

Binomial and trinomial versions of the ATTRIVAR control chart

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ABSTRACT

The ATTRIVAR control chart uses attribute and variable sample data to control the process mean of a quality characteristic X . The trinomial version of the ATTRIVAR chart, presented in this paper, uses, firstly, two discriminating limits to classify the sample items into three excluding categories. Items can be classified in categories A, B and AB in which the items have the X dimension, respectively, lower than the lower discriminating limit, greater than the upper discriminating limit and between the lower discriminating limit and the upper discriminating limit. The number of sample items in each category lead to one of three decisions: the process is stated as in-control, the process is stated as out-of-control or the X dimension is measured for all the sample items. If the decision made is this last one, the sample mean of the X values is used to decide the state of the process. The binomial version classify the items into two excluding categories, defective (A and B of the trinomial) and not defective (AB of the trinomial). The value of Average Run Length (ARL) were used to evaluate the efficiency of this control chart. The trinomial version of the ATTRIVAR chart is more efficient than the binomial version in detecting a shift in the process mean.

KEYWORDS: Control Chart. ATTRIVAR. Trinomial Version.

Paper topics: EST – Estatística, MP – Modelos Probabilísticos

1. Introduction

The control chart for attributes was firstly designed to control processes in which the monitored quality characteristics cannot be measured on a continuous numerical scale. The number of defectives manufactured items from samples of size 50 is a common example. However, the focus of several recent studies lies on the control of variable-type quality characteristics with attribute charts. In 2008, the first study to propose an attribute chart to control the mean of a continuous variable X was published, [Wu and Jiao 2008]. This article inspired many other studies, in which attribute charts are used to control the mean, the mean vector, the variance and the covariance matrix, [Wu et al. 2009], [Ho and Costa 2015], [Ho and Quinino 2016], [Haridy et al. 2014], [Quinino et al. 2017], [Melo et al. 2016], [Aparisi et al. 2018], [Bezerra et al. 2018], [Machado et al. 2018], [Mosquera et al. 2018], [Zhou 2019].

The strategy to use attribute and variable sample data (ATTRIVAR data) was introduced in 2014, [Sampaio et al. 2014]. In this study, the samples are split in two subsamples, in such a way that, the items of the first subsample are submitted to a go/no-go gauge test, and the items of the second subsample are measured. It is simple to obtain the properties of the ATTRIVAR charts when the items submitted to the attribute-type inspection are not the same as the ones submitted to the variable-type inspection. This sampling strategy was also adopted in other studies, [Ho and Quinino 2016], [Ho and Aparisi 2016], [Aparisi and Ho 2018]. However, in 2015 a study was developed, in which, the sample items are first submitted to the attribute-type inspection and, if the attribute sample information is not conclusive enough to decide the state of the process, they are also submitted to the variable-type inspection [Aslam et al. 2015]. In this case, the sample mean of the X observations is used to decide the state of the process. It is important to evidence that the attribute-type inspection always truncates the X probability distribution function and, due to that, the mathematical development to obtain the distribution of a sample mean is not trivial. The ARL is the average number of samples taken from the process that the control chart requires to signal a shift in the process mean.

In this paper, the named binomial version of the ATTRIVAR chart is described in section 2 accordingly to the studies in the literature. In section 3, the trinomial version is described and, in section 4, the ARL values are calculated and the efficiency of both versions of the ATTRIVAR chart are compared for different scenarios. The conclusions are presented in section 5.

