5 is needed so that the beginning of service of each vessel (HI_n) can only be initiated after its arrival in the port. This constraint does not only allow a vessel to be moored in only one berth ($At_{n,b}$, binary variable), but also ensures that each vessel is moored at least and no more than once.

The constraints 7 and 8 work together so that vessel moored are not concomitant in the same berth. If the vessel i is served previously to the vessel j ($Or_{i,j}$, binary variable), this vessel's final hour of service i (HF_i) must be lesser than initial hour of vessel j attendance, and vice versa otherwise. Since the combined constraints are commutative, only the occasions on which the vessel's index i is strictly smaller than the vessel index j are necessary.

The existence of M values (a very high value that would disable the constraint) on the constraints indicates that in a few combinations of indexes i and j some of them will be activated. If the vessels i and j are both moored in the same berth, one of the M values will be obstructed, allowing one of the two constraints to be active. It is worth noting that the same logic makes both constraints obsolete if vessels i and j are moored in separate berths, ensuring concurrent attendance between berths. It is also necessary to define the order of service of the vessels, which can be observed by the impact of the variable $Or_{i,j}$ in the set of constraints. If the vessel i is processed before the vessel j , the variable $Or_{i,j}$ will be active and will invalidate the value M in the first constraint, while maintaining the value M in the second constraint, and vice versa otherwise.

The constraint 9 is necessary so that the servicing time of each vessel is linked to the values of the final and initial hour of attendance. Only this set of constraints ensures the operation of the BAP itself.

4.1.2. BAP objective functions

There are many variants of possible objective functions for BAP. This research is not intended to find the definition or choice of more comprehensive or correlated objective functions, but rather the necessary computational effort to model them. In order to consider specific objective function, the addition of some variables or constraints may be necessary.

The classic objective function for this problem is the service time, characterized from the beginning of the service until the moment of departure of a vessel. Usually, one can minimize (i) the sum of the service times of vessels and (ii) the longest service time. Due to the simple BAP (without QCAP and QCSP) uses the fixed service times, or parameterized, there is no objective function to apply, returning to be measured later in the more complex models.

The minimization of the final moment of service (completion) of a vessel (objective function 1) is also part of the classic objective functions. It is possible to measure (i) the sum and (ii) the latest. The sum of completion time service does not require the addition of any variables or constraints, since it uses the structural variables of the problem. The minimization of the latest completion time service, known as makespan, adds 1 auxiliary variable (Mak) and n constraint (constraint 10), where n is the amount of vessels.

Also, in the measurement of time intervals, there is a minimization of the total time of staying of a vessel in the port, or total time in the system (objective function 2), characterized from the arrival of the vessel to the port until the moment of its completion. Similar to those already mentioned, the total time of a vessel in the port can be found in the approaches (i) the sum and (ii) the longest. The sum of the system time does not demand the addition of variables or constraints. The longest system time demands the addition of 1 variable (MTS) and n constraints (constraint 11).

Another classic, and perhaps most important, objective function is related to delays. Delays of two types can be found, like (i) delay to start the service, characterized by the difference of the moment of starting service and the arrival of the vessel to the port (objective function 3); and (ii) delay to end service, characterized by the difference of the moment of the completion and the estimated departure time (objective function 4). These delays can be measured in three approaches, (i) the sum of the delays, (ii) the longest delay, and (iii) the occurrence of delays. The sum of the delays to start the service does not require additional variables or constraints. The sum of the delays to departure requires n variables (AF_n) and n constraints (constraint 14). For both cases the longest delay approach adds of 1 variable (MAI and MAF) and n constraints (constraints 12 and 15). The approach that verifies the occurrence of delays for both cases adds n variables (Aib_n and AFb_n , binary variables) and n constraints (constraints 13 and 16).

4.2. QCAP formulation

Advancing in the detailing of the port problem BAP, the determination of the amount of quay cranes used by each berth during the time horizon. The definition of the problem itself requires the discretization of the units of time t , previously continuous, bringing more complexity to the model. From the previous resolution, the old variables $Or_{i,j}$ and $At_{n,b}$ will be the parameters of the problem, making fixed the vessels allocation and mooring order. That operation still allow some flexibility of schedules of service times.

