

LINEAR PROGRAMMING AND METHODS OF MULTIVARIATE REGRESSION APPLIED TO THE PREDICTION OF THE REFRACTORY CAMPAIGN DURATION IN A STEEL COMPANY

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ABSTRACT

When it comes to steel processes, it is known that refractory materials are responsible for a significant portion of the steel production costs. For this reason, this work aimed to understand the high variability and the low durability of the refractory campaign that compose a process of continuous casting in a large LD mill in the state of Minas Gerais, identifying the relationship between the process variables so it was possible to make estimates about the duration of its refractory campaign. For the selection of the explanatory factors, a variation of the method Stepwise was used. In each step of the algorithm, a model based on linear programming was responsible for the calculations of the linear regression coefficients. In the end, a prediction model was obtained for the duration of the campaign containing 12 explanatory factors and 97.66% of statistical significance.

KEYWORDS. Refractory Materials, Multivariate Regression, Linear Programming.

Paper topics (POI, EST&MP, OA)

1. Introduction

The crisis in the international financial system, which has its roots in the largest global power, the United States, not only affects the monetary side of the economy but also directly influences the real economy. Among the sectors most affected are those that have assets and/or liabilities in foreign currency, especially in dollars. The steel sector is a sector of great representation in the Brazilian economy and has a remarkable proportion of rights and obligations registered in foreign currency in its equity structure [Francisco et al., 2013].

In the process of improving the productivity of steel making processes, it is not only satisfactory to invest heavily in the acquisition of machines and new technologies, and thus, in-depth studies of the dynamics of production processes are required, distinguishing the phases and factors that most impact the efficiency of results.

This article aims to study on the stage of transformation of liquid steel into ingots. At this stage of the steelmaking process, equipment called “tundishes” are used, which, according to [Zimmer et al., 2004], are replaced and sent to the maintenance industry with the occurrence of wear on the refractory lining .

In this way, understanding the durability behavior of refractory materials is a fundamental action for the moderation of the costs of this sector. However, the high diversity of dynamic factors that simultaneously affect this process makes its prediction a complex task. For [Ritzman et al., 2004], changes in process conditions makes pressure on a company’s ability to generate accurate forecasts, which are necessary for an organization to know which features are essential for scheduling its activities over the time.

The present study analyzed the factors affecting the useful life of refractories of steel distributors in an LD (Linz-Donawitz) mill located in a large steel mill in the state of Minas Gerais. For the definition of critical factors, an adaptation of the method Stepwise multiple regression model was applied. A model based on linear programming was applied to each step of the Stepwise method for the calculation of linear regression estimators. Estimates were generated from sample data provided by the company.

It is noteworthy that the company studied does not have an assertive plan for the forecast of the duration, purchase or material exchange plan for these equipment, since there are no reliable estimates as to the actual duration of the refractory materials campaign.

2. Literature Review

2.1. Multivariate Statistics in Steelmaking Processes

Multivariate analysis of data refers to all statistical methods that simultaneously analyze multiple measures that work on individuals or objects under investigation [Hair et al., 2009]. And in the case of steelmaking processes, the application of multivariate statistical occurs mostly related to study physical, mechanical and chemical compositions of the final product components. There are not many studies in the literature relating the life time of equipment and its components in industrial processes.

When we consider the multivariate statistics applied to the context of steelmaking processes, studies can be found for the calculation of the mass balance of blast furnaces [Ruuska et al., 2017], estimation of the energy conservation potential in the Chinese industry [Rama Mohana Rao et al., 2013], early detection of segregation in continuous casting processes [Nieto et al., 2015], among others. We can also point out the use of different application methods for the analysis of multivariate statistics in steel processes, such as logistic regression in [Xu e Zhao, 2005], and robust regression methods [Mondal, 2016], for example. A systematic characterization of multivariate statistical methods is presented by [Peres e Fogliatto, 2018].

2.2. Multiple Linear Regression

A regression analysis is represented by an equation that describes the statistical relationship between one or more predictors and their response variable, as well as to predict new observations. For [Hair et al., 2009] multiple regression analysis is a multivariate statistical technique that can be used to analyze the cause and effect relationship between a single dependent variable and several independent, predictive or explanatory variables.