2. Binomial version of the ATTRIVAR control chart

The Binomial version of the ATTRIVAR chart (B-ATTRIVAR) monitors a process mean by taking samples of size n regularly. When the Binomial ATTRIVAR chart is in use, the sample is submitted, firstly, to an attribute-type investigation, in such a way that, if the item's quality characteristic X is lower than a lower discriminating limit LDL or is greater than an upper discriminating limit UDL , the item is classified as defective and, if the quality characteristic X is between LDL and UDL , the item is classified as not-defective. This first part of the Binomial ATTRIVAR chart's monitoring procedure consists in classifying, by a go/no-go gauge test, which of the n items of the sample are stated as defective or not-defective and, if the quantity of defective items Y_D are small, the process is considered to be in-control, if the quantity of defective items are greater than or equal to a warning limit Wy but smaller than a control limit CLy , the same sample is submitted to the second part of the monitoring procedure, if the quantity of defective items are greater than or equal to CLy , the process is stated to be out-of-control. The second part of the Binomial ATTRIVAR chart's monitoring procedure consists in measuring the values of the quality characteristic X of all the n items and computing the sample mean $\bar{X}$. The process is considered in-control if $\bar{X}$ is between a lower control limit LCL and an upper control limit UCL and it is considered out-of-control if $\bar{X}$ is greater than UCL or lower than LCL . This control chart is named Binomial ATTRIVAR as the first stage only classifies the items of a

sample in two categories, defective or not-defective. It is also named, in the literature, as ATTRIVAR-1, [Ho and Aparisi 2016].

Figure 1 shows the flowchart that explains how the binomial version of the ATTRIVAR chart works to monitor a process mean.

Figure 1: Flowchart of the binomial version of ATTRIVAR chart

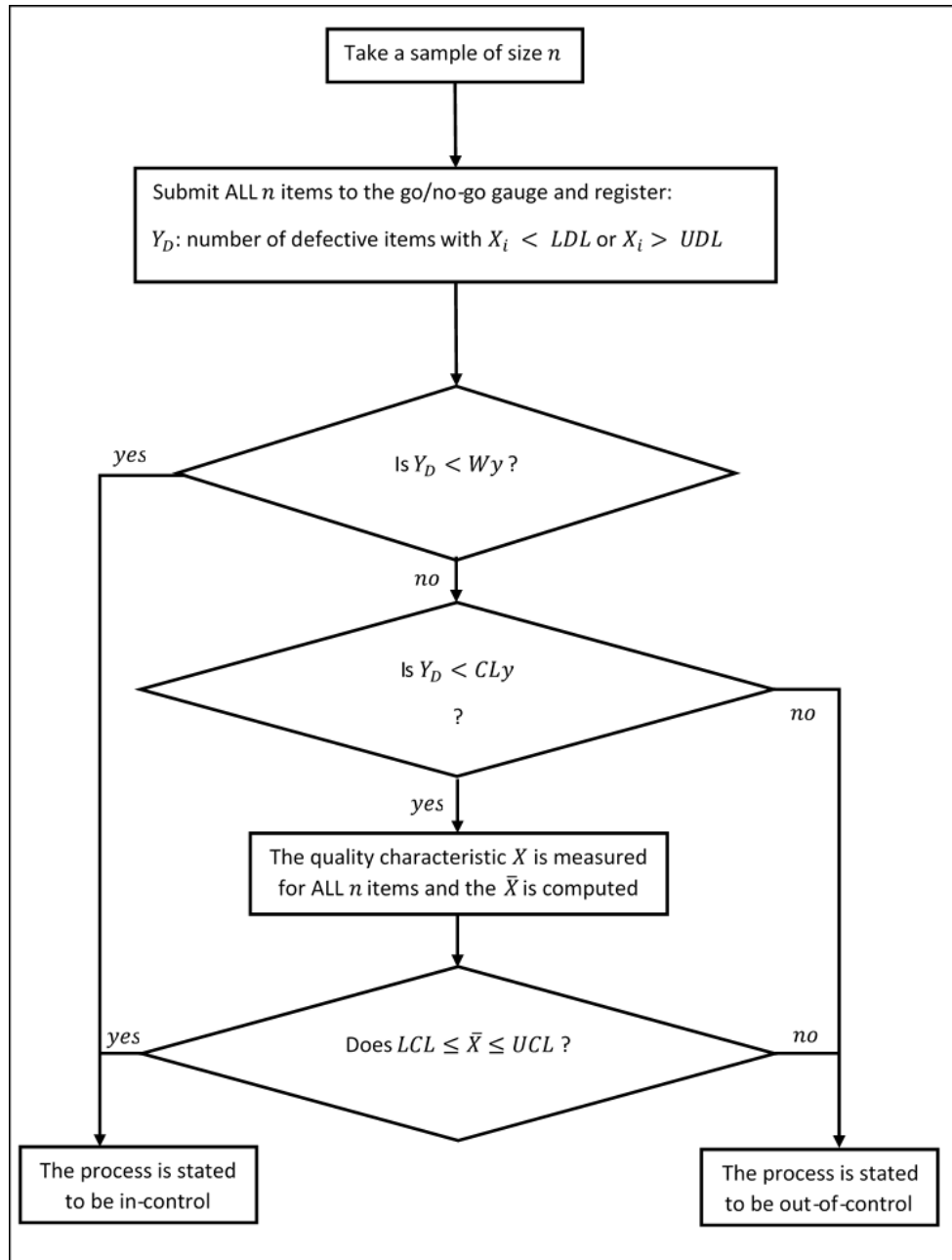
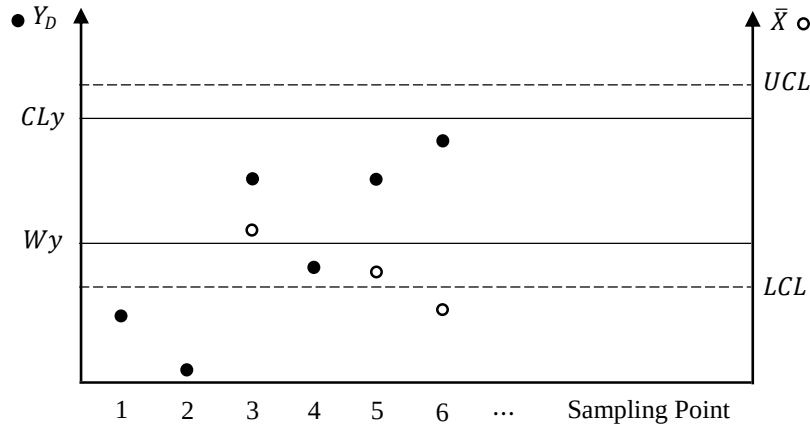