The solution of this problem, acquiring as parameters the result of the previous problem characterizes, according to the notation proposed by [Bierwirth e Meisel, 2015], the model $BAP \rightarrow QCAP$ on detailing *disc | dyn, due | QCAP | misc*. There is no classification for the description considered by the QCAP model. However the modeling used here resembles an adapted simpler QCSP model and a description can be made about *bay, prmp | ready | cross, safe | move*, where each vessel is characterized by a single bay that contains all the load.

(QCAP)

$$\text{Min } \sum_{n \in N} \sum_{t \in T} (t.HF_{n,t} - t.HI_{n,t}) + MTA + \quad (17)$$

$$\sum_{n \in N} \sum_{t \in T} t.HF_{n,t} + Mak + \quad (18)$$

$$\sum_{n \in N} (\sum_{t \in T} t.HF_{n,t} - Ch_n) + MTS + \quad (19)$$

$$\sum_{n \in N} (\sum_{t \in T} t.HI_{n,t} - Ch_n) + MAI + \sum_{n \in N} AIB_n + \quad (20)$$

$$\sum_{n \in N} AF_n + MAF + \sum_{n \in N} AFB_n + \quad (21)$$

$$\sum_{t \in T} \sum_{b \in B} QCTrEB_{b,t}; \quad (22)$$

$$\text{s.t.1 } Ch_n \leq \sum_{t \in T} t.HI_{n,t}; \quad n \in N \quad (23)$$

$$\sum_{t \in T} t.HF_{i,t} \leq \sum_{t \in T} t.HI_{j,t} + M(1 - Or_{i,j}) + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (24)$$

$$\sum_{t \in T} t.HF_{j,t} \leq \sum_{t \in T} t.HI_{i,t} + MOr_{i,j} + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (25)$$

$$Be_{b,t} = Be_{b,t-1} + \sum_{n \in N} Ca_n At_{n,b} HI_{n,t} - NQCB_{b,t} + FA_{b,t}; \quad b \in B, t \in T \quad (26)$$

$$Be_{b,t} \leq M(2 - HF_{n,t} - At_{n,b}); \quad n \in N, b \in B, t \in T \quad (27)$$

$$\sum_{t \in T} t.HI_{n,t} \leq \sum_{t \in T} t.HF_{n,t}; \quad n \in N \quad (28)$$

$$\sum_{t \in T} HI_{n,t} = 1; \quad n \in N \quad (29)$$

$$\sum_{t \in T} HF_{n,t} = 1; \quad n \in N \quad (30)$$

$$\sum_{b \in B} NQCB_{b,t} = NQC; \quad t \in T \quad (31)$$

$$NQCB_{b,t} \leq NQCMaxB; \quad b \in B, t \in T \quad (32)$$

$$\text{s.t.2 } \sum_{t \in T} (t.HF_{n,t} - t.HI_{n,t}) \leq MTA; \quad n \in N \quad (33)$$

$$\sum_{t \in T} t.HF_{n,t} \leq Mak; \quad n \in N \quad (34)$$

$$\sum_{t \in T} t.HF_{n,t} - Ch_n \leq MTS; \quad n \in N \quad (35)$$

$$\sum_{t \in T} t.HI_{n,t} - Ch_n \leq MAI; \quad n \in N \quad (36)$$

$$\sum_{t \in T} t.HI_{n,t} - Ch_n \leq M.AIB_n; \quad n \in N \quad (37)$$

$$\sum_{t \in T} t.HF_{n,t} - PE_n \leq AF_n; \quad n \in N \quad (38)$$

$$AF_n \leq MAF; \quad n \in N \quad (39)$$

$$AF_n \leq M.AFB_n; \quad n \in N \quad (40)$$

$$QCTrEB_{b,t} \geq \sum_{bb \in b} (NQCB_{bb,t} - NQCB_{bb,t+1}); \quad b \in B, t \in T \quad (41)$$

$$QCTrEB_{b,t} \geq \sum_{bb \in b} (NQCB_{bb,t+1} - NQCB_{bb,t}); \quad b \in B, t \in T \quad (42)$$

4.2.1. QCAP constraints

A notable change is found in the continuous variables that stored the value of the initial moment of service ($HI_{n,t}$, binary variable) and completion time ($HF_{n,t}$, binary variable) of a vessel. These n variables, previously continuous, are replaced by $n.t$ binary variables, where t is the number of units of the time horizon considered. The altered constraints are presented in the equations 18, 19, 20, 23, 24, 25, 34, 35, 36, 37 e 38.