The multiple regression analysis aims to estimate the impact of the increment of each independent variable, which translates into the weight of each independent variable, on the respective variation of the dependent variable [Fávero e Fávero, 2016]. The set of these independent variables, for [Sarmiento, 2010], forms the regression statistical variable, that is, a linear combination of the independent variables that best explain the dependent variable. Thus, the model representing the multiple regression is given by equation 1:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \epsilon \quad (1)$$

The variable y is the phenomenon under study; β_0 represents the intercept; $\beta_k, \forall k \in N$ are the angular coefficients; x_k are the explanatory variables and ϵ is the error term, i.e., the difference between the measured value and the value predicted by the model for each observation. The ϵ error, also called residue, may represent possible variables that were not included in the model but also contribute to the explanation of y .

2.2.1. The Stepwise Method

The Stepwise regression is presented as the strategy chosen for an exploratory approach. By using this type of regression, there is no consistent theory about the phenomena studied, and there is only an interest in describing little-known relationships among variables [Abbad e Torres, 2002]. The method has an automatic tool used in the exploratory stages of modeling to identify a useful subset of predictor variables. In the process, the most significant variable is added systematically during each step.

The standard Stepwise method adds predictors as needed for each step. The process ends when all variables that are not in the model have values of p-value greater than the value of the specified insert α , all variables in the model having values of p-value less than or equal to the value of removal α specified.

Since the model contains j explanatory variables in the final formulation, the calculation of the F statistic for any variable x_r is given by expression 2:

$$F = \frac{(SSE_{(j-x_r)} - SSE_j)}{DF_{x_r} \times MSE_j} \quad (2)$$

Being that:

- (a) $SSE_{(j-x_r)}$: SS error after remove x_r from the equation;
- (b) $SSE_{(j)}$: SS error before remove x_r from the equation;
- (c) DF_{x_r} : degrees of freedom of x_r ;
- (d) $MSE_{(j)}$: MS error before remove x_r from the equation;

If the p-value for each variable is greater than the one entered at the significance level, the variable with the highest p-value is excluded and a new equation is generated. In the case of predictors to be added, if the method can not exclude a predictor, the value of F and a p-value is

calculated for each independent variable that is not in the equation. If the model has j variables, then F , for a predictor x_a , is given by the following expression 3:

$$F = \frac{(SSE_j - SSE_{(j+x_a)})}{DF_{x_a} \times MSE_{(j+x_a)}} \quad (3)$$

Being that:

- (a) $SSE_{(j)}$: SS error before add x_a to the equation;
- (b) $SSE_{(j+x_a)}$: SS error after add x_a to the equation;
- (c) DF_{x_a} : degrees of freedom of x_a ;
- (d) $MSE_{(j+x_a)}$: MS error after add x_a to the equation;

If the p-value satisfies the value of F with the lowest significance level for any predictor, the method includes the variable with the lowest p-value to the model. The method terminates when it is not possible to enter any variables.

3. Methods and Case Study

According to [Turrioni e Mello, 2012], scientific research can be classified as to nature, objectives, approach and its procedures. The present work is characterized as an applied, exploratory and quantitative research, since it was based on the problem, allowing the applicability of the proposed plan, as well as the use of statistical and optimization approaches for its formulation.

3.1. Data Collect

The data used in this study were obtained through the software Trace, which stores the process control information of continuous casting in the steelworks surveyed. These data are continuously collected through field measurements, automated by a PLC (Programmable Logic Controller) system and, when there are collection failures, the data is manually rectified by the operators.

The duration data of the campaigns and the factors selected for the accomplishment of this research were extracted between the periods of January 2016 and December 2017.

After the data collection, they were preprocessed in order to exclude items with unusual patterns, missing data or inconsistencies. This action found 63 rows to be deleted, out of a total of approximately 17,000 rows of data.

3.2. Company Scenario

The variability in the duration of the refractory campaign, presented in Figure 1, generates recurrent problems, especially in relation to the planning of purchases. As a result of this variability, the company has the problem of the lack of accuracy between the forecast of purchases and the actual consumption. This leads to high acquisition costs, compared to emergency purchases, or high inventory maintenance costs, compared to unnecessary volumes purchased.

The Figure 2 shows a comparison between the actual and expected consumption of the refractory materials for the period between December 2016 and May 2017.

Thus, it is sought the identification of a function that correlates the existing factors in the process with the duration of the campaign, providing an aid in the decision making. Having said this, it is necessary to understand the factors intrinsic to the process in order to relate the duration of the campaign to the operational aspects.