Figure 2 illustrates a representation of the ATTRIVAR chart. In Figure 2, the black points illustrated represent the values of Y_D and use the scale in the left-hand side of the chart and limits W_y and CL_y , and the white points illustrated represent values of $\bar{X}$ and use the scale in the right-hand side of the chart and limits LCL and UCL . Figure 2 shows that the samples, of which the values of Y_D are plotted below the attribute warning limit W_y , are considered in-control and,

thus, the values of the quality characteristic X are not measured and also there are no cost with the second stage of the monitoring procedure for those samples.

Figure 2: The ATTRIVAR chart



The frequency of samples that go to the second part of the monitoring procedure for the binomial version of the ATTRIVAR chart, when the process is in-control, is dimensioned by $\% \bar{X}_B$ and this value depends on W_y , CL_y , LDL and UDL . For the binomial version of the ATTRIVAR chart, the value $\% \bar{X}_B$ can be calculated with equation (1).

$$\% \bar{X}_B = \sum_{i=W_y}^{CL_y-1} \frac{n!}{i!(n-i)!} \times (p)^i \times (q)^{(n-i)} \quad (1)$$

Where $p = \left[1 - \Phi \left(\frac{UDL - \mu_0}{\sigma} \right) \right] + \Phi \left(\frac{LDL - \mu_0}{\sigma} \right)$, $q = 1 - p$, μ_0 is the process mean and σ is the process standard-deviation. In Figure 2, it is also shown that the ATTRIVAR chart signals in sample number 6 as the value of $\bar{X}$ falls below LCL .