In the constraint of service time also several changes are noticed. The parameter that restricted the service time in the BAP is no longer considered. It was transformed into a parameter of load amount of the vessel (Ca_n). Attendance time is now determined by a set of constraints that simulates, in each unit of time, the port loading / unloading operation. The constraints are presented in the equations 26, 27, 28, 29 and 30.

The equation 26 was inspired by the inventory balance constraint that exists in the classical modeling of lot sizing problems. It can be observed that each berth b ($Be_{b,t}$) has its value updated in the time horizon, plus the sum of the cargoes of the vessels and decreases the amount of quay cranes available for the berth ($NQCB_{b,t}$, integer variable) in moment t . An attendance slack variable ($FA_{b,t}$) is also required. The constraint 27 gets the moment of undocking.

As the nature of this problem indicates that it is possible to choose the amount of quay cranes assigned to a berth in the time horizon. It must also be ensured that these values are always connected to the total amount of quay cranes in the port (constraint 31) and restricted to the maximum amount of quay cranes allowed in each berth (constraint 32). It is worth noting that the maximum amount of quay cranes per berth is a previously calculated value considering the vessel's internal divisions (bays) and the safety distance between the quay cranes, which is not yet modeled (QCSP), but appropriately considered.

4.2.2. QCAP objective functions

The QCAP makes it possible to vary the time of service of the vessels, so the classic objective function of minimizing service time can be considered (objective function 17). The sum of the service times does not require the addition of variables or constraints. The longest service time demands the addition of 1 auxiliary variable (MTA) and n constraints (constraint 33).

The QCAP modeling brings a new objective function, minimizing the number of movements made by the quay cranes. For this, it is necessary to introduce $b.t$ of variables ($QCTrEB_{b,t}$) and $2.b.t$ constraints (constraints 41 and 42), where b the number of berths and t the planning horizon.

Due to the choice of this model does not use quay crane discrimination as indexes, the objective function developed only minimizes the sum of movements made by quay cranes. The constraints verify if $NQCB_{b,t}$ changes its value at subsequent times, where the value difference between t and $t - 1$ is taken from the progressive sum of $NQCB_{b,t}$.

4.3. QCSP formulation

Follow to a next level in the detailing of the port problem BAP $\rightarrow$ QCAP, not only the determination of the amount of quay cranes is demanded by each berth, but also the exact positioning of these inside the vessel and its movements during the horizon time. Like as QCAP, the QCSP initiates a new set of index, requiring the discretization of the internal units of a vessel, which will be called a bay, bringing greater complexity to the model. There is also the addition of a extra bay between the berths. A port with b berths and y bays per berth are made by $b.y + (b - 1)$ bays in its full extent.

In addition to the parameters $Or_{i,j}$, and $At_{n,b}$, the QCSP receives as input the old variables $NQCB_{b,t}$. This detailing restricts the problem to choose the best schedule of quay cranes into the time horizon and builds a complete solution for the objective functions of the BAP.

The solution of this problem, acquiring as parameters the result of the previous problems characterizes, according to the notation proposed by [Bierwirth e Meisel, 2015], the model BAP $\rightarrow$ QCAP $\rightarrow$ QCSP by the approach of integration and *disc* | *dyn*, *due* | *QCAP*, *QCSP* | *misc* by the modeling approach. The QCSP Model still is *bay*, *prmp* | *ready* | *cross*, *safe* | *move* by the modeling approach.

(QCSP)

$$\text{Min} \quad \sum_{n \in N} \sum_{t \in T} (t.HF_{n,t} - t.HI_{n,t}) + MTA + \quad (43)$$

$$\sum_{n \in N} \sum_{t \in T} t.HF_{n,t} + Ma + \quad (44)$$

$$\sum_{n \in N} (\sum_{t \in T} t.HF_{n,t} - Ch_n) + MTS + \quad (45)$$

$$\sum_{n \in N} (\sum_{t \in T} t.HI_{n,t} - Ch_n) + MAI + \sum_{n \in N} AIb_n + \quad (46)$$

$$\sum_{n \in N} AF_n + MAF + \sum_{n \in N} AFb_n + \quad (47)$$

$$\sum_{b \in B} \sum_{y \in Y} \sum_{t \in T} QCTrAB_{b,y,t} + \sum_{b \in B} \sum_{t \in T} QCTrAB2_{b,t}; \quad (48)$$