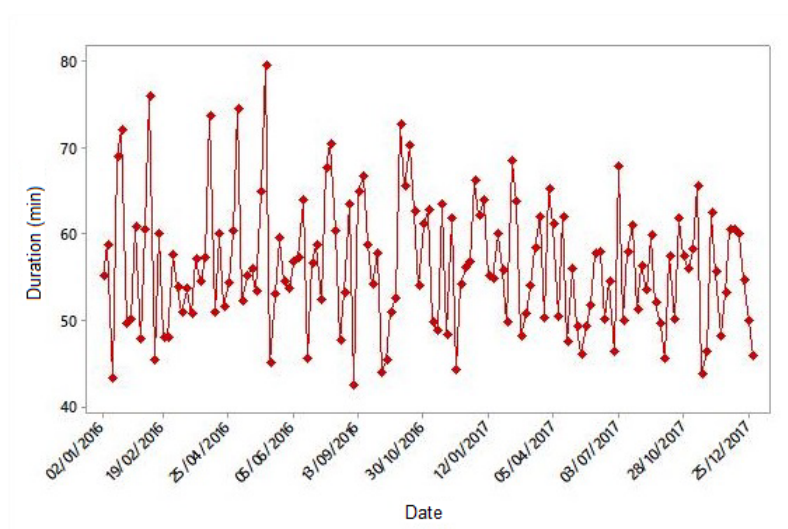


Figure 1: Equipment campaign duration.

3.3. Variables in the Continuous Casting Process

The factors of the continuous casting process that are in direct contact with the equipment and that were tested as possible predictors were obtained by means of reviews in the literature on the subject, being:

- (a) T : Temperature in degrees Celsius. Thermomechanical stresses and stresses influence the life of the tool [Lee e Zhang, 2004].
- (b) Chemical composition: C , Mn , Cr , Si , S , P , Cu , Ni , N_2 , Mo e Al . Chemical corrosion generates loss of mass in the coating, influencing the life of the refractory [Bragança, 2012].
- (c) V_m : Average casting speed. Erosion, impact and mechanical wear can reduce the life of refractory materials [Lee e Zhang, 2004].
- (d) Pr : Production of billet in tonnes. An increase in the duration of the campaign raises the productivity of the company [Carvalho, 2005].

4. Proposed Methodology and Descriptions

An optimization approach based on mathematical modeling was developed for the definition of slope values for each factor influencing the process. The exact model, based on linear programming, is solved by the open source solver LpSolve, thus avoiding investments in commercial solvers such as IBM-CPLEX and Gurobi Optimizer. Constructed modeling is performed during the iterations of addition and removal of explanatory variables to the prediction model, which are performed by the Stepwise method. All the codes were implemented in Python 2.7.12. The hardware used was a notebook with processor Intel(R) Core(TM) i5-4210U with 1.70GHz frequency, 8 GB of RAM memory and using the Ubuntu 16.04 operating system.

4.1. Linear Programming Model

The linear programming model proposed in this study contains the following sets, parameters and decision variables:

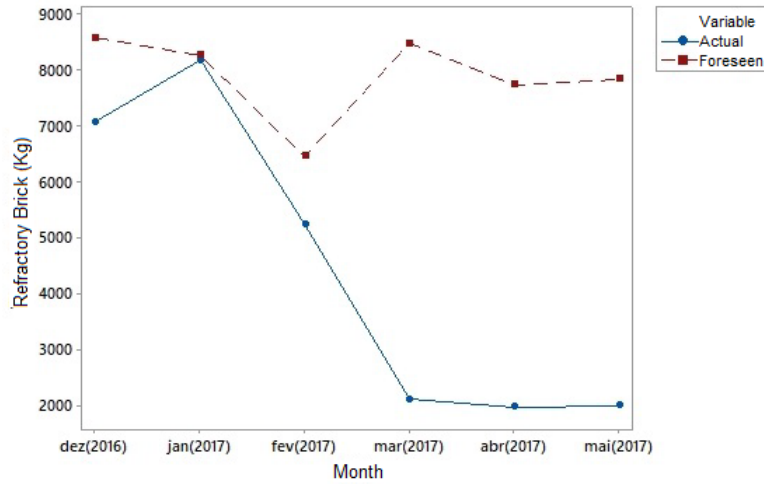


Figure 2: Actual and predicted consumption of refractory brick.

- τ : set of influencers in the duration of the refractory campaign.
- κ : set of samples to be analyzed.
- y_i : duration of the campaign in a given sample $i \in \kappa$
- x_{ij} : value obtained for a given influencing factor $j \in \tau$ in a campaign $i \in \kappa$.
- β_j : the slope value assigned in the prediction curve for the influencer $j \in \tau$.
- β_0 : the intercept value assigned in the prediction curve.
- ϵ_i : residual value found in the sample prediction $i \in \kappa$.