3. Trinomial version of the ATTRIVAR control chart

In this section, the Trinomial ATTRIVAR chart is described with the idea of a version of the ATTRIVAR chart rule even more economic and more efficient in detecting a shift in the process mean. When the trinomial version of the ATTRIVAR chart (T-ATTRIVAR) is in use, it works similarly to the B-ATTRIVAR chart but instead of sorting the sample items in two categories (defective or not-defective), in the first part, the T-ATTRIVAR chart sort the sample items in three excluding categories. When the T-ATTRIVAR chart is in use, a sample of size n is taken and all of those n items are submitted to a go/no-go gauge test that register the defective items of category A, in which $X \leq LDL$, the defective items of category B, in which $X \geq UDL$, and the items of category AB, in which $LDL < X < UDL$. The quantity of defective items of category A is represented as Y_A , the quantity of defective items of category B is represented as Y_B , and the quantity of items of category AB is represented as Y_{AB} . If there are defective items of category A and category B in the same sample, the process is considered to be in-control. If there are defective items of only one category (A or B) in the sample but the quantity of defective items is lower than W_y , than the process is also considered to be in-control. If there are defective items of only one category (A or B) in the sample and the quantity of defective items are greater than or equal to CL_y , than the process is considered to be out-of-control. If there are defective

items of only one category (A or B) in the sample and the quantity of defective items are greater than or equal to Wy and lower than CLy , than the quality characteristic X is measured for all of those n items and $\bar{X}$ is calculated and, then, if $\bar{X}$ is between the $\bar{X}$ chart's control limits, LCL and UCL , than the process is considered to be in-control, otherwise the process is considered out-of-control. Figure 3 shows the flowchart that explain how T-ATTRIVAR chart works.