$$\text{s.t.1} \quad Ch_n \leq \sum_{t \in T} t.HI_{n,t}; \quad n \in N \quad (49)$$

$$\sum_{t \in T} t.HF_{i,t} \leq \sum_{t \in T} t.HI_{j,t} + M(1 - Or_{i,j}) + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (50)$$

$$\sum_{t \in T} t.HF_{j,t} \leq \sum_{t \in T} t.HI_{i,t} + MOr_{i,j} + M(2 - At_{i,b} - At_{j,b}); \quad b \in B, (i, j) \in N : i < j \quad (51)$$

$$Be_{b,y,t} = Be_{b,y,t-1} + \sum_{n \in N} Can,y At_{n,b} HI_{n,t} - QC_{b,y,t} + FA_{b,y,t}; \quad b \in B, y \in Y, t \in T \quad (52)$$

$$\sum_{y \in Y} Be_{b,y,t} = M(2 - HF_{n,t} - At_{n,b}); \quad n \in N, b \in B, t \in T \quad (53)$$

$$\sum_{t \in T} t.HI_{n,t} \leq \sum_{t \in T} t.HF_{n,t}; \quad n \in N \quad (54)$$

$$\sum_{t \in T} HI_{n,t} = 1; \quad n \in N \quad (55)$$

$$\sum_{t \in T} HF_{n,t} = 1; \quad n \in N \quad (56)$$

$$\sum_{b \in B} \sum_{y \in Y} QC_{b,y,t} + \sum_{b \in B} QCET_{b,t} = NQC; \quad t \in T \quad (57)$$

$$\sum_{y \in Y} QC_{b,y,t} \leq NQCB_{b,t}; \quad b \in B, t \in T \quad (58)$$

$$QC_{b,y,t} + QC_{b,y+1,t} \leq 1; \quad b \in B, y \in Y, t \in T \quad (59)$$

$$QC_{b,y,max,t} + QCET_{b,t} \leq 1; \quad b \in B, t \in T \quad (60)$$

$$QCET_{b,t} + QC_{b+1,1,t} \leq 1; \quad b \in B, t \in T \quad (61)$$

$$\text{s.t.2} \quad \sum_{t \in T} (t.HF_{n,t} - t.HI_{n,t}) \leq MTA; \quad n \in N \quad (62)$$

$$\sum_{t \in T} t.HF_{n,t} \leq Ma; \quad n \in N \quad (63)$$

$$\sum_{t \in T} t.HF_{n,t} - Ch_n \leq MS; \quad n \in N \quad (64)$$

$$\sum_{t \in T} t.HI_{n,t} - Ch_n \leq MAI; \quad n \in N \quad (65)$$

$$\sum_{t \in T} t.HI_{n,t} - Ch_n \leq M.AIb_n; \quad n \in N \quad (66)$$

$$\sum_{t \in T} t.HF_{n,t} - PE_n \leq AF_n; \quad n \in N \quad (67)$$

$$AF_n \leq MAF; \quad n \in N \quad (68)$$

$$AF_n \leq M.AFb_n; \quad n \in N \quad (69)$$

$$QCTrAB_{b,y,t} \geq \sum_{yy \in Y} (QC_{b,yy,t} - QC_{b,yy,t+1}) + \quad b \in B, y \in Y, t \in T \quad (70)$$

$$\sum_{bb \in b-1} (QCET_{bb,t} - QCET_{bb,t+1}) + \sum_{bb \in b-1} \sum_{yy \in Y} (QC_{bb,yy,t} - QC_{bb,yy,t+1});$$

$$QCTrAB_{b,y,t} \geq \sum_{yy \in Y} (QC_{b,yy,t+1} - QC_{b,yy,t}) + \quad b \in B, y \in Y, t \in T \quad (71)$$

$$\sum_{bb \in b-1} (QCET_{bb,t+1} - QCET_{bb,t}) + \sum_{bb \in b-1} \sum_{yy \in Y} (QC_{bb,yy,t+1} - QC_{bb,yy,t});$$

$$QCTrAB2_{b,t} \geq \sum_{bb \in b} (QCET_{bb,t} - QCET_{bb,t+1}) + \sum_{bb \in b} \sum_{yy \in Y} (QC_{bb,y,t} - QC_{bb,y,t+1}); \quad b \in B, t \in T \quad (72)$$

$$QCTrAB2_{b,t} \geq \sum_{bb \in b} (QCET_{bb,t+1} - QCET_{bb,t}) + \sum_{bb \in b} \sum_{yy \in Y} (QC_{bb,y,t+1} - QC_{bb,y,t}); \quad b \in B, t \in T \quad (73)$$

4.3.1. QCSP constraints

The adaptations made to the QCSP are mainly related to the change in dimensionality in the berth attendance variable. An index y indicating the discretization of bays inside each berth is added.