When assigning the slope values to the influencing factors, we try to minimize the total errors found. In this way, the assignment can be solved as presented in the equations [4 - 8].

$$\min \sum_{i \in \kappa} |\epsilon_i| \quad (4)$$

$$\epsilon_i = y_i - (\beta_0 + \sum_{j \in \tau} \beta_j x_{ij}) \quad \forall i \in \kappa \quad (5)$$

$$\epsilon_i \text{ free} \quad \forall i \in \kappa \quad (6)$$

$$\beta_j \text{ free} \quad \forall j \in \tau \quad (7)$$

$$\beta_0 \text{ free} \quad (8)$$

The equation 4 minimizes the magnitude of the residuals found, while the constraint exposed by inequality 5 guarantees the attribution of the sample residues found in the prediction

model. Finally, the constraints 6, 7 and 8 inform the domain of the decision variables. However, to solve the problem using linear programming algorithms, we must first make some adjustments in the presented model.

The first adjustment takes place by replacing the free variables of the problem (variables that can receive positive or negative values) by variables having domain greater than or equal to zero. To perform this adjustment, the variables were replaced using the relations presented in the equations [9 - 11].

$$\epsilon_i = (\epsilon_i^+ - \epsilon_i^-) \quad \forall i \in \kappa \quad (9)$$

$$\beta_j = (\beta_j^+ - \beta_j^-) \quad \forall j \in \tau \quad (10)$$

$$\beta_0 = (\beta_0^+ - \beta_0^-) \quad (11)$$

Thus, taking into account the relations [9 - 11], we can rewrite the equations 4 and 5 as follows:

$$\min \sum_{i \in \kappa} |(\epsilon_i^+ - \epsilon_i^-)| \quad (12)$$

$$(\epsilon_i^+ - \epsilon_i^-) = y_i - [(\beta_0^+ - \beta_0^-) + \sum_{j \in \tau} (\beta_j^+ - \beta_j^-) x_{ij}] \quad \forall i \in \kappa \quad (13)$$

Since the domains of the new decision variables are expressed by equations [14 - 19].

$$\epsilon_i^+ \geq 0 \quad \forall i \in \kappa \quad (14)$$

$$\epsilon_i^- \geq 0 \quad \forall i \in \kappa \quad (15)$$

$$\beta_j^+ \geq 0 \quad \forall j \in \tau \quad (16)$$

$$\beta_j^- \geq 0 \quad \forall j \in \tau \quad (17)$$

$$\beta_0^+ \geq 0 \quad (18)$$

$$\beta_0^- \geq 0 \quad (19)$$

The second adjustment that is necessary is the linearization of the module used in the objective function of the problem. To make this adjustment, the equation 12 has been replaced by the equations given in [20 - 23].

$$\min \sum_{i \in \kappa} \epsilon_i^M \quad (20)$$

$$\epsilon_i^M \geq (\epsilon_i^+ - \epsilon_i^-) \quad \forall i \in \kappa \quad (21)$$

$$\epsilon_i^M \geq (\epsilon_i^- - \epsilon_i^+) \quad \forall i \in \kappa \quad (22)$$

$$\epsilon_i^M \geq 0 \quad \forall i \in \kappa \quad (23)$$

4.2. Calculation of Isolated Linear Correlations

For the beginning of the proposed method, we first need to know the correlation indices between the duration of the campaign and each influencing factor. For this evaluation, the indexes r , R^2 and R_{aj}^2 were used. Pearson's Degree of Correlation (r) measures the degree of linear correlations between two quantitative variables, while the determination coefficients R^2 and R_{aj}^2 measure the fit percentage of the model to the data. Table 1 shows the correlations between the factors that are candidates to enter the model with the response variable “duration”.

Table 1: Degree of correlation between variables.

Variables	r	R^2	R_{aj}^2
T	-0,107	70,46%	59,00%
C	0,031	90,29%	75,53%
Mn	0,162	89,38%	73,01%
P	0,091	83,10%	68,29%
S	0,022	80,54%	68,65%
Si	0,273	84,77%	69,48%
Al	-0,137	78,75%	65,20%
Cu	0,041	85,53%	67,32%
Ni	0,064	81,05%	65,78%
Mo	0,008	73,58%	63,72%
N2	-0,025	72,08%	64,98%
Pr	0,979	98,45%	97,93%
Vm	0,025	79,61%	65,18%
Cr	0,186	89,03%	70,83%

Based on the values in Table 1, it is noted that the higher values of r , R^2 and R_{aj}^2 are observed in the “production” factor. These values indicate a strong positive linear correlation. It is also highlighted that there are factors with linear inverse correlation, as is the case of “nitrogen”, “temperature” and “aluminum”.