Figure 3: Flowchart of the trinomial version of the ATTRIVAR chart

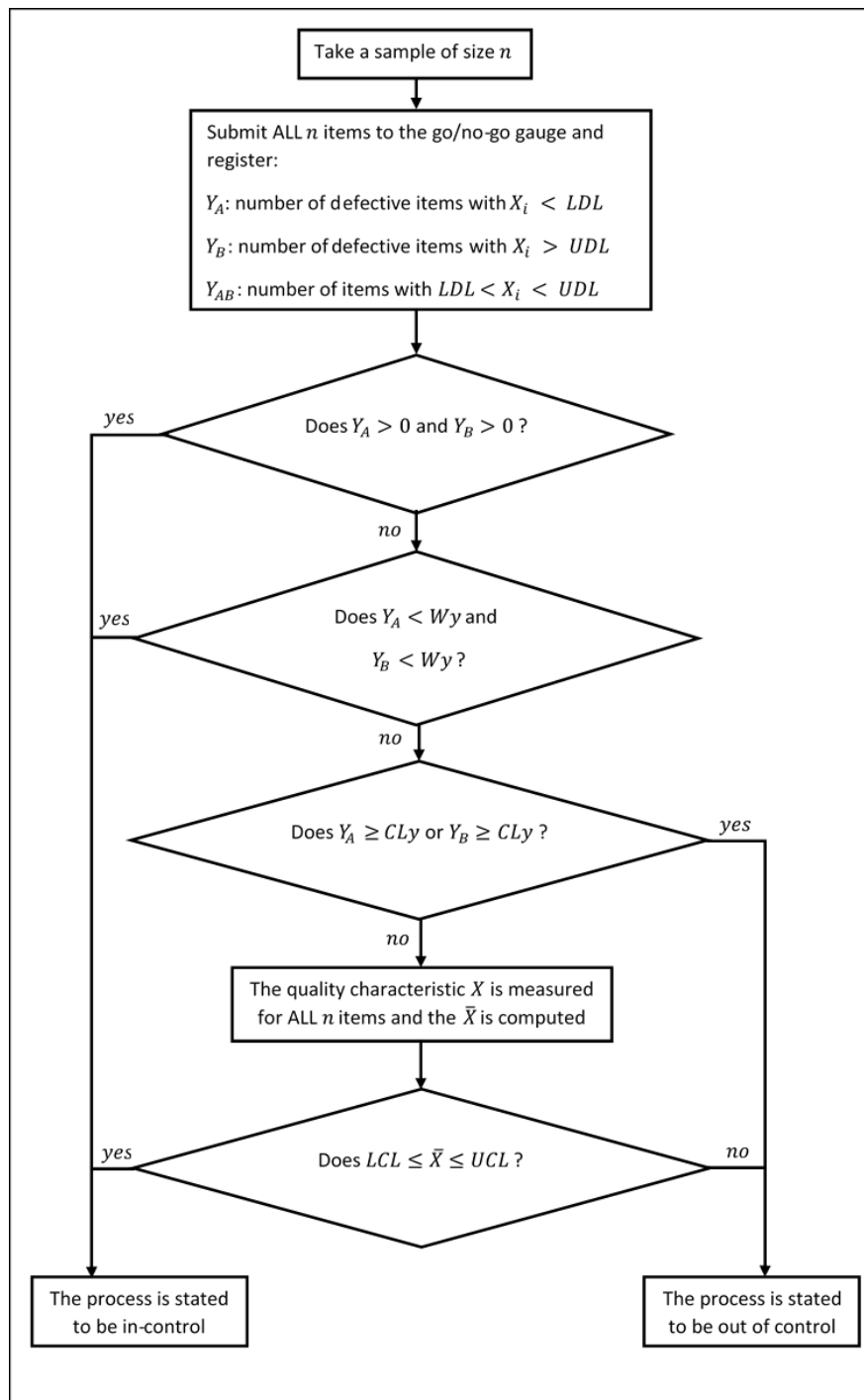


Figure 2 can also represent the T-ATTRIVAR chart if it is considered that when $Y_A > 0$ and $Y_B = 0$, then $Y_D = Y_A$, and when $Y_B > 0$ and $Y_A = 0$, then $Y_D = Y_B$ and when $Y_A > 0$ and $Y_B > 0$, then $Y_D = 0$.

This trinomial version rule suggests that real alarms will only appear in samples with quality characteristic $\bar{X}$ out of the limits of one specific side of the central line, that is, all defective items classified in category A, or all of defective items classified in category B. With this classification of items in a sample, it could happen to have, in the same sample, items classified as category A and category B, which in the Binomial version of ATTRIVAR chart, both would simply be considered in the same category, as defective, and if they sum up to a quantity greater than or equal to Wy it would lead to the second stage of monitoring procedure and the evaluation of the $\bar{X}$ value, but with the trinomial rule proposed in this section, it does not happen. Two observations, in the same sample, classified as category A and category B evidence that the process mean is centered in the target value and, thus, this sample should not be taken to the evaluation of the $\bar{X}$ value as it could probably result in a false alarm. When, in first stage of the T-ATTRIVAR chart, at least one observation is classified as category A and one observation is classified as category B, in the same sample, the Y_D value for this sample is considered zero, and the monitoring process continue to the next sample.

For the T-ATTRIVAR chart the probability $\% \bar{X}_T$ of a sample to go to the second part of the monitoring procedure is calculated with equation (2).

$$\% \bar{X}_T = \sum_{i=Wy}^{CLy-1} \frac{n!}{i!(n-i)!} \times P_A^i \times P_{AB}^{(n-i)} + \frac{n!}{i!(n-i)!} \times P_B^i \times P_{AB}^{(n-i)} \quad (2)$$

Where $P_A = \Phi\left(\frac{LDL-\mu_0}{\sigma}\right)$ and $P_B = 1 - \Phi\left(\frac{UDL-\mu_0}{\sigma}\right)$ and $P_{AB} = (1 - P_A - P_B)$. The second part of the T-ATTRIVAR chart, the variable chart, is similar to the B-ATTRIVAR chart.