In order to service the berths ($Be_{b,y,t}$), the loading parameters of a vessel become more detailed. Now it contains the load information of the vessel in distributed bays ($Ca_{n,y}$). The constraints then are expressed with new index as seen in the equations 52 and 53. The service is made by the binary variable $QC_{b,y,t}$ with indexes in berth, bay and time. The berth slack also undergoes the same dimensionality changes, since its physical meaning becomes the crane slack ($FA_{b,y,t}$).

In QCAP the total and maximum amount of quay cranes on the port and per berth were adapted constraints that meant the safety distance between the quay cranes in a model that still did not consider the detailing of bays on a vessel. Modeling this new index and new variables, they can be rewritten as seen in equations 57 e 58. Finally, it is necessary, in addition to the number of quay cranes per berth, to ensure the physical safety constraint and not collision, as seen in the constraints 59, 60 e 61. The maximum number of bays per berth was represented as y_{max} . The introduction of these constraints ensures that two quay cranes are not in adjacent bays at the same time, keeping at least 1 bay as the safety margin.

4.3.2. QCSP objective functions

The new set of indexes discretize the internal detail of a vessel, allowing now measuring and optimizing the number of movements between bays made by the quay cranes into the time horizon. This new objective function requires the addition of $b.(y+1).t$ of variables ($QCTrAB_{b,y,t}$ and $QCTrAB_{b,t}$, binary variables) and $2.b.(y+1).t$ constraints (constraints 70, 71, 72 and 73). The operation of these constraints are similar to the equations 41 and 42 of QCAP model.

5. Computational experiment and analysis

The experiments planned for this paper are only to demonstrate the operation of mathematical models, such as the impact of the objective functions solved separately. A toy problem was used for illustration and understanding. A fictitious port with two berths and three service quay cranes and a three vessels horizon to be served. Each vessel has three bays with distributed loads: (2, 3, 2), (5, 1, 1) and (1, 3, 3); three arrival times: 1, 3 and 5; and three estimated departure times: 6, 8 and 11 respectively. An example solution is presented in Table 1. This is a simple example of resolution and how the associated problems and deci-

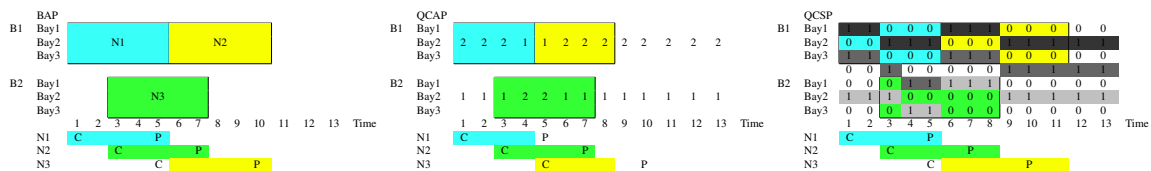


Table 1: (a) left: BAP solution. (b) center: QCAP solution. (c) right: QCSP solution.

sions can be understood. For each objective function the solving algorithm was implemented like section 3 for the coupled models presented in section 4. The results are presented in Table 2. The values found for

	BAP	QCAP	QCSP		AF	AFb	MAF	AI	AIb	MAI	TA	MTA	TS	MTS	TF	Mak
AF	3	0	2	AF			b	b	b	b	b	b	bas	ba	bas	b
AFb	2	1	1	AFb												
MAF	1	0	2	MAF	b			b	b	bas	b	b	b	b	b	b
AI	1	0	1	AI	b		b		bas	bas	b	b	b	b	b	b
AIb	1	0	1	AIb	b		b	bas	bas	bas	b	b	b	b	b	b
MAI	1	0	1	MAI	b		b	bas	bas		b	b	b	b	b	b
TA	15	9	14	TA	b		b	b	b	b	bas	bas	b	bs	b	b
MTA	5	3	5	MTA	b		b	b	b	b	bas		b	bs	b	b
TS	16	10	15	TS	bas		b	b	b	b	b	b	ba	ba	bas	b
MTS	6	4	6	MTS	ba		b	b	b	b	bs	bs	ba		ba	b
TF	25	19	24	TF	bas		b	b	b	b	b	b	bas	ba		b
Mak	11	8	11	Mak	b		b	b	b	b	b	b	b	b	b	