4.3. Adapted Stepwise Method

For the regression model adjustment using the Stepwise method, the α value for input and removal of influencing factors was set to $\alpha = 0,05$. In performing the prediction model adjustment steps, twelve iterations were performed by the Stepwise method, and at each iteration a new variable was added to the model. The data collected were randomly separated into two groups: the first group is used in the step of defining the slope indexes of the influencing factors, performed by the proposed linear modeling; the second group of data is used for the validation of the insertions or removals made in the prediction curve, performed during the iterations of the Stepwise method by the calculations of statistical strength and correlation.

The Algorithm 1 describes the pseudocode of the proposed method. Table 2 shows the predictive (independent) factors inserted in each iteration, as well as the process of updating the

error and correlation values S , R^2 , R_{aj}^2 and R_{pred}^2 . Finally, the prediction curve for the duration of the refractories campaign found at the end of the method is presented by Equation 24.

```

LP-SW ():
1  Calculate correlation for every factor  $j \in \tau$ 
2   $P_v = P_v^N = P_v^A = \infty$ 
3   $\theta = \{\}$ 
4  repeat
5       $P_v = P_v^N$ 
6      Sort set  $\tau$  in descending order by correlation level
7      for ( $j \in \tau$ ) do
8          Insert the factor  $j$  into the prediction model
9           $P_v^A = LP(\beta, \epsilon)$ 
10         if ( $P_v^A < P_v$ ) then
11             Remove the factor  $j$  from set  $\tau$ 
12             Add the factor  $j$  to set  $\theta$ 
13              $P_v^N = P_v^A$ 
14             break
15         end
16         else Remove the factor  $j$  from the prediction model
17     end
18     Sort set  $\theta$  in ascending order by correlation level
19     for ( $j \in \theta$ ) do
20         Remove the factor  $j$  into the prediction model
21          $P_v^A = LP(\beta, \epsilon)$ 
22         if ( $P_v^A < P_v^N$ ) then
23             Add the factor  $j$  to set  $\tau$ 
24             Remove the factor  $j$  from set  $\theta$ 
25              $P_v^N = P_v^A$ 
26             break
27         end
28         else Add the factor  $j$  to the prediction model
29     end
30 until ( $P_v^N > P_v$ )
31 end
32 return Prediction model found

```

Algorithm 1: Proposed adaptation to the Stepwise prediction algorithm.

$$\begin{aligned}
 D = & -397,8 + (0,301 \times T) + (42,41 \times C) + (11,47 \times Mn) - (523,7 \times P) \\
 & + (199,1 \times S) + (30,32 \times Si) + (198,5 \times Al) + (14,59 \times Cr) \\
 & - (100,1 \times Cu) - (71,7 \times Mo) + (0,3871 \times Pr) - (37,22 \times Vm)
 \end{aligned} \quad (24)$$

It is noted that the method did not perform the insertion of the predictive factors N_2 and Ni . It is possible to observe that, as expected, with each step the value of S decreases, while

Table 2: Inserts executed between steps in the Stepwise method.

Step	Inserted Variable	S	R^2	R_{aj}^2	R_{pred}^2
1	Production	31,994	95,84%	95,84%	95,83%
2	Average Casting Speed	26,166	97,21%	97,21%	97,19%
3	SI	25,293	97,39%	97,39%	97,38%
4	C	24,893	97,47%	97,47%	97,46%
5	S	24,653	97,52%	97,52%	97,51%
6	P	24,252	97,60%	97,60%	97,58%
7	Cr	24,163	97,62%	97,62%	97,60%
8	Temperature	24,092	97,64%	97,63%	97,61%
9	Mn	24,015	97,65%	97,65%	97,63%
10	Cu	24,974	97,66%	97,65%	97,64%
11	Al	23,939	97,67%	97,66%	97,64%
12	Mo	23,928	97,67%	97,66%	97,64%

an inverse relationship occurs with the values of R^2 , R_{aj}^2 and R_{pred}^2 . It is also worth noting that the final value of R_{aj}^2 , 97.66%, indicates that the model generated has a strong adjustment to the reference data.