4. Efficiency calculation and comparison

The efficiency of control charts to detect a shift in the monitored process mean is commonly measured by the value of Average Run Length (ARL) as found in the literature, [Ho and Aparisi 2016], [Machado et al. 2018]. The Average Run Length represent the number of samples needed to signal a shift in the process mean by a control chart. Thus, the lower the value of ARL, the more efficient is the control chart. To make a fair comparison of efficiency between the control charts, the ARLs should be compared in the same conditions of all set of parameters, including the chance of a false alarm also know in the literature as error type I.

The calculation of the ARL values for ATTRIVAR charts is not simple due to the fact that the first part of the monitoring procedure, the attribute-type inspection of the sample, truncates the probability distribution of the X dimension, and thus, to calculate the sample mean in the second part of the monitoring procedure, it is used a very complex calculation to sum the different kinds of truncated normal distribution functions, [Ho and Aparisi 2016]. A computational code, using R programming language, was developed, in this study, to make the theoretical calculations of the ARL values for both versions of the ATTRIVAR chart. Those theoretical ARL values (ARLc) were validated by computing ARL values obtained by simulation (ARLs). The simulation was also made with R programming language.

Table 1 shows the ARL values obtained by the computational program for two cases of sample size $n=3$ and $n=5$, and several cases of mean shift (δ) and percentage of samples that go to the second part of the monitoring procedure ($\% \bar{X}$). In Table 1, the values in the column "Sim-Error" represent the percentual simulation error between the ARLc and ARLs values and it is calculated in percentage with equation (3).

$$\text{Sim_Error} = \frac{|\text{ARLc} - \text{ARLs}|}{\text{ARLc}} \times 100 \quad (3)$$