Table 2: (a) left: Values of objective functions. (b) right: Resolutions with same answers.

each model and each objective function (Table 2 (a)) present the impact that the details of the subsequent

problems (QCAP and QCSP) generate in the final solution found, since the values greatly change between the stages. The objective functions that have obtained the same solution at some stage are listed in Table 2 (b). It is noted that several resolutions obtained the same solution for the BAP problem (solutions marked with *b*, in light blue), except for *AFb* case. Some of these cases still had the same solution for the QCAP (solutions marked with *ba*, in yellow), as *TS*, *MTS* and *TF*. Other solutions were able to obtain different results for QCAP, but returned to the same solution in QCSP, ending the resolution with the same answer (solutions marked with *bs*, in light green), like *MTS*, *TA* and *MTA*. Some cases had the very equal path to resolution, obtaining the same solution in all stages (solutions marked with *bas*, in dark green). The *bas* cases are: (i) *AI*, *AIb* and *MAI*; (ii) *TA* and *MTA*; and (iii) *AF*, *TS* and *TF*.

For the *bs* case, we had the *MTS* gathering to the (ii) *bas* case. The impact of these objectives are presented in the Table 3.

	MTS		TA		MTA	
	Amount	Type	Amount	Type	Amount	Type
Variables	1	continuous	none	-	1	continuous
Constraints	n	inequality	none	-	n	inequality

Table 3: Impact of dimensionality on models by objective functions of *bs* and (ii) *bas* cases.

For the (i) *bas* case, we had the *AI*, *AIb* and *MAI*. The impact of these objectives are presented in the Table 4.

	AI		AIb		MAI	
	Amount	Type	Amount	Type	Amount	Type
Variables	none	-	n	binary	1	continuous
Constraints	none	-	n	inequality	n	inequality

Table 4: Impact of dimensionality on models by objective functions of (i) *bas* case.

For the (iii) *bas* case, we had the *AF*, *TS* and *TF*. The impact of these objectives are presented in the Table 5.

	AF		TS		TF	
	Amount	Type	Amount	Type	Amount	Type
Variables	n	continuous	none	-	none	-
Constraints	n	inequality	none	-	none	-

Table 5: Impact of dimensionality on models by objective functions of (iii) *bas* case.

As noted in Table 3 the sum of service time (*TA*) was the simplest model and it was able to find the same solution than *MTS* and *MTA*. The same happens to the sum of initial delays (*AI*) in Table 4. In this case the impact is more important whereas *AIb* demanded an addition of binary variables. For the last case in Table 5 the sum of system times (*TS*) and the sum of completion (*TF*) were better than *AF*.

6. Conclusion

A integrated coupled model was proposed by the mathematical model BAP → QCAP → QCSP, with detailing *disc | dyn, due | QCAP, QCSP | misc* and *bay, prmp | ready | cross, safe | move*. This model is unprecedented in literature and it was noted functional to a controlled toy problem. Now it can be used for other larger instances and port configurations. A brief analysis was presented, showing how the choice of a specific objective function can impact the dimensionality of the model.

An analysis was also made of equal solutions found by different objective functions. Identifying complex objective functions, which can result at the same solutions to those found by simpler objective functions, is important for using an efficient model and still consider all the alternatives.

Of course that the answer found works only for this toy problem, but the difference among the solutions initialize a discussion about the choice of performance measures. This analysis is not exclusive to port problems and can be done for any problems with multiple objectives.

For future research it is indicated the use of models with deep integration. These complex models will be able to bring even more credibility to the solutions found. The proposed analysis made for the toy problem should be repeated on a larger scale in order to find new patterns of correlation. The patterns of correlation can be a indication of subclass of problems, where it can be varied the amount of berths, quay cranes, bays and even the load distribution.

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