5. Results Obtained

In order to legitimize the model proposed in this study, the prediction calculations were performed by inserting the original values of the process into the prediction equations, using the reference data provided by the company. Figure 3 shows the comparative graphs between the actual and expected campaign durations.

Figures 5[a] and 5[b] show the comparisons for the period from January 2016 to December 2017 for two different distributors. It is possible to analyze that the predicted values have adherence to the real values, even for different equipment, generating evidence that the model has assertive adjustment. Figures 5[c] and 5[d] refer to the same distributors, but restricted to a period of six months (June 2017 to December 2017), in order to highlight more clearly the adjustment of the prediction curve .

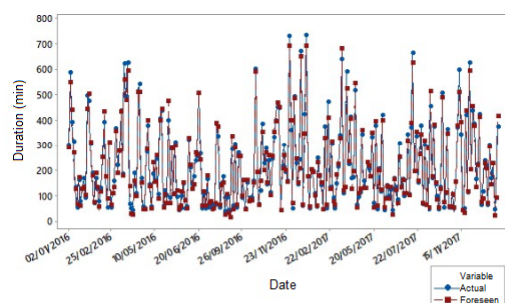
Considering the data equivalent to the two years of collection, the average residual value obtained for the data has a value of 16 minutes. This value was obtained from the equation 25.

$$\bar{\epsilon} = \frac{\sum_{i \in \kappa} \epsilon_i^M}{|\kappa|} \quad (25)$$

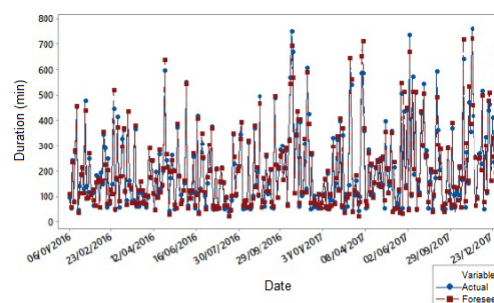
Given the proximity of the prediction curve found, it is understood that it is possible to characterize the behavior of the campaign duration of the stock distributors by means of the factors raised in the literature and considered in this study.

6. Concluding Remarks

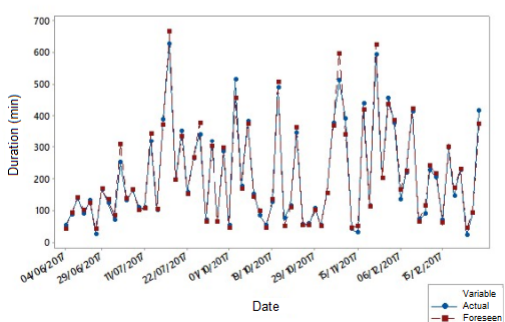
In observing the aspects analyzed in the literature review, it was noticed that the refractories in a steel industry are subject to numerous factors of the process. Such aspects can lead to the premature degradation of the coatings and the high variability in the duration of the campaigns, which motivated the interest in carrying out this study. Considering that this situation is a problem in the company studied and is related to the management of the process, it was decided to propose a



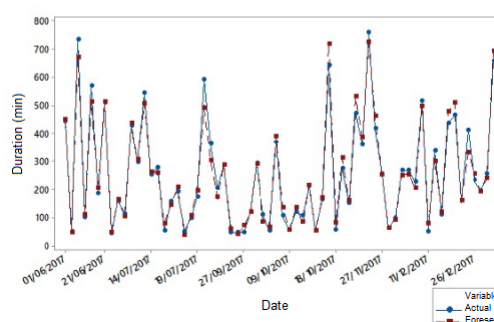
(a) 2 years - Tundish 1



(b) 2 years - Tundish 2



(c) 6 months - Tundish 1



(d) 6 months - Tundish 2

Figure 3: Prediction function adjustment against the actual duration of the refractory.

method of prediction for the aid in making decisions that depend on the information of the duration of the company, such as the purchase of inputs, for example.

To understand the behavior of the duration of the campaign, it was proposed the elaboration of a forecast model for the variable of the process “duration”. The combination of the Stepwise method and an algorithm based on linear programming were used to adjust the prediction curve. The result found by the proposed algorithm was satisfactory, reaching a value higher than 97% for all correlation evaluations.

A proposal for subsequent works would be the elaboration of a supply plan based on prediction models, being explicit the quantities of materials demanded and also the moments in which they should be acquired. In addition, it is also proposed an evaluation on the maintenance of equipment coatings, in order to evaluate if there are wastages and failures during the accomplishment of the work by the operating sector.

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