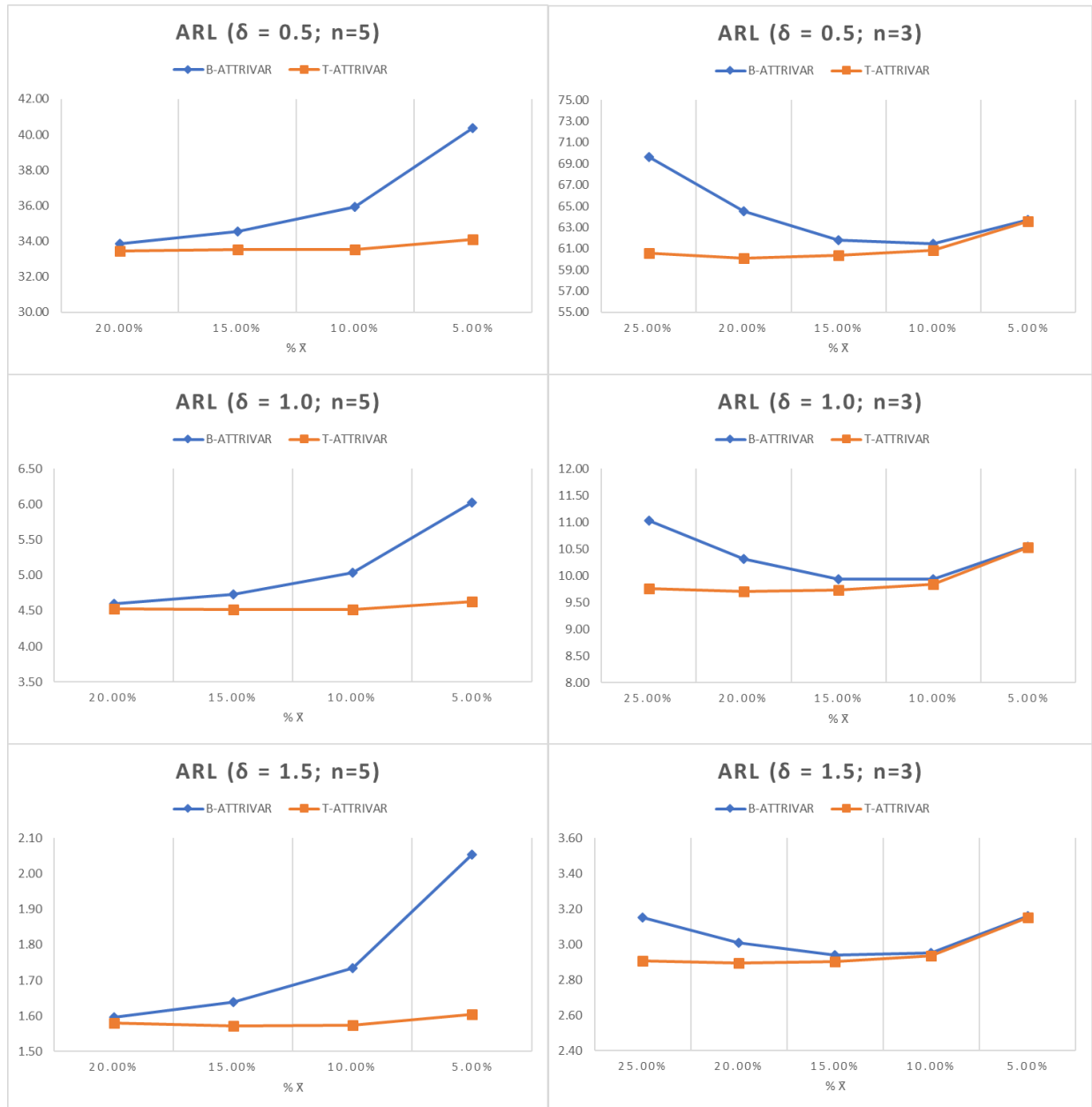
Table 1: ARL comparison

n	% $\bar{X}$	δ	B-ATTRIVAR			T-ATTRIVAR		
			ARLc	ARLs	Sim - Error	ARLc	ARLs	Sim - Error
3	25	0.25	199.98	202.08	1.05%	183.44	184.50	0.58%
3	25	0.5	69.60	69.88	0.40%	60.59	60.62	0.05%
3	25	1	11.03	11.00	0.27%	9.76	9.75	0.10%
3	25	1.25	5.48	5.48	0.00%	4.95	4.96	0.17%
3	25	1.5	3.15	3.15	0.00%	2.91	2.90	0.24%
3	25	2	1.53	1.53	0.00%	1.47	1.47	0.21%
3	15	0.25	185.72	186.26	0.29%	183.40	182.23	0.64%
3	15	0.5	61.81	61.99	0.29%	60.40	60.55	0.26%
3	15	1	9.94	9.94	0.00%	9.73	9.71	0.16%
3	15	1.25	5.03	5.05	0.40%	4.94	4.95	0.12%
3	15	1.5	2.94	2.94	0.00%	2.90	2.90	0.03%
3	15	2	1.48	1.48	0.00%	1.47	1.47	0.03%
3	5	0.25	188.59	188.84	0.13%	188.49	187.98	0.27%
3	5	0.5	63.68	63.83	0.24%	63.60	63.75	0.23%
3	5	1	10.54	10.57	0.28%	10.53	10.56	0.30%
3	5	1.25	5.38	5.38	0.00%	5.37	5.37	0.07%
3	5	1.5	3.16	3.16	0.00%	3.15	3.14	0.27%
3	5	2	1.57	1.57	0.00%	1.57	1.57	0.02%
5	20	0.25	131.62	134.06	1.86%	133.20	132.66	0.41%
5	20	0.5	33.86	33.86	0.01%	33.20	33.45	0.76%
5	20	1	4.60	4.61	0.36%	4.53	4.52	0.20%
5	20	1.25	2.44	2.44	0.01%	2.41	2.41	0.02%
5	20	1.5	1.60	1.60	0.22%	1.58	1.58	0.17%
5	20	2	1.08	1.08	0.06%	1.08	1.08	0.05%
5	15	0.25	135.40	136.03	0.46%	133.57	133.34	0.17%
5	15	0.5	34.55	34.62	0.20%	33.55	33.49	0.18%
5	15	1	4.73	4.74	0.27%	4.52	4.54	0.46%
5	15	1.25	2.51	2.51	0.09%	2.40	2.40	0.02%
5	15	1.5	1.64	1.64	0.06%	1.57	1.57	0.13%
5	15	2	1.10	1.10	0.30%	1.08	1.08	0.04%
5	5	0.25	146.90	146.93	0.02%	134.67	133.85	0.61%
5	5	0.5	40.37	40.31	0.13%	34.11	34.33	0.66%
5	5	1	6.02	6.03	0.14%	4.63	4.62	0.18%
5	5	1.25	3.22	3.22	0.12%	2.46	2.46	0.16%
5	5	1.5	2.05	2.05	0.18%	1.61	1.60	0.46%
5	5	2	1.24	1.24	0.07%	1.09	1.09	0.02%

It is possible to note that, the theoretical values of ARL can be validated as they are very similar to the ARL values obtained by simulation, for both versions of the ATTRIVAR chart, as the simulation error is lower than 2% for all the scenarios in Table 1.

To compare the efficiency of the two versions of the ATTRIVAR chart, Figure 4 was made by plotting a graph with the information in Table 1 to make it easier to visualize the difference between the ARL values.

Figure 4: Graphs of ARL comparison



In Figure 4, it is possible to note that, for all the studied scenarios, the trinomial version of the ATTRIVAR chart is more efficient than the binomial version. As also shown in Figure 4, when $n=5$, the advantage in efficiency, by using the trinomial version instead of the binomial version, is bigger with lower values of $\% \bar{X}$. The explanation to this is that with a bigger sample there are more possible rules, that is, combinations of the attribute control chart limits (W_y and CL_y), and the chances to have, in the same sample, defective items classified as category A and category B is higher even with small values of $\% \bar{X}$. Also, when using higher values of $\% \bar{X}$ with a bigger sample size, ARL values approximate to Shewhart's $\bar{X}$ chart which makes the

performance between B-ATTRIVAR and T-ATTRIVAR very similar. Then, the opportunity to benefit from the T-ATTRIVAR chart's efficiency when $n = 5$, are in scenarios where the maximum tolerated $\% \bar{X}$ is 15% which makes this trinomial version of the ATTRIVAR chart even more economical in matter of sampling procedure. For $n = 3$, the T-ATTRIVAR chart is significantly more efficient than the B-ATTRIVAR chart when $\% \bar{X} = 20\%$ and $\% \bar{X} = 25\%$. When the sample size $n = 3$ and $\% \bar{X}$ is a lower value, the ARL values for T-ATTRIVAR and the B-ATTRIVAR chart are not so different because the discriminating limits are very wide and the chance to have two items being classified as category A and category B in the same sample is very low, and then, having no difference between the monitoring rules, but when the values of $\% \bar{X}$ increases, the discriminating limits gets tighter and this chance increases, showing smaller values of ARL for the T-ATTRIVAR chart making more evident the efficiency of this chart to avoid those false alarms.

5. Conclusions

The computational program, developed in a R programming language code, was successful built to execute the complex mathematical procedure to evaluate the ARL values of ATTRIVAR charts. The values calculated by the computational program were validated, by computer simulation, as the simulation errors were all below 2%.

The described trinomial version of the ATTRIVAR control chart shows more efficiency in detecting shifts in the process mean and is more economical than the binomial version of the ATTRIVAR chart in several scenarios, proved by the ARL values comparison. For a sample size $n = 3$, the T-ATTRIVAR chart is more efficient when $\% \bar{X}$ is greater than 15% and, for a sample size $n = 5$, the T-ATTRIVAR chart is more efficient when $\% \bar{X}$ is lower than 15%.

6. Acknowledgements

The research for this paper was supported by project 2018/0147-0 of the FAPESP – Fundação de Amparo a Pesquisa do Estado de São Paulo.